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A Problem with Circular Round-trips under the Special Theory of Relativity

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Abstract

The paper explores some idiosyncrasies of the special theory of relativity using multiple sets of three inertial reference frames. These frames, by means of imagined clock handoffs around regular polygons, are used to approximate the effects of special relativity for circular round-trips. One not-too-surprising finding is that clocks on the fronts of spaceships that are traveling the same speed but always in opposite directions on identical circular orbits will have the same elapsed times when they return to their starting positions, but their clocks will be slower than that of a stationary observer. As a consequence, however, the distance traveled by the spaceships (as determined by the spaceships) is shorter than their orbits as measured by the stationary observer. This leads to a paradox (which is similar to the Ehrenfest paradox), since the orbits of the spaceships were designed to coincide with the orbits seen by the stationary observer but are too short to coincide. A more serious problem occurs, however, since star distances measured by parallax will be shorter when determined by the spaceships than when determined by the stationary observer.

Keywords: Special Relativity, Circular round-trips, Ehrenfest Paradox

1.0 Introduction

A paradox in the special theory of relativity occurs when the elapsed time between two events in one inertial reference frame differs from that of another. It is usually the "traveling" reference frame that has the slower clock. This seems paradoxical, because either reference frame could be considered the "traveling" reference frame. One approach to understanding the paradox is to introduce a third inertial reference frame. That is the approach taken here.

So in the paper I will (1) illustrate several idiosyncrasies of the special theory while using a third inertial reference frame, (2) ask whether spaceships going the same speed but always in opposite directions on identical circular round-trips will have different travel times from each other or that of a stationary observer, and (3) ask whether this leads to any contradictions in the special theory.

2.0 Two Spaceships with a Stationary Observer

Consider the two spaceships, whose initial positions are shown by the arrows in figure 1. Assume the two spaceships travel at the same constant speed in identical counter-clockwise circular orbits about A and C. Also assume that the spaceships' orbits are stationary in the same plane, and that there is a stationary observer, B, who can make observations midway between the centers of the orbits.

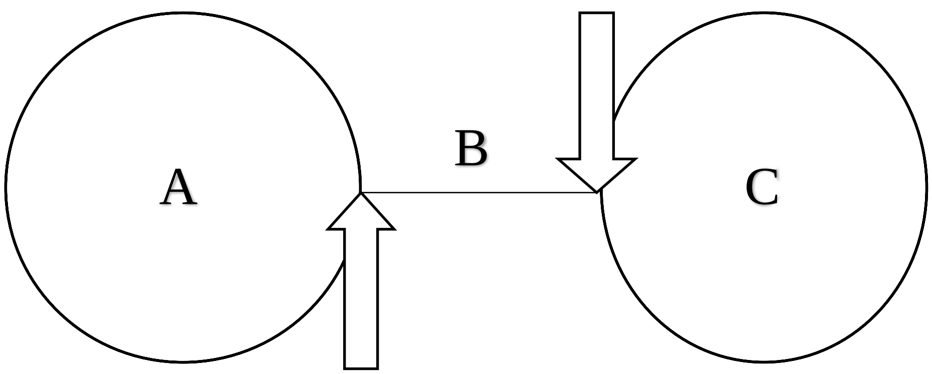

Figure 1

Assume the two spaceships simultaneously cross a line that passes between the centers of the two orbits and that when they cross that line they are going in opposite directions perpendicular to the line. Also assume that when the two spaceships first cross the line the clocks in the spaceships as well as that of observer B are set to zero. The question is what will be the times on the three clocks when the spaceships complete one orbit.

Observer B is aware that the two spaceships are not in inertial reference frames. He/she assumes therefore that it will be necessary to use both the general theory of relativity and the special theory to calculate what the arrival times will be according to the clocks on the spaceships. Observer B notes that the two spaceships will always be pointed in exactly opposite directions as they make their way around their orbits.[1] So the effects of the special theory should be the same throughout their circuits (at least in terms of the time differences between the clocks on spaceship A and spaceship C). And the effects of the general theory would also be uniform, since to maintain their orbits the two spaceships (or some mechanism outside the spaceships) would need to apply constant centripetal forces to keep the spaceships from flying off in directions tangent to their intended orbits.

Since the spaceships' orbits and motion are identical except for their starting directions and positions, observer B concludes that the clocks for the spaceships will be the same when they complete their next circuits. However, the spaceship clocks could differ from observer B's clock, due both to special relativity effects and general relativity effects.

Would observers on the spaceships come to a different conclusion? They are always traveling in opposite directions. So using only the special theory, an observer on one of the spaceships might think that the clock on the other would differ when their spaceships return to the starting line.

Since the effects of general relativity may be difficult to untangle here, let us first consider a similar problem where only the special theory is involved.

3.0 One Stationary Reference Frame and Two Inertial Reference Frames Moving in Opposite Directions

Consider the two trains, A and C in figure 2, going in opposite direction, with part of a stationary inertial reference frame, B, shown between them. Figure 2 shows the fronts of the trains about to cross a line that is perpendicular to the paths of the trains. This represents event 1. An event is defined here an occurrence that has a single set of space coordinates (although we ignore the y and z coordinates here) and a single time coordinate in a single reference frame. There would be a corresponding set of space and time coordinates for each of the other inertial reference frames for this same event. Since A and C are crossing perpendicularly to the direction of motion, we may call their crossing simultaneous in all three reference frames, so long as we are at the starting line (or on a plane perpendicular to the directions of motion that contains the starting line). At the time of the crossing all three clocks are set to zero.

[1] An easy way to show this is to move orbit C with its arrow (and without any rotation) and place it on top of orbit A. Next draw a line through the center of the orbits to the tips of the arrows. This line will be perpendicular to the arrows (since the center lines of the arrows are tangent to the orbits). Imagine now that this perpendicular line will follow the arrows as the spaceships complete their orbits. From this it is easy to see that the arrows will always point in opposite directions.

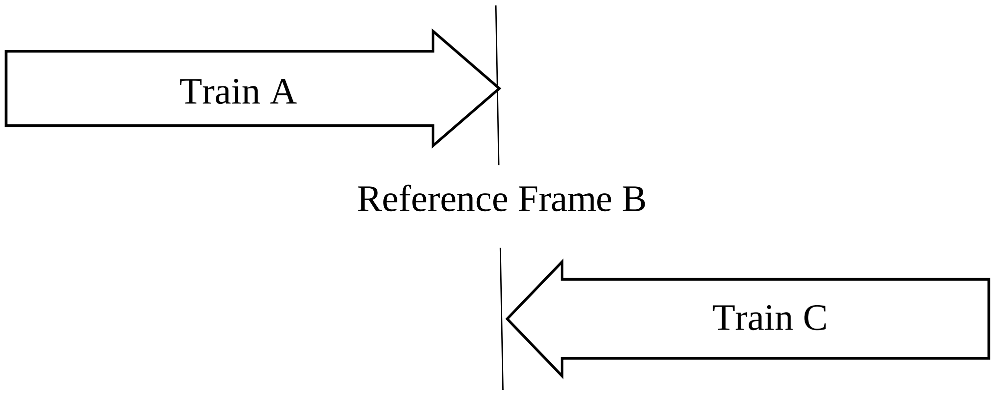

Figure 2

Event 2 occurs when the ends of the trains simultaneously reach the same line. We assume the trains are moving at the same speed (with respect to the stationary observer) and have the same length (as measured by the stationary observer). The origin for reference frame B is the starting line. The origins for the trains are at the tips of their arrows. The question is: what are the times on the various clocks when event 2 occurs (i.e. when the ends of the trains reach the starting/finishing line).

First, let us consider how the clocks for train C and reference frame B differ when the end of train C crosses the starting/finishing line. Let x and t be the space and time coordinates on train C, while x' and t' are the corresponding coordinates for reference frame B, and x'' and t'' are the corresponding coordinates for train A. All clocks on all reference frames are set to zero when the trains first cross the line, and everything is measured in meters. (Meters for time is the distance that light would travel in a second; so it is usually called light travel-time.)

Let us assume that the trains are quite long: half the distance that light would travel in a second (as measured on reference frame B). Note that trains will be somewhat longer on reference frames A and C due to the Lorentz contraction. Assume the trains are moving at one fourth the speed of light (relative to reference frame B). The trains need two seconds to cross the observation line (according to observers at the origin of reference frame B). So when the end of train C crosses the line, $t_2' = 5.99585 \times 10^8$ meters or two seconds. The subscript 2 is for event 2, when the ends of trains A and C cross the observation line. At that point x_2' still equals 0. To get the corresponding event 2 coordinates for train C we use the Lorentz transformation shown below.

$$x_2 = (1 - \beta^2)^{-1/2}\ x_2' + \beta\,(1 - \beta^2)^{-1/2}\ t_2'$$
$$t_2 = \beta\,(1 - \beta^2)^{-1/2}\ x_2' + (1 - \beta^2)^{-1/2}\ t_2'.$$

β represents reference frame C's (or A's) speed (relative to that of reference frame B) as a ratio to the speed of light. So it will be .25. So substituting .25, 0, and 5.99585×10^8 for β, x_2', and t_2' in the above equations. We see that

$x_2 = 1.548 \times 10^8$ meters
$t_2 = 6.192 \times 10^8$ meters (of light travel-time).

The ratio of train C's clock to that of observer B = $t_2/t_2' = 1.033$. So observer B's clock appears to be running slower than that on train C, at least in this comparison. However, the reason for this may be because we have implicitly assumed a non-zero time for when end of train C shares an event with B at $t_0' = 0$ and $x_0' = - 1.499 \times 10^8$ meters. (I have not discussed event 0 yet, but will do so shortly.)

On the other hand, the value for x_2 is what we might expect. The Lorentz contraction factor of $(1 - \beta^2)^{1/2}$ applied to the 1.548×10^8 meters yields the 1.499×10^8 meters we assume for the apparent length of train C in stationary reference frame B.

Note also that $x_2/t_2 = .25$. This is the value we set for β, and is the same for train C as it is for observer B. And, as might be expected x_2/(observer B's measure of the train length) also equals 1.033. This is the reciprocal of the Lorentz contraction factor of 0.968.

To get the corresponding coordinates for train A we use the inverse Lorentz transformation as shown below, but still use .25 for β. The inverse Lorentz transformation has minus signs where β appears (and is not squared) because observer B is moving in a negative direction (relative to train A).[2]

$$x_2'' = (1 - \beta^2)^{-1/2} x_2' - \beta (1 - \beta^2)^{-1/2} t_2'$$
$$t_2'' = -\beta (1 - \beta^2)^{-1/2} x_2' + (1 - \beta^2)^{-1/2} t_2'$$

This gives the coordinates in train A's frame, x_2'' and t_2'', at the end of train A for when the ends of trains A and C reach the observation line. This is still event 2, since its physical location is on a plane perpendicular to the direction of motion and where $x_2' = 0$ in reference frame B. This will occur when the coordinates for observer B are $t_2' = 5.99555 \times 10^8$ meters (2 seconds) and where $x_2' = 0$. So substituting .25, 0, and 5.99585 x 10^8 in this equation for β, x_2', and t_2', we get

$x_2'' = -1.548 \times 10^8$ meters
$t_2'' = 6.192 \times 10^8$ meters (of light travel-time).

This is the same as for train C, except that x_2'' is a negative value in train A's coordinates (since train A's origin, at the tip of its arrow, is to the right). Also the train length (according to A)/train length (according to observer B) = $-x_2''$/(1/2 the distance light travels in a second) = 1.033. Also $-x_2''/t_2'' = .25$. This is the value we set for β, which is same for trains A and C as it is for observer B. These and other results are shown in Table 1 below.

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[2] Some writers call my first transformation the inverse, and the other the non-inverse. The versions of the Lorentz transformation shown in this paper are taken from Taylor and Wheeler (1966, p. 42 & p. 58).

Table 1
Coordinates for the Three Inertial Reference Frames for Six Events

Event	Train C position x (meters/ 10^8)	Train C time t (meters/ 10^8)	Frame B position x'(meters/ 10^8)	Frame B time t'(meters/ 10^8)	Train A position x"(meters/ 10^8)	Train A time t"(meters/ 10^8)
0: t' = 0, x' = -1 train length	NA	NA	-1.499	0	- 1.548	0.387
1: all coordinates set to zero	0	0	0	0	0	0
2: at B's origin, B's t' = 2 seconds	1.548	6.192	0	5.996	- 1.548	6.192
3: B's time same as #2, x' = 1 train length	3.096	6.580	1.499	5.996	0	5.805
4: B's time same as #2, x' = -1 train length	0	5.805	- 1.499	5.996	- 3.096	6.580
5: A's time same as #3, x'' = -1.451 x 10^8	NA	NA	0	5.621	- 1.451	5.805

One way observers on reference frame B can measure the train lengths is by using two events that are simultaneous in B's reference frame. So events 3 and 4 have been computed letting an observer on B predict where the trains' origins should be at a particular time. These are shown in table 1. Event 3 occurs in observer B's reference frame at the same time as event 2 (according to B). Its position is one train length to the right (in observer B's reference frame): 1.499 x 10^8 meters. Event 4 in Table 1 has the same time as event 3 (in reference frame B), but is positioned one train length to the left of observer B's origin.

We can see that train A has the expected x'' coordinate for event 3, namely zero, which is at train A's origin. This shows that at the same time in B, the end of the train A is at B's origin (event 2) and the front of train A is 1.499 x 10^8 meters to the right (according to B in event 3). Train A thinks its length is 1.548 x 10^8 meters, while B thinks A's length is 1.499 x 10^8 meters. The ratio of 1.548/1.499 = 1.033, exactly what we would expect using the Lorentz contraction. Train C's coordinates for event 3 are also shown in table 1.These were obtained using the Lorentz transformations on frame B's coordinates for event 3.

For event 3, it may be seen that reference frame B has a faster clock than train A, but a slower clock than train C. For event 4, reference frame B has a faster clock than train C, but a slower clock than train A. The lengths of trains A and C as measured on reference frame B is 1.499 x 10^8 meters.

So why is train A's time shorter for event 3 than it is for event 2? Part of the problem is that when you look at the time on train A from the stationary platform, the time you see (if you could see it) depends not only when you look (according to B's time), but where you are (on frame B) when you look. (This is also true if you are on one of the trains. The time you see on B depends not only on when you look but also on where you are on the train when you look.) Also event 2 and event 3 are asking different questions. For event 2 we are asking (for A) how long it takes one train-length on train A (as measured on A) to pass a point on platform B. While for event 3 we are asking (for A) how long it takes a twice Lorenz contracted version of A's length (as seen by A) to pass a point on platform B. This is because the Lorentz contracted length of A that has been measured on B (as 1.499 x 10^8 meters) can be seen as further Lorentz contracted on frame A.

To clarify this explanation I have added what I call events 0 and 5. Event 0 looks at where an observer on reference frame B expects to find the end of train A at time 0 (on reference frame B). Since the difference between event 0 and event 1 is spacelike, it is not appropriate to use the Lorentz transformation to find the event 0 coordinates for train A. However, the end of the train is no different than the front of the train when we consider the movement of a point on the train that passes a length of 1.499 x $10^{\wedge 8}$ meters as defined by the stationary observer. So the event 0 coordinates were obtained by assuming the time and space differences between event 0 and event 2 are the same as those between event 1 and event 3 for either A or B.

From this we see that while an observer on reference frame B at time 0 and at B's origin (event 1) may think that the trains have just started to pass, someone on reference frame B at time 0 and -1.499 x 10^8 meters (event 0) sees the end of A (as expected), but notices that A has been moving for 0.387 x 10^8 meters (of light travel-time) after all clocks on frame A were set to zero.

So what we can conclude is: (1) that 1.548 x 10^8 meters is the length of train A (or C) as it would be measured on train A (or C), (2) that it takes 6.192 x 10^8 meters (of light travel-time on train A) for all of train A to pass any point on frame B, and (3) it takes only 5.805 x 10^8 meters (of light travel-time) on train A for any point on train A to pass a length measured on frame B as 1.499 x 10^8 meters.

This makes sense if we say that train A need only pass a Lorentz contracted portion of the 1.499 x 10^8 meters on frame B. If so, between event 1 and event 3, a length of 1.451 x 10^8 meters on train A (as measure on A) would have passed the starting line on frame B.

To verify this argument consider event 5. Event 5 has the same time on train A as that of event 3: 5.805 x 10^8 meters (of light travel-time). Its position on train A is – 1.451 x 10^8 meters. To get B's coordinates for event 5, we use the Lorentz transformation on the train A coordinates. The train B coordinates are:

$$x_5' = 0$$
$$t_5' = 5.621 \times 10^8 \text{ meters}$$

So event 5 is at observer B's origin. But not all of train A has passed yet. Both train A and the stationary observer agree on that. Train A thinks that 1.451 x 10^8 meters of its length has passed B's origin for event 5. But at the same time as event 5 (on train A), event 3 shows the front of train A to be across from B, where B's position is 1.499 x 10^8 meters (as measured by B).

Let us reiterate the double Lorentz contraction explanation. The length of train A as carefully measured on train A is 1.548 x 10^8 meters. The length as it appears on reference frame B is 1.499 x 10^8 meters. This is the distance between the two ends of train A (as measured by B) when they appear at the same time on frame B. The length of the 1.499 x 10^8 meters on B can be thought as being further Lorentz contracted on train A. This length of 1.451 x 10^8 meters is the length on A that has just passed the starting line on B in event 5. And the time that train A needs to pass this part of its length between event 1 and event 3 is 5.805 meters (of light travel-time). And, as expected, the ratio of 1.451/5.805 = .25.

In the next two sections I will concentrate on the results of events 3, 4, and 5. These are needed to determine the travel Δt's and Δx's that are appropriate for when the front (and only the front) of one of the trains (or spaceships) passes a measured distance on reference frame B.

4.0 A Back-and-forth Round-trip Time

So are the shorter times in the fronts of the spaceships (or trains) real, or are they merely a matter of perspective? Lord Halsbury and others[3] suggested a thought experiment that might help. Take one spaceship (or train) out, and then hand off the clock (or imagine an instantaneous, momentum-free handoff or signal) to a returning spaceship. For our purposes here let us assume we take the tip of train A out, and then return on the front of a second train, which is a train length behind the end of the first train on the same track as train C.

Event 3 ends the first part of the journey and we already have its coordinates in all three reference frames. For the stationary observer the coordinates are:

[3] I will refer to this subsequently as an imagined handoff or simply a handoff. It is attributed to "Halsbury and others" in an article from Wikipedia.org/wiki/Twin_paradox#History, accessed 2/13/2023.

$x_{3'} = 1.499 \times 10^8$ meters
$t_3' = 5.996 \times 10^8$ meters (of light travel-time).

Using this we can find the coordinates for event 6, which is when a second train on train C's track arrives at the origin of the reference frame of observer B. We know that in frame B, the second train on train C's track must travel 1.499×10^8 meters (according to observer B) to reach the origin (of observer B), and that β is still .25. So $t_6' = t_3' + 1.499 \times 10^8 / \beta$. Consequently,

$x_6' = 0$ meters
$t_6' = 11.992 \times 10^8$ meters

To get the A-then-C round-trip time, where we use train A as the first leg and use a second train on train C's track as the second leg, we want $t_{6rt} = t_3'' + (t_6 - t_3)$, where the subscript 'rt' signifies the round-trip time. To get x_6 and t_6, we use the Lorentz transformation on x_6' and t_6'. So we use the Lorentz transformation on $\beta = .25$, $x_6' = 0$, and $t_6' = 11.992 \times 10^8$. This give us:

$x_6 = 3.100 \times 10^8$ meters
$t_6 = 12.385 \times 10^8$ meters

So $t_{6rt} = t_3'' + (t_6 - t_3) = (5.805 + (12.385 - 6.580)) \times 10^8 = 11.611 \times 10^8$ meters

So this is twice as long as t_3'' (5.805×10^8), but this makes sense since the trip is twice as long. Also in this comparison, the elapsed time of the stationary observer ($t_6' = 11.992 \times 10^8$) is larger, and by the same ratio we found in the event 3 comparison. The ratio, 1.033 here, equals $1/(1 - \beta^2)^{1/2}$.

Using imagined handoffs one can also compare a zigzag path with a straight path. Of course, one must be careful to use values for the x coordinates and for β that reflect the relative motions of the inertial reference frames for each zig or zag.

5.0 A Trip Around a Circle or a Regular Polygon with an Even Number of Sides

To find what happens for a circular round-trip let us start out with a square, as shown in figure 3. Spaceship A starts at the center of node 1 and ends at the center of node 1. Spaceship C begins and ends at the center of node 5. These starting and finishing positions allow us to start and finish at B's origin. Each spaceship has a time handoff at each node. Spaceships A and C (and their respective handoff successors) are also always going in opposite directions on parallel paths.

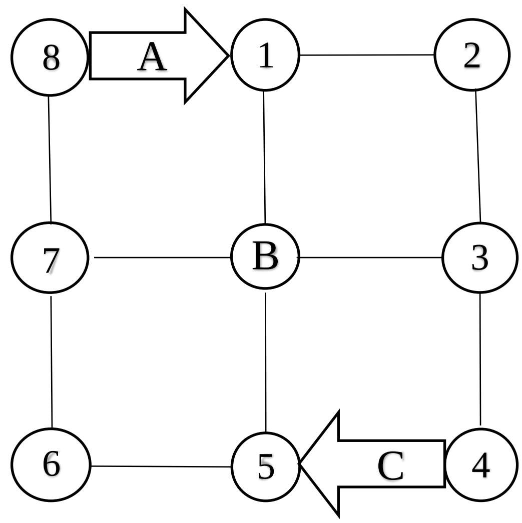

Figure 3

Next we generalize to a regular polygon with 2n sides. Then we let n become infinite. The result is a circle. And we may then use the same results we found in section 3.

Besides being a square, figure 3 makes some changes from figure 1 (in section 2). The spaceships orbits are on top of each other with B in the center of both. I have also changed the direction of the spaceships so that the results in Table 1 can be more easily seen.

Let us see how this works out using round-trips around the square shown in figure 3.

There are eight nodes. The odd numbered nodes are midway between the even numbered nodes. As mentioned, spaceship A starts out at node 1, moves clockwise, and finishes at node 1. Spaceship C starts out at node 5, also moves clockwise, and finishes at node 5. Observer B has two lines going through the center of the orbits and can use whichever one is appropriate at a particular time. One will be parallel to the direction of motion of the spaceships. The other will be perpendicular.

Step 1. The clocks are set to zero as the tips of A and C leave nodes 1 and 5 respectively. B can also set his/her clock to zero at this time, since he/she is on a plane that is perpendicular to the directions of motion and passes through the centers of nodes1 and 5. We also imagine that when B sets his/time that the whole stationary reference frame also resets their time to zero. (This would be possible if everyone on the stationary reference frame knew when B was going to reset his/her time, and at that time everyone set their clocks to zero.) The spaceships are moving at a constant speed of .25 times the speed of light with respect to the stationary observer. Then A reaches node 2 and C reaches node 6. Let us use observer B's definition of where nodes 2 and 6 are. Node 2 is 1.499 x 10^8 meters to B's right. While node 6 is 1.499 x 10^8 meters to B's left. These are the same numbers we have seen in table 1. They could, of course, be rescaled without changing the relative times or distances. So spaceship A will arrive at node 2 with a frame-A time of 5.805×10^8 meters, as shown for event 3 in table 1. The corresponding frame B time for event 3 is 5.996×10^8 meters. The ratio t3'/t3'' = 1.033. So the stationary observer has the faster clock. Also spaceship C will arrive at node 6 with a frame-C time of 5.805×10^8 meters, as shown for event 4 in table 1. Again the ratio $t_4'/t_4 = 1.033$, and the stationary observer has the faster clock.

Step 2. At this point A and C must hand off their clocks. So A hands off his/her clock to the front of a spaceship going vertically downward (in figure 3) and just reaching the center of node 2. C hands off his/her clock to a spaceship going vertically upward (in figure 3) and just reaching the center of node 6. The receiving spaceships enter these times in their logs. We can also hand off the distances traveled for the spaceships. However, the latter is not really needed since we can obtained these distances by multiplying the time by β. Then A and C set their clocks to zero.

Observer B does not really need to hand off his/her time, but let us assume that he/she does, so that we can keep track of his/her times. So let us assume that at node 2, for example, four things happen simultaneously: (1) A hands off his/her clock, as well as the distance traveled (according to A). (2) The times on the clocks are recorded as are the distances traveled. (3) All clocks are the set to zero, and (4) The two lines going through the handoff node switch roles: the parallel line becomes the perpendicular line and the perpendicular line becomes the parallel line. All these occurrences are simultaneous at any corner handoff node because they occur at the same time at a single physical location.

A person stationary at the center of the square could not claim all these actions are simultaneous. But such a person could claim that the activities that occur on reference frame B are simultaneous for him/her. So this person at the center of reference frame B could record the event 3 results and set his/her time to zero every time there is a time handoff (according to a reference frame B source or by knowing when the handoffs will occur). At node 2, for example, the recording would then show: (1) spaceship A has traveled 1.499×10^8 meters, (2) the platform time is 5.996×10^8 meters, (3) the frame B time is now zero and (4) B has a new set of lines that are parallel and perpendicular to the new direction of motion. So if B keeps a log of these times, their sum would tell us what his/her times would be if there were no interruptions in his/her clock time. Knowing β and the elapsed time, B could also calculate the distances traveled.

The same things happen for the remaining nodes for A and B. There will be a parallel set of nodes for C and B. We do not really need to have a handoff at the middle nodes for the sides of the square, since the handoff could be to the same spaceship. But for our purposes here let us assume we do. Then the total travel times around the square are as follows:

A's time = $8 \times 5.805 \times 10^8 = 4.644 \times 10^9$ meters
B's time = $8 \times 5.996 \times 10^8 = 4.797 \times 10^9$ meters
C's time = $8 \times 5.805 \times 10^8 = 4.644 \times 10^9$ meters

So A and C have the same times, while the B/A and B/C time ratios = 1.033.

Next we may obtain the distances from the spaceship or stationary observer logs or by multiplying the times by β. Using the table 1 values for events 3, 4, and 5 they are:

A's distance = 8 x 1.451 x 10^8 = 1.161 x 10^9 meters
B's distance = 8 x 1.499 x 10^8 = 1.199 x 10^9 meters
C's distance = 8 x 1.451 x 10^8 = 1.161 x 10^9 meters

So A and C have the same distances, while the B/A and B/C distance ratios = 1.033.

For a polygon with more than four sides, the spaceships' paths going into the vertex handoff nodes would no longer be at right angles to those coming out of the nodes. However, we would still have rectangles whose shorter sides would be the paths of motion (with a parallel line for observer B), and whose the longer sides lines would be perpendicular to the paths of motion that pass through the relevant observation positions. This leads to the same ratios. So as the number of sides becomes infinite (and we have circular orbits), we still have the same ratios.

A circular orbit is especially interesting, because no handoffs are needed. However, for our purposes here I am still assuming we have handoffs. The advantage of keeping the handoffs is that we can assume there is no transfer of momentum or force at the handoff nodes. That allows us to claim there are no general relativity effects.

6.0 The Problem

From the preceding section we may conclude that not only are the orbital times of the spaceships shorter than those of the stationary observer, so are the distances.

But if the size of the orbits are smaller for the spaceships than that of the stationary observer, it is difficult to see how the orbits could coincide. And we have described the handoffs in such a way that the orbits would necessarily coincide. So this is a problem.

And if the orbits are shorter for the spaceships than for the stationary observer, so are the diameters of those orbits. As a consequence, star distances measured by parallax using those diameters as scaling factors will also be shorter when determined by the spaceships than when determined by the stationary observer. This is a more serious problem. I would call it a contradiction.

We have, in effect, determined the orbital size for the spaceships with the special theory using imagined handoffs along the sides of a regular polygon with 2n sides. For each of these sides each pair of reference frames (A and B or C and B) would dispute the length of those sides. Under the special theory they will not agree. As n becomes infinite, we have a circular orbit whose diameter can be determined by B. A and B will not agree on the diameter of the orbit, since they do not agree on the lengths of the sides of a regular polygon with an arbitrarily large number of sides. Since A and B do not agree on the diameters of A's orbit, they will not agree on star distances measured by parallax where the diameters are used as scaling factors. The source of their disagreement is the special theory. So the special theory must be wrong, unless, of course, there is some mistake in my calculations or reasoning.

Normally under the special theory we allow certain disagreements about length because while A can measure lengths on spaceship A that are parallel to its direction of motion, B cannot do so directly. The stationary observer can simultaneously mark points on B that are directly across from the front and back of a spaceship. B can then measure the distance between those two marks. If B knows β, he/she can also see how long it takes a length on spaceship A to pass. B can then multiply that time by β to get length. Both approaches yield the same length of A as measured by B. But neither approach will satisfy A, since what is simultaneous on B is not necessarily simultaneous on A. For this reason we permit certain disagreements about lengths that are parallel to the direction of motion. However, we do not permit disagreements about lengths perpendicular to the direction of motion. That would violate an assumption of the special theory that the laws of physics are the same in all inertial reference frames.

7.0 The Ehrenfest Paradox

In 1909 Paul Ehrenfest described a paradox which is similar to the unequal-orbit-lengths problem described in section 6. Ehrenfest[4] considered what would happen to the outer perimeter of a rigid disc that is rotating fast enough that there are relativistic effects. The radius at rest, R_0, which is perpendicular to the direction of motion for any segment of the perimeter, should remain unaffected by special relativity effects. But $2\pi R_0$ will be larger than the Lorentz-contracted perimeter. Ehrenfest concluded that "Born rigidity"[5] is therefore incompatible with special relativity.

I confess, I am not familiar with Born rigidity. So I will discuss only what I think is the essence of the paradox. We have a rigid body that is rotating and an observer at the center of the disc who is not rotating with that disc. My interpretation is that there are an infinite number of pairs of inertial reference frames, one pair for each point on the outer edge of the disc. And all are operating at the same time. However, if we approximate this with non-rotating regular polygons where we are considering only one moving point per side, then we have the same situation as was used in this paper, except that in this paper the polygonal sides were considered serially, while in the Ehrenfest paradox they were considered simultaneously. In both cases we are arguing that what is true for the differential segment is true for the whole circle.

In any case, the same (or at least a similar) paradox arises. In this paper the unequal-orbit lengths arise because the spaceships measure only Lorentz-contracted versions of the

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[4] This is described in Wikipedia.org/wiki/Ehrenfest_paradox, accessed 2/18/2023
[5] Ibid. "Born rigidity" is discussed in the same article.

orbit defined by the stationary observer. Also in this paper we can imagine that no momentum or forces are transferred at the handoffs. Also we should note, that the same paradox occurs when the path of the hand-off orbit is a square.

8.0 How Spaceships A and C See Each Other

So how do the spaceships see each other when they are on opposite sides of a circular round-trip? To answer this I have copied parts of table 1 into table 2 below.

Table 2: The Coordinates for Three Inertial Reference Frames for Two Events

Event	Train C position x (meters/ 10^8)	Train C time t (meters/ 10^8)	Frame B position x'(meters/ 10^8)	Frame B time t'(meters/ 10^8)	Train A position x''(meters/ 10^8)	Train A time t''(meters/ 10^8)
3: B's time same as #2, x' = 1 train length	3.096	6.580	1.499	5.996	0	5.805
4: B's time same as #2, x' = -1 train length	0	5.805	- 1.499	5.996	- 3.096	6.580

We can see from table 2 that spaceship C could, in theory, see spaceship A's handoff in event 3 at 5.805 x 10^8 meters (of light travel-time). In event 4 spaceship A could, in theory, see that spaceship C's handoff at 5.805 x 10^8 meters (of light travel-time). So there is no problem with the handoff times we have previously assumed.

As expected, however, the times of the handoffs for the viewing spaceships are much larger than the times handed off. Spaceship C's clock shows 6.580 x 10^8 meters (of light travel-time), when it views spaceship A's handoff time. Similarly, spaceship A's clock shows 6.580 x 10^8 meters (of light travel-time), when it views spaceship C's handoff time. And something of this sort is what we expected.

The stumbling block for us is that someone on A sees C's handoff long after spaceship A has made its own handoff. Similarly someone on C sees A's handoff long after spaceship C has made its own handoff. While this is counter-intuitive, it is consistent with the special theory. It also explains why we might suspect (incorrectly) that spaceship A and spaceship C would have different arrival times in a circular round-trip.

9.0 Summary

The paper explores some idiosyncrasies of the special theory of relativity using three inertial reference frames. It uses imagined clock handoffs to see what happens on circular round-trips. The effects of the general theory can be ignored if we assume any time handoffs are momentum-free and force-free.

One finding is that clocks on the fronts of spaceships that are traveling the same speed but always in opposite directions on identical circular orbits will have the same elapsed times when they return to their starting positions, but their clocks will be slower than that of a stationary observer.

A second finding is that the distances traveled by the spaceships (as determined by the spaceships) is shorter than their orbits as measured by the stationary observer. As a consequence, the diameter of the orbital plane will be shorter when determined by one of the spaceships than when determined by the stationary observer. This leads to a contradiction, since star distances measured by parallax, using the diameters of the orbits as scaling factors, will be shorter when determined by the spaceships than when determined by the stationary observer.

A related problem has to do with the orbits themselves. If the times are shorter for the spaceships than for the stationary observer, so are the circumferences of their orbits. We have designed the orbits in such a way that they coincide. However, the special theory leads us to the conclusion that the lengths of those orbits differ. This problem is similar to that of the Ehrenfest paradox.

References

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2. Taylor, E. F. and Wheeler, J. A.; *Spacetime Physics*; W.H. Freeman and Co. (San Francisco, 1966)
3. Wikipedia.org/wiki/Ehrenfest_paradox (accessed February 18, 2023)
4. Wikipedia.org/wiki/Twin_paradox#History (February 13, 2023)