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The Ives-Stilwell Experiment

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Abstract:

The Doppler shift of light from moving canal rays, as predicted by the Larmor-Lorentz theory and the emission theory, is computed, and compared to the observed result of the Ives-Stilwell Experiment.

Introduction:

The principal objective of the Ives-Stilwell experiment is to search for the transverse Doppler effect as predicted by the Larmor-Lorentz theory.

The idea of using canal rays to search for transverse Doppler effect was first suggested in a debate between A. Einstein and W. Ritz. However, their imagined experiment is not feasible. That is because one can never be sure of observing canal rays at right angles to their direction of motion.

Furthermore, the expected Doppler shift is very tiny and the slightest deviations, from the angle of 90^o, introduce conventional Doppler shifts of similar magnitude to that predicted by theory.

For these reasons, Ives and Stilwell have decided to observe in the direction of maximum blue Doppler shift and the direction of maximum red Doppler shift around the angle of 0^o and 180^o, respectively.

However, Doppler measurements around the angles of 0^{o} and 180^{o} are not exactly the same as Doppler measurements done around the angle of 90^{o}.

And, furthermore, there is an emerging consensus among researchers, in this field, in recent years that Ives and Stilwell have actually discovered a new type of Doppler effect caused mainly by an intrinsic asymmetry in the observed longitudinal Doppler effect for equal positive and negative velocities along the line of sight (*Ref. #5*).

1. The Experiment:

The following is a summary of the Ives-Stilwell experimental arrangement (*Ref. #1.a & #1.b*):

1. A canal-ray tube of the Dempster type is used to generate the hydrogen ions.

2. A small concave mirror is installed at about 7^{o} from the center to reflect light emitted in the backward direction by the canal rays.

3. An AC rectifier capable of delivering up to 30,000 volts is used to maintain the high negative potential applied to the accelerating electrode.

4. A spectrograph of 10.87Amm-1 is used to disperse the spectrum from the canal rays on photographic plates of III-J Eastman type.

5. A measuring microscope is used to obtain the displacement of $H\beta$ due to Doppler effect with respect to the rest wavelength of the second principal line in the Balmer series.

6. The velocities of the canal rays are computed from the voltage used and the charge-to-mass ratio, e/m.

By comparing the observed values to the computed values of Doppler shift, Ives and Stilwell have concluded that the prediction of the Larmor-Lorentz Theory is verified.

The Ives-Stilwell experiment along with their follow-up experiment in 1941 has a number of unsatisfactory aspects; and its experimental results are deemed inconclusive in a comprehensive review by Wallace Kantor, a seasoned experimenter in this field, (*Ref. #3*). And furthermore, according to Daniel Y. Gezari, Ives and Stilwell have measured only a slight asymmetry in the observed longitudinal Doppler effect for equal positive and negative velocities along the line of sight; and their experimental finding has nothing to with the transverse Doppler effect (*Ref. #5*).

Nevertheless, the Ives-Stilwell experiment is an inspiring and interesting experiment. And regardless of any interpretations, its experimental result is correct.

2. The Doppler Equations:

Since the treatment of Doppler effect is theory-dependent, its formulas are necessarily different for different theories. Moreover, each theory has two sets of equations for computing the effect, one for frequency and one for wavelength. For the comparison between two or more theories to be meaningful, only one of these two sets of equations should be used for the theories in question. Here, the wavelength formulas will be used throughout this discussion.

It should be noted, however, that the term 'Doppler shift' is defined differently in physical optics and astronomical spectroscopy. In the former, Doppler shift is defined as $(\lambda' - \lambda)$, and in the latter, as $(\lambda' - \lambda)/\lambda$, where λ' is the observed wavelength, and λ is the rest wavelength. The definition of Doppler shift as $(\lambda' - \lambda)$ will be used in the discussion about the Ives-Stilwell Experiment.

Although nanometer (nm) is now the fashionable wavelength unit in the optical regime, Angstrom (A) as used by these two experimenters is the appropriate unit in this context.

In the original report of Ives-Stilwell experiment, the observed results are compared to the computed values of the classical wave theory and the Larmor-Lorentz theory. In this discussion, however, the same observed results are compared to the values predicted by the Larmor-Lorentz theory. and the emission theory.

Only the motion of the light source is involved in the experiment under discussion. For a clear exposition, therefore, W. Kantor's notation is employed, and the plus and minus signs are implemented directly into the Doppler formulas within the range:

$$0^o \leq \theta \leq 180^o$$

in the two quadrants.

The following are the Doppler equations of the three theories:

A. The Doppler Formulas of the Classical Wave Theory:

Let *b* = *v/c*, where *v* is the velocity of the source of light with respect to the observer, and *c* is the velocity of light relative to the reference frame in which the source is at rest.

For the approaching source of light, therefore,

$$\lambda_a = \lambda\left(1 - \beta\ \cos\theta\right) \qquad 2.1$$

where λ_a is the wavelength as measured by the observer, λ is the rest wavelength, and θ is the angle made to the observer's line of sight by the velocity vector of the light source.

For the receding source of light,

$$\lambda_r = \lambda\left(1 + \beta\ \cos\theta\right) \qquad 2.2$$

where λ_r is the observed wavelength.

It should be noted that the same quantity $\lambda\beta cos\theta$ is added and subtracted in the two cases, respectively. From this, it follows, at once, that the classical theory predicts a null result for the Ives-Stilwell experiment. However, the frequency equations of the classical wave theory do give a positive result of about $\frac{1}{2}\gamma\beta 2$ for the current experiment.

B. The Doppler Formulas of the Larmor-Lorentz Theory:

Let β, λ_a, λ_r, λ, and θ be as defined above.

And let $\gamma = [1 - v^2/c^2]^{-1/2}$.

For the approaching source of light, therefore,

$$\lambda_a = \lambda\gamma\left(1 - \beta\ \cos\theta\right) \qquad 2.3$$

And for the receding source of light,

$$\lambda_r = \lambda\gamma\left(1 + \beta\ \cos\theta\right) \qquad 2.4$$

From these two equations, we calculate the average wavelength Λ,

$$\Lambda = \tfrac{1}{2}\left(\lambda_a + \lambda_r\right) = \lambda\gamma \qquad 2.5$$

Let $\Delta\lambda = \Lambda - \lambda = \lambda(\gamma - 1)$, and hence,

$$\Delta\lambda \approx \tfrac{1}{2}\lambda\beta^2 \qquad 2.6$$

which is the value predicted by the Larmor-Lorentz theory for the Ives-Stilwell Experiment.

C. The Doppler Formulas of the Emission Theory:

Let c_a denote velocity of light from the approaching source,

$$c_a = c\sqrt{1 - \frac{v^2}{c^2}\sin^2\theta} + v\cos\theta \qquad 2.7$$

and let c_r denote velocity of light from the receding source,

$$c_r = c\sqrt{1 - \frac{v^2}{c^2}\sin^2\theta} - v\cos\theta \qquad 2.8$$

For the approaching source of light, we compute the observed period, T_a,

$$T_a = \frac{Tc_a - Tv\cos\theta}{c_a} \qquad 2.9$$

$$T_a = T\left[\frac{c\sqrt{1 - \frac{v^2}{c^2}\sin^2\theta}}{c\sqrt{1 - \frac{v^2}{c^2}\sin^2\theta} + v\cos\theta}\right] \qquad 2.10$$

From this equation, we obtain the observed wavelength λ_a,

$$\lambda_a = \lambda\left[\frac{c\sqrt{1 - \frac{v^2}{c^2}\sin^2\theta}}{c\sqrt{1 - \frac{v^2}{c^2}\sin^2\theta} + v\cos\theta}\right] \qquad 2.11$$

where λ is the rest wavelength.

For the receding source of light, we compute the observed period, T_r,

$$T_r = \frac{Tc_r + Tv\cos\theta}{c_r} \qquad 2.12$$

$$T_r = T\left[\frac{c\sqrt{1-\frac{v^2}{c^2}\sin^2\theta}}{c\sqrt{1-\frac{v^2}{c^2}\sin^2\theta} - v\cos\theta}\right] \qquad 2.13$$

From this equation, we obtain the observed wavelength λ_r,

$$\lambda_r = \lambda\left[\frac{c\sqrt{1-\frac{v^2}{c^2}\sin^2\theta}}{c\sqrt{1-\frac{v^2}{c^2}\sin^2\theta} - v\cos\theta}\right] \qquad 2.14$$

For $\theta = 0^o$, Equation #2.11 is reduced to this equation:

$$\lambda_a = \lambda\left(1 - \frac{v}{c+v}\right) \qquad 2.15$$

and for $\theta = 180^o$, Equation #2.14 is reduced to this equation:

$$\lambda_r = \lambda\left(1 + \frac{v}{c-v}\right) \qquad 2.16$$

3. *The Prediction of the Emission Theory:*

Let β, λ_a, λ_r, λ, and θ be as defined above.

And let $\gamma = [1 - v^2/c^2]^{-1/2}$.

For a directly approaching source of light, therefore,

$$\lambda_a = \lambda \left(1 - \frac{v}{c + v} \right) \quad 3.1$$

And for a directly receding source of light,

$$\lambda_r = \lambda \left(1 + \frac{v}{c - v} \right) \quad 3.2$$

From these two equations, we calculate the average wavelength ***Λ,***

$$\Lambda = \tfrac{1}{2} \left(\lambda_a + \lambda_r \right) = \lambda \gamma^2 \quad 3.3$$

By dividing Equation #3.3 of the emission theory by Equation #2.5 of the Larmor-Lorentz theory, we obtain:

$$\frac{\Lambda_{ET}}{\Lambda_{LLT}} = \frac{\lambda \gamma^2}{\lambda \gamma} = \gamma \quad 3.4$$

and accordingly, the computed prediction of the emission theory is higher by the factor of γ than the computed prediction of the Larmor-Lorentz theory.

It follows, therefore, that the emission theory predicts the following result for the Ives-Stilwell experiment:

$$\Delta \lambda_{ET} = \Delta \lambda_{LLT} \gamma \approx \tfrac{1}{2} \lambda \gamma \beta^2 \quad 3.5$$

where $\Delta\lambda_{ET}$ is the shift in the wavelength as predicted by the emission theory, and $\Delta\lambda_{LLT}$ is the shift in the wavelength as given by Equation #2.6 of the Larmor-Lorentz theory.

For source velocities less than *0.1c* (about 30,000kms^{-1}), Equation #2..6 of the Larmor-Lorentz theory and Equation #3.5 of the emission theory yield similar results, as shown in Table #3.1, for

l = 4849.32 A.

The Emission Theory *½λγβ² A*	**The Larmor-Lorentz Theory** *½λβ² A*	**Source Velocity** *kms-1*
0.0325986	0.0325982	1.1×10^3
0.1683907	0.1683791	2.5×10^3
0.6737037	0.6735166	5.0×10^3
3.2686037	3.2598207	1.1×10^4
10.824375	10.776267	2.0×10^4
16.955665	16.837917	2.5×10^4
24.491515	24.246600	3.0×10^4

Table #3.1

In Table #3.2, the predictions of the two theories are compared to the Ives-Stilwell data of 1938 and 1941 for the ions of H_2.

Predictions of The Emission Theory	**Predictions of Larmor-Lorentz Theory**	**Observed Values of Δλ**
0.0202001	0.0202	0.0185
0.0243002	0.0243	0.0225
0.0280003	0.0280	0.0270
0.0360005	0.0360	0.0345
0.0478009	0.0478	0.0470
0.0670018	0.0670	0.0670
0.0686019	0.0686	0.0675
0.0724021	0.0724	0.0800
0.0869031	0.0869	0.0900
0.1054045	0.1054	0.1145

Table #3.2

The velocity range of the canal rays in the Ives-Stilwell experiment is between 850 kms^{-1} and 2100 kms^{-1}. Within this low velocity range, the predictions of the two theories, as shown in Table #3.2, are identical, and hence the experimental evidence is inconclusive. However, the emission theory uses, in its predictions, straightforward Galilean transformations and without any helper hypotheses; while the Larmor-Lorentz theory uses complicated Lorentz transformations along with additional hypotheses, to make the same predictions. On the basis of Ockham's razor, therefore, the latter theory loses out.

4. Concluding Remarks:

- To compute the velocities of the canal rays, Ives and Stilwell use this equation for the kinetic energy of the hydrogen ions,

$$eV = m_H c^2 (\gamma - 1) \qquad 4.1$$

  where e is the charge, V is the voltage, and m_H is the mass of the ion. But since the term $(\gamma - 1)$ in this equation is the same term $(\gamma - 1)$ in the equation of transverse Doppler effect, this clear mutual dependency necessarily makes their experimental method appear circular, and reduces the evidential value of the observed results considerably; unless an independent method for determining the velocities of the canal rays is found to verify it.

- It is not quite accurate to conclude from the Ives-Stilwell experimental result that the classical wave theory fails in this test. And that is because, even though computed wavelengths, on the basis of the classical wave theory, are symmetrical, there is always a slight asymmetry in computed frequencies according to that theory.

- Ives and Stilwell give no specific reason technical or otherwise for their choice of measuring only wavelengths, in this experiment, instead of performing measurements on frequencies, which are easier in the optical regime and more suitable for checking experimentally the rates of atomic clocks in accordance with their Larmor-Lorentz theory.

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4. *Waldron, R.A., (1977).* "The Wave and Ballistic Theories of Light : A Critical Review", London, F. Muller.

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Addendum:

1. What is the physical mechanism for the transverse Doppler effect according to the emission theory of light?

According to this equation for computing the speed resultant of light in the reference frame of the moving light source:

$$c' = \sqrt{c^2 + v^2 + 2cv\cos\theta'}$$

where θ' is the angle between the velocity vector c and the velocity vector v; the direction of every computed speed resultant, in the reference frame of the moving source, must be shifted by some amount into the forward direction of the moving source, except, of course, the direction of the speed resultant of c + v and the direction of the speed resultant of c - v.

From this, It follows, at once, that the angle of incidence θ, as measured in the reference frame of the stationary observer, is always less than the angle of emission θ' , as measured in the reference frame of the moving source. And the difference, Δθ, between these two angles can be calculated by using the following equation:

$$\sin(\Delta\theta) = \frac{v}{c}\sin\theta$$

where θ is the angle between the velocity vector c' and the velocity vector *v*, as measured in the reference frame of the stationary observer.

It's precisely this difference that causes the transverse Doppler effect, according to the emission theory. For example, in the case of light received at an observed angle of 90^{o} in the the reference frame of the stationary observer, the actual angle of emission is greater than 90^{o} by Δθ, where:

$$\sin(\Delta\theta) = \frac{v}{c}$$

and that necessarily produces a minute red shift in the wavelength at the angle of 90^{o} as measured in the reference frame of the stationary observer; i.e. transverse Doppler effect Δλ.

2. ***Does the motion of the observer relative to a stationary source of light produce a red transverse Doppler shift?***

The observer's motion with respect to a stationary source of light will, certainly, produce the red transverse Doppler shift, if the effect of light aberration on the true position of the light source is ignored.

The motion of the observer always shifts the position of the stationary source towards the forward direction of the velocity vector of the moving observer by an amount Δθ, which depends on the velocity of the moving observer v and the position of the stationary source with respect to the velocity vector of the observer:

$$\sin(\Delta\theta) = \frac{v}{c}\sin\theta$$

where

$$\Delta\theta = \theta' - \theta$$

and where θ' and θ are the true position and the apparent positions of the light source respectively

And since,

$$\theta' = \theta + \Delta\theta$$

it's clear that if the angle of the apparent position θ, as measured in the reference frame of the moving observer, is inserted directly, and without any correction for light aberration, into the Doppler equations of any theory, then it must lead to a sinusoidal function for all the red shifts of the transverse Doppler effect due to the velocity of the observer and as measured by the same moving observer.

3. ***Does the classical wave theory predict a transverse Doppler shift?***

Clearly, the classical wave theory does predict a transverse Doppler shift in the frequency of light.

Here is its Doppler equation for approaching light sources:

$$f_a = f\left(1 + \frac{v}{c - v}\right)$$

where f and f_a are the emitted and observed frequencies respectively.

And here is its Doppler equation for receding light sources:

$$f_r = f\left(1 - \frac{v}{c + v}\right)$$

where f and f_r are the emitted and observed frequencies respectively.

By using the above Ives-Stilwell method, the transverse Doppler shift in the light frequency can be computed on the basis of the classical wave theory:

$$\Lambda = \tfrac{1}{2}\left(f_a + f_r\right) = f\gamma^2$$

And therefore,

$$\Delta f \approx \tfrac{1}{2} f\gamma\beta^2$$

which is the value predicted by the classical wave theory for the Ives-Stilwell experiment.

Notice that the amount of the blue shift in the frequency f_a is larger than the amount of the red shift in the frequency f_r as calculated on the basis of the classical wave theory. But that poses no problem in this regard. Since, by definition, theories that assume the speed of light to be equal to c in vacuum, necessarily give a larger amount of Doppler blue shift and smaller amount of red Doppler shift for equal speeds of approach and recession.

Nevertheless, how can this theory account for the fact that the observed blue Doppler shift in the frequency is less than the calculated blue Doppler shift, and the observed red Doppler shift in the frequency is greater than the calculated red Doppler shift?

It's very simple; and exactly in the same way, the Larmor-Lorentz theory accounts for the fact that the observed blue Doppler shift in the wavelength is less than the calculated blue Doppler shift, and the observed red Doppler shift in the wavelength is greater than the calculated red Doppler shift; and that is to say:

The observed blue Doppler shift in the frequency is less than the calculated blue Doppler shift, because of the added amount of the red transverse Doppler shift:

$$f_{observed}(blue) = f_{calculated}(blue) + \Delta f(red)$$

and the observed red Doppler shift in the frequency is greater than the calculated red Doppler shift, because of the added amount of the red transverse Doppler shift:

$$f_{observed}(red) = f_{calculated}(red) + \Delta f(red)$$

And that is the same procedure used within the framework of the Larmor-Lorentz theory:

The observed blue Doppler shift in the wavelength is less than the calculated blue Doppler shift, because of the added amount of the red transverse Doppler shift:

$$\lambda_{observed}(blue) = \lambda_{calculated}(blue) + \Delta\lambda(red)$$

and the observed red Doppler shift in the wavelength is greater than the calculated red Doppler shift, because of the added amount of the red transverse Doppler shift:

$$\lambda_{observed}(red) = \lambda_{calculated}(red) + \Delta\lambda(red)$$

4. ***Does Einstein's Special theory of relativity predict transverse Doppler blue shifts in light frequencies?***

According to the following Doppler equation given by Einstein in his 1905 paper entitled: "*On the Electodynamics of Moving Bodies*":

$$\nu' = \nu \frac{1 - \cos\phi . v/c}{\sqrt{1 - v^2/c^2}}$$

where φ is defined explicitly by Einstein as being the angle between the observer's line of sight and the velocity vector v; and υ and ν' are the emitted the observed frequencies;

Special relativity does predict a transverse Doppler blue shifts in light frequencies at the angle of 90o.

However, as in the case of the classical wave theory and the Larmor-Lorentz theory, this theory too can apply the aforementioned Ives-Stilwell method to account for the fact that the observed blue Doppler shift in the frequency is less than the calculated blue Doppler shift; and that is to say:

The observed blue Doppler shift in the frequency is less than the computed blue Doppler shift, because of the added amount of the red transverse Doppler shift:

$$\nu_{observed}(blue) = \nu_{computed}(blue) + \Delta\nu(red)$$

and the observed red Doppler shift in the frequency is greater than the calculated red Doppler shift, because of the added amount of the red transverse Doppler shift:

$$\nu_{observed}(red) = \nu_{computed}(red) + \Delta\nu\,(red)$$

and that is it.

5. ***What is the main difference between this equation:***

$$c' = \sqrt{c^2 + v^2 + 2cv\cos\theta'}$$

and this equation:

$$c' = c\sqrt{1 - \frac{v^2}{c^2}\sin^2\theta} + v\cos\theta$$

for computing the speed resultant of light on the basis of the emission theory?

The equation:

$$c' = \sqrt{c^2 + v^2 + 2cv\cos\theta'}$$

is used to calculate the speed resultant of light in the reference frame in which the the light source is at rest; and where the angle between the velocity vector c and the velocity vector v can be measured.

And, by contrast, the equation:

$$c' = c\sqrt{1 - \frac{v^2}{c^2}\sin^2\theta} + v\cos\theta$$

is used to compute the speed resultant of light in the reference frame in which the measuring observer is at rest; and where the angle between the velocity vector c and the velocity vector v cannot be observed; and only the angle between the vector of the velocity resultant of light and the vector of the velocity of the light source can be measured. And that is because, by definition, light from moving sources always travels towards the measuring observer along the direction of the velocity resultant of c and v.