Relativity – Gravitoinertial Correction

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2026, 17 May

ABSTRACT: This article introduces a new mathematical model for relativistic correction based on a neutralization of the gravitational potential of space. Any object that has velocity or acceleration looses a quantity of matter and gains a quantity of energy, and the consequence is a lost of materiality. Based on inertial and non-inertial frames, the full relativistic correction is derived for both reference frames. So, this mathematical model explains the cause of relativistic correction.

KEYWORDS: Lorentz factor, special relativity, inertial reference frame, general relativity, non-inertial reference frame, relativistic correction, gravitoinertial correction.

Contents

1 Symbology

In this text we will use the following symbols with its abbreviated units of measure:

N = Newton, kg = kilogram, m = meter, s = second, rad = radians.

G = Gravitational field intensity [N kg^{-1}] [m s^{-2}];
V_G = Gravitational potential [N m kg^{-1}] [m^2 s^{-2}];

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q_G = Gravitational charge (mass) [kg];
k_G = Gravitostatic constant (Universal Gravitational Constant) = $6.6739 \cdot 10^{-11}$ N m^2 kg^{-2};
γ_0 = Gravitational permeability of vacuum $\gamma_0 = 1/(4\pi k_G)$ = $1.1924 \cdot 10^{9}$ kg^2 N^{-1} m^{-2};
I_I = Inertial current [m^2 s^{-2}];
F = Force [N] [kg m s^{-2}];
a = Acceleration [m s^{-2}];
v = Velocity [m s^{-1}];
ω = Angular velocity [rad s^{-1}];
r = Radial length [m];
d = Distance [m];
t = Time [s].

2 Introduction

To describe physical phenomena, we must have a frame of reference that includes a coordinate system to indicate the spatial position of a particle, and clocks with fixed coordinates within the frame to indicate time. Frames of reference in which a particle has constant velocity, if the particle is not acted upon by external forces, are called inertial frames of reference. All inertial frames of reference move with constant relative velocities to each other; therefore, the particle possesses the property of inertia, that is, it has uniform motion under zero net force. [1]

A transformation in space and time is necessary to determine the relationship between the position ***(x, y, z)*** at time ***t*** measured by an observer in a particular frame of reference and the corresponding position ***(x’, y’, z’)*** at time ***t’*** for the same particle, but measured by a second observer in a different inertial frame of reference. Only after obtaining this mathematical structure of transformation is it possible to fully describe physical phenomena.

2.1 Search for the Invariance of Physics Laws

The first person to conduct a systematic study of mechanics was Isaac Newton in his book "Principia" in 1687. One property of Newton's laws of motion is that they are invariant under Galilean transformations, that is, the mathematical expressions of the laws of mechanics have the same form for all observers whose relative motion is described by a constant velocity. In other words, with regard to the laws of mechanics, all inertial frames of reference are completely equivalent.

As an example, let's consider a set of Cartesian coordinates ***(x', y', z')*** for an inertial frame of reference ***F’***, say a train moving with constant velocity $\boldsymbol{v} = (v_x, v_y, v_z)$ relative to the inertial frame of reference ***F*** (the ground) with Cartesian coordinates ***(x, y, z)***. Then the coordinate transformation (Galilean transformation) between the two frames of reference, considering that the origins of ***F*** and ***F’*** coincide at time ***t = 0*** and that the axes are parallel to each other, is given by:

$$r' = r - vt \quad \text{and} \quad t' = t \quad .$$

Since the mechanical laws of motion are the same in both inertial frames of reference, it is not possible to use observations of mechanical phenomena to determine which frame of reference is "at rest" and which is "in motion". This is the primitive principle of relativity for mechanics, sometimes called the Galilean principle of relativity.

In the 19th century, the study of electrical and magnetic phenomena led to the discovery of electromagnetic waves and their association with the oscillation of fields. The similarity to mechanical oscillations was so great that it led researchers to believe that these waves could be transmitted through empty space using a medium called "Luminiferous Æther". The theory of a stationary "æther" permeating all space and matter and providing a medium for the oscillations of fields made perfect sense in analogy with the propagation of mechanical waves.

2.1.1 Special Relativity

At the beginning of the 20th century, after the failure of the Michelson-Morley (M-M) experiments to prove the Earth's motion through this medium, Hendrik Lorentz obtained his transformation (Lorentz transformation) through a mathematical approach based on specific physical hypotheses about the "æther", attempting to salvage the theory of the "Luminiferous Æther" and explain why experiments like M-M's failed. The main points of how he arrived at the transformations were: [1]

- **The Need for Invariance in Maxwell’s Equations**
  Lorentz realized that the laws of electromagnetism did not follow Galileo's principle of relativity. He sought a coordinate transformation that would keep Maxwell's equations invariant (with the same form) when moving from a stationary frame of reference in the "æther" to another frame of reference in motion.
- **Lorentz-Fitzgerald Contraction Hypothesis**
  To explain the null result of the Michelson-Morley experiment, Lorentz proposed that bodies moving through the “æther” underwent a contraction in the direction of motion. He developed this idea from mathematical considerations about the properties of molecular electromagnetic forces.
- **Introduction of “Local Time”**
  To accommodate the constancy of the speed of light that emerged in his equations, Lorentz introduced a modified time variable, called "local time" $t' = t - v x / c^2$ (a form of relativistic time), which depended on the position of the moving body.
- **Mathematical Procedure**
  Through mathematical manipulation, Lorentz discovered that spatial coordinates and time were not absolute, but rather interconnected when changing reference frames. He introduced the Lorentz factor $\gamma = 1/\sqrt{1 - v^2/c^2}$ to correct Galilean transformations.

The derivation of the relationships between inertial frames of reference started from the analysis of a frame of reference ***S***, with coordinates ***(x, y, z, t)*** at rest, and a frame of reference ***S’***, with coordinates ***(x’, y’, z’, t’)*** and constant velocity ***v*** along the X-axis relative to ***S***, as shown in the figure below (for simplification we have ***y//y’***, ***z//z’*** and at ***t = t’ = 0*** the origins ***O*** and ***O’*** coincide).

Equations of motion for a light ray that starts from the origin ***O = O’*** at time ***t = t’ = 0*** are:

$$-c^2 t^2 + x^2 + y^2 + z^2 = 0 \quad \text{and} \quad -c^2 t'^2 + x'^2 + y'^2 + z'^2 = 0 \quad .$$

Assuming the transformations are linear, we have:

$$x' = A(x - v t) \quad \text{and} \quad t' = B t + C x \quad .$$

Substituting into the previous equation and solving the linear system, we obtain the Lorentz factor γ:

$$A = B = \gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \quad \text{and} \quad C = -\frac{v}{c^2} B \quad .$$

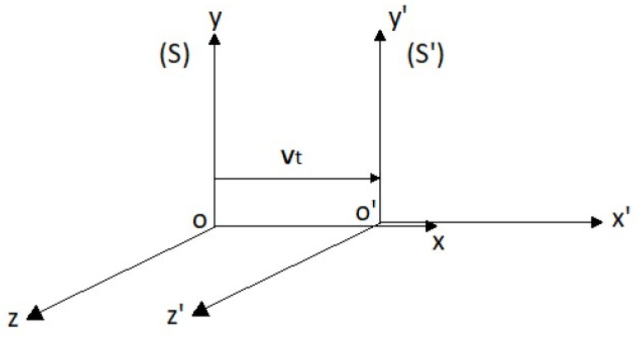

Figure 1: Inertial frames of reference (S) and (S').

And Lorentz’s Transformations:

$$x'=\gamma(x-vt) \quad , \quad y'=y \quad , \quad z'=z \quad \text{and} \quad t'=\gamma\left(t-\frac{\vec{v}\cdot\vec{x}}{c^2}\right) \quad .$$

Their inverses are:

$$x=\gamma(x'+vt') \quad , \quad y=y' \quad , \quad z=z' \quad \text{and} \quad t=\gamma\left(t'+\frac{\vec{v}\cdot\vec{x}'}{c^2}\right) \quad .$$

Let's consider a clock in reference frame ***S'***, moving with velocity ***v*** relative to ***S***, and another clock in ***S***. In the reference frame of the clock in ***S'***, it is at rest at ***x' = 0*** (it is at the center of the coordinate axes) and marks a proper time[2] ***t'***. If ***t*** is the time marked by the clock in ***S***, we have:

$$t=\gamma\left(t'+\frac{\vec{v}\cdot\vec{x}'}{c^2}\right)=\gamma t' \quad \Rightarrow \quad \Delta t=\gamma\Delta t'=\frac{\Delta t'}{\sqrt{1-\frac{v^2}{c^2}}} \quad .$$

Since $\gamma > 1$, then $\Delta t > \Delta t'$, that is, a time interval marked by a clock in reference frame ***S*** is greater than the interval marked by another clock in ***S’***, provided there is a relative velocity ***v*** between them. In other words, the time interval marked by an observer at rest is greater than the interval marked by an observer in motion. This phenomenon is called Time Dilation, and it shows that time is not an absolute entity.

Although Lorentz obtained the mathematical equations, he still relied on the existence of a stationary "æther". In 1905, Albert Einstein published an article that reinterpreted these transformations removing the need for the "æther", defining the theory now known as the Theory of Special Relativity, based on two postulates: [2]

1. **Principle of Relativity**
   The laws of nature are the same in all inertial frames of reference, which possess relative velocities. Time and space are relative, not absolute, and interconnected in "space-time".
2. **Constancy of the Light Speed**
   The speed of light in empty space is a constant of nature and is independent of the motion of the emitting body.

The other required assumption is that space is homogeneous and isotropic, that is, no region of space is intrinsically different from any other and there is no preferred direction in space.

2.1.2 General Relativity

In 1915, Albert Einstein developed the Theory of General Relativity, which expands the theory to accelerated and gravitational systems, proposing that gravity is not a classical force, but a geometric consequence of the curvature of "space-time," and can be considered a "fictitious" force. Its main concepts are:

1. **Space-Time Curvature**

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[2] In physics, the proper value (eigenvalue) of a quantity is defined as the value of that quantity measured in a frame of reference in which the body associated with it is at rest..

Massive bodies distort the four-dimensional fabric of the universe (three spatial directions and time), dictating the trajectory of objects and light (gravitational lensing). Time passes more slowly in intense gravitational fields (gravitational time dilation).

2. **Principle of Equivalence**
   There is no physical distinction between an accelerated system and a uniform gravitational field.

To observe how a gravitational field can affect the running of a clock, let's consider an enclosure in free fall with acceleration ***–g***. At the moment the fall begins, a device inside (at ***F***) emits a monochromatic light beam, with frequency f_0, vertically upwards, at a height ***h*** from the ceiling. The Figure illustrates this situation.

According to the principle of equivalence, for an observer in reference frame ***S'***, the light ray reaches the ceiling (at ***P***) at the instant $t=h/c$ and its frequency remains f_0. However, for an observer in reference frame ***S***, who experiences the gravitational field ***g***, the ray moves away from him at a velocity $v=gt$ ($v_0 = 0$). Therefore, the frequency ***f*** of the light measured in reference frame ***S*** undergoes a Doppler effect, given by:

$$f=f_0\left(1-\frac{gt}{c}\right)=f_0\left(1-\frac{gh}{c^2}\right) \quad \Rightarrow \quad \frac{f}{f_0}=\left(1-\frac{gh}{c^2}\right) \quad .$$

The equation shows that the frequency deviation f/f_0 is determined by the difference in gravitational potential $V_G=gh$ between the ceiling and the floor of the enclosure divided by the speed of light squared. Taking the Newtonian gravitational potential at a distance ***r*** from a body with gravitational charge (mass) q_G relative to the distance at infinity, one can write:

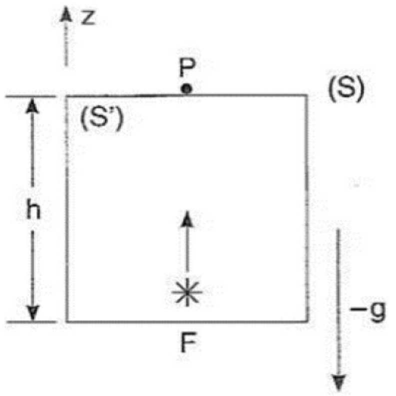

Figure 2: Free-falling enclosure.

$$\Delta t=\Delta t_\infty\left(1-k_G\frac{q_G}{r c^2}\right) \quad ;$$

$$V_G(r)=-k_G\frac{q_G}{r}=-\frac{1}{4\pi\gamma_0}\frac{q_G}{r} \quad \Rightarrow \quad \Delta t=\Delta t_\infty\left(1+\frac{V_G(r)}{c^2}\right) \quad .$$

With:

Δt = Time interval measured at the distance r from the center of mass [s];
Δt_∞ = Time interval measured at infinity [s];
q_G = Gravitational charge (mass) of Earth = $5{,}976 \cdot 10^{24}$ kg;
V_G = Gravitational potential [$m^2\ s^{-2}$] [$N\ m\ kg^{-1}$];
k_G = Gravitostatic constant (Universal Gravitational Constant) = $6{,}6739 \cdot 10^{-11}\ N\ m^2\ kg^{-2}$;
γ_0 = Gravitational permeability of space [$kg^2\ N^{-1}\ m^{-2}$];
r = Distance from object to center of planet [m];
c = Light speed in vacuum = $2.998 \cdot 10^8\ m\ s^{-1}$.

2.2 A New Model for Relativistic Correction

The Lorentz transformation was developed based on the principle of the existence of a medium through which electromagnetic waves travel – the "Luminiferous Æther". The speed of light would depend only on the electromagnetic characteristics of this medium, and the constancy of its speed would allow the electromagnetic equations to be invariant. The importance of this consideration is not only mathematical but also physical; that is, disregarding the existence of this medium, what would be the chance of arriving at the same result using another calculation methodology and/or starting from other physical considerations? For this reason, Einstein established the postulate of the "constancy of the light speed", indirectly validating the hypothesis of the existence of a medium that determines the speed at which light (electromagnetic waves) propagates.

The addition of gravitational potential for relativistic correction in non-inertial systems, due to the principle of equivalence between acceleration and gravitational field, showed that gravitational fields or accelerated bodies cause a temporal alteration. However, as demonstrated in the chapter Gravitational Potential Induction of the article Gravitoinertial Fields [3], the squared velocity that occurs in the equilibrium of satellite orbits also induces a gravitational potential. Therefore, we can conclude that time and space are altered by gravitoinertial fields.

Given that the adoption of postulates is a provisional measure until the reason for the need for relativistic correction is understood, and because both corrections, for inertial and non-inertial reference frames, stem from gravitoinertial phenomena, we propose a new approach based on known physical phenomena to unify the corrections through a single mathematical procedure. To this end, we will consider the existence of the Inertial Field [4] and its interaction with the Gravitational Field [5], and we will show that the relativistic correction is a consequence of the neutralization of the spatial gravitational potential; therefore, we could call it Gravitoinertial Correction.

3 Correction for Gravitational Charge (Mass)

When a gravitational charge (mass) is subjected to a gravitational potential from a central field, the associated potential energy is:

$$U = q_G V_G = q_G G r \quad .$$

With:

U = Potential energy [J] [N m];
q_G = Gravitational charge (mass) [kg];
V_G = Gravitational potential [$m^2\ s^{-2}$] [N m kg^{-1}];
G = Gravitational field [N kg^{-1}] [m s^{-2}];
r = Distance from object to center of planet [m].

In calculating satellite orbits, the gravitational force is equated to the centripetal acceleration force due to the satellite's uniform circular motion:

$$F = \frac{U}{r} = q_G G = q_G \frac{v^2}{r} = q_G \omega^2 r \quad .$$

With:

F = Force [N];
U = Potential energy [J] [N m];
q_G = Gravitational charge (mass) [kg];
G = Gravitational field [N kg^{-1}] [m s^{-2}];
v = Object velocity around the planet [m s^{-1}];
ω = Object angular velocity [rad s^{-1}];
r = Distance from object to center of planet [m].

We see that, in orbital equilibrium, the gravitational field G is proportional to the inertial current I_I [4]:

$$G=\frac{V_G}{r}=\frac{v^2}{r} \quad \text{, so} \quad V_G=Gr=v^2=I_I \quad .$$

As seen in the chapter Gravitational Potential Induction of the article Gravitoinertial Fields [3], when the gravitational potential induced by the satellite's inertial current reaches the value of the satellite's gravitational potential (relative to the center of the planet), the gravitational field force on the satellite becomes zero and the satellite remains in a circular orbit, that is, its height relative to the center of the planet remains constant. It is as if the gravitational force, which corresponds to the weight of the satellite, reduces in proportion to the square of its velocity, its inertial current.

The idea of a relativistic correction for extremely high speeds is that the same phenomenon of weight neutralization occurs in a moving body, but in relation to the gravitational potential of space the body loses matter. We can understand that the speed of light squared is equivalent to a spatial gravitational potential, as demonstrated by the equation above, and when the body's speed reaches the speed of light, then this spatial potential will be neutralized and this body will have zero mass.

In the gravitation domain, the spatial gravitational potential $V_{G0}=c^2=I_{I0}$ indicates the existence of a spatial gravitational field $G_0=V_{G0}/r=c^2/r$ that determines the limit of bodies' materiality. The equation $E=mc^2=q_G c^2$ tells us that the amount of matter in any object, upon reaching the speed of light, transforms into energy. Taking a body to the speed of light does not produce a chain reaction that disintegrates matter; the body is still there, but in an immaterial, radiant form, luminous like photons, and in this form we do not measure its mass.

Thus, any speed to which a body is subjected will provide a degree of immateriality proportional to the square of its speed, that is, a decrease in matter and an increase in energy. If two objects of the same mass are positioned in inertial reference frames that have different speeds, the lower speed reference frame will perceive the object in the higher speed reference frame as having less matter, whose mass can be measured.

There are two ways to approach relativistic (or gravitoinertial) correction for a gravitational charge: self-propulsion or external propulsion. In the first case, the energy required to accelerate the object and maintain its final velocity does not undergo inertial correction because there is no relative velocity between the object and the force of the field that accelerates it. In the second case, since propulsion is a consequence of the action of an external field, there will be correction for this field because there is relative velocity between the object and the field. The second case occurs in cyclotrons; the accelerating force on the object decreases as the object increases its speed, therefore there is relativistic correction.

3.1 Gravitational Charge with Constant Speed

If the gravitational charge's velocity vector is perpendicular to the gravitational field vector, as in the case of satellites, the inertial current $I_I=v^2=V_G$ of the gravitational charge will neutralize this amount of static gravitational potential of the planet's gravitational field, and the gravitational charge will be subjected to a smaller gravitational force. In this condition, we might (erroneously) understand that the amount of gravitational charge (mass) decreases and therefore weighs less, but what actually happens is that the gravitational field acting on the charge decreases in proportion to the square of its velocity. The inertial energy $U=q_G V_G=q_G v^2=q_G I_I$ associated with this velocity can be considered, through the relation $E=q_G c^2$, as a gravitational charge (mass) that will be subtracted from the original gravitational charge. The result can be calculated using the relation:

$$q_{Gi}c^2 - q_{Gi}v^2 = q_{Gi}(c^2 - v^2) = q_{Gf}c^2 \quad \Rightarrow \quad q_{Gf} = q_{Gi}\frac{c^2 - v^2}{c^2} = q_{Gi}\left(1 - \frac{v^2}{c^2}\right) \quad .$$

With:

q_{Gi} = Initial gravitational charge (mass) [kg];
q_{Gf} = Final gravitational charge (mass) [kg];
c = Light speed in vacuum = $2.998*10^8$ m s^{-1};
v = Velocity of the gravitational charge (mass) [m s^{-1}].

In this situation, because the gravitational charge decreases, we can associate the correction with the inverse square of the Lorentz factor γ[3], also known as the gamma factor.

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}} \quad \Rightarrow \quad \gamma^2 = \frac{1}{1 - v^2/c^2} \quad \Rightarrow \quad \frac{1}{\gamma^2} = 1 - \frac{v^2}{c^2} \quad \Rightarrow \quad q_{Gf} = \frac{1}{\gamma^2} q_{Gi} \quad .$$

In fact, the gravitational charge (mass) does not decrease; what happens is that the gravitational force $F = q_G G$ is weaker, and since the planet's gravitational field ***G*** remains constant, one might get the false impression that the gravitational charge (mass) $\boldsymbol{q_G}$ decreases. What occurred was a neutralization of the planet's gravitational potential with a gravitational potential induced by an inertial current $I_I = v^2 = V_G$. Therefore, the gravitational energy associated with a gravitational charge (mass) $\boldsymbol{q_{G1}}$ that has velocity ***v***, considering that the gravitational field is created by the gravitational charge $\boldsymbol{q_{G2}}$, will be:

$$U = q_{G1f}V_G = q_{G1i}G r\left(1 - \frac{v^2}{c^2}\right) = k_g\frac{q_{G1i}q_{G2}}{r}\left(1 - \frac{v^2}{c^2}\right) = \frac{1}{4\pi\gamma_0}\frac{q_{G1i}q_{G2}}{r}\left(1 - \frac{v^2}{c^2}\right) \quad .$$

With:

U = Gravitational energy [J] [N m];
q_{G1i} = Initial gravitational charge (mass) [kg];
q_{G1f} = Final gravitational charge (mass) [kg];
q_{G2} = Gravitational charge (mass) [kg];
V_G = Gravitational potential [m^2 s^{-2}] [N m kg^{-1}];
G = Gravitational field [N kg^{-1}] [m s^{-2}];
r = Distance from object to the center of q_{G2} [m];
v = Gravitational charge (mass) velocity [m s^{-1}];
c = Light speed in vacuum = $2.998*10^8$ m s^{-1};
k_G = Gravitostatic constant = $6{,}6739*10^{-11}$ N m^2 kg^{-2} [m^3 kg^{-1} s^{-2}];
γ_0 = Gravitational permeability of space = $1.1924*10^9$ kg s^2 m^{-3} [kg^2 N^{-1} m^{-2}].

3.2 Gravitational Charge Driven with Constant Speed

When an artificial gravitational field is produced to accelerate a gravitational charge to a specific speed, the same neutralization of the gravitational field seen above occurs. An increasingly gravitational field intensity will be needed because the action of the gravitational force on the gravitational charge will be less as the gravitational charge increases in speed. It is as if the mass of the gravitational charge were greater because the speed of the gravitational charge produces a gravitational potential that neutralizes part of the gravitational force that accelerates it. The result can be calculated through the relationship:

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[3] The Greek letter used for the Lorentz factor is γ (gamma) and should not be associated with the gravitational permeability of vacuum γ_0.

$$q_{Gf}c^2-q_{Gf}v^2=q_{Gf}(c^2-v^2)=q_{Gi}c^2 \quad \Rightarrow \quad q_{Gf}=q_{Gi}\frac{c^2}{c^2-v^2}=q_{Gi}\frac{1}{1-v^2/c^2} \quad .$$

With:

q_{Gi} = Initial gravitational charge (mass) [kg];
q_{Gf} = Final gravitational charge (mass) [kg];
c = Light speed in vacuum = $2.998*10^8$ m s^{-1};
v = Velocity of the gravitational charge (mass) [m s^{-1}].

As the gravitational charge accelerates, the speed is constantly increasing and the gravitational force is constantly decreasing.

Using the Lorentz factor γ, we have:

$$\gamma=\frac{1}{\sqrt{1-v^2/c^2}} \quad \Rightarrow \quad \gamma^2=\frac{1}{1-v^2/c^2} \quad \Rightarrow \quad q_{Gf}=q_{Gi}\frac{1}{(1-v^2/c^2)}=\gamma^2 q_{Gi} \quad .$$

The gravitational energy associated with a gravitational charge (mass) $\boldsymbol{q_{G1}}$ that has velocity $\boldsymbol{v}$, considering that the gravitational field is created by the gravitational charge $\boldsymbol{q_{G2}}$, will be:

$$U=q_{G1f}V_G=q_{G1i}Gr\left(1-\frac{v^2}{c^2}\right)^{-1}=k_g\frac{q_{G1i}q_{G2}}{r}\left(1-\frac{v^2}{c^2}\right)^{-1}=\frac{1}{4\pi\gamma_0}\frac{q_{G1i}q_{G2}}{r}\left(1-\frac{v^2}{c^2}\right)^{-1}$$

This also occurs for particles accelerated by electric and/or magnetic fields, as is the case with cyclotrons: the accelerating force on the particle decreases as its speed increases, and the particle's mass tends (erroneously speaking) to infinity when it approaches the speed of light.

4 Correction for Space and Time

As we have seen, the relativistic correction of gravitational charge (mass) for inertial reference frames is done using the Lorentz Factor squared. The explanation is that the calculation of this value takes into account the square of the velocity, and since the Lorentz Factor was originally calculated considering systems with constant velocities (inertial systems), squaring the velocity also squares the Lorentz Factor.

This means that applying the Lorentz Factor to the mass correction without squaring it is an error, which is understandable because this correction was not deduced from known phenomena, but from hypotheses that, at the time, seemed reasonable. Einstein adopted the coordinate transformations obtained by Hendrik Antoon Lorentz, now called Lorentz Transformations, establishing his postulates to support his new interpretations, considering that Lorentz took into account the light speed invariance in his calculations.

However, the calculation of the relativistic correction for space and time uses the Lorentz Factor without squaring it. In the calculations developed here, the explanation for this can be deduced from the equation for the induction of gravitational potential through inertial current:

$$V_G=Gr=v^2=I_I \quad \Rightarrow \quad v^2=\left(\frac{d}{t}\right)^2=V_G=\frac{1}{4\pi\gamma_0}\frac{q_G}{r} \quad \Rightarrow \quad v=\frac{d}{t}=\sqrt{V_G}=\sqrt{\frac{1}{4\pi\gamma_0}\frac{q_G}{r}} \quad .$$

With:

V_G = Gravitational potential [m^2 s^{-2}] [N m kg^{-1}];
G = Gravitational field [N kg^{-1}] [m s^{-2}];
r = Distance from object to center of mass [m];
v = Gravitational charge (mass) velocity [m s^{-1}];

I_I = Inertial current [$m^2\ s^{-2}$];
d = Distance [m];
t = Time [s];
q_G = Gravitational charge (mass) [kg];
γ_0 = Gravitational permeability of space = 1.1924×10^9 kg s^2 m^{-3} [$kg^2\ N^{-1}\ m^{-2}$].

Here, we are interested in the corrections to the distance ***d*** and time ***t*** corresponding to the velocity of the gravitational charge which decreases its charge (mass) as a function of the square of its velocity. Isolating the velocity, we have:

$$v=\frac{d_f}{t_f}=\sqrt{V_{Gf}}=\sqrt{\frac{1}{4\pi\gamma_0}\frac{q_{Gf}}{r}} \quad ; \quad q_{Gf}=\frac{1}{\gamma^2}q_{Gi} \quad \Rightarrow \quad v=\frac{d_f}{t_f}=\frac{1}{\gamma}\sqrt{\frac{1}{4\pi\gamma_0}\frac{q_{Gi}}{r}}=\frac{1}{\gamma}\sqrt{V_{Gi}}=\frac{1}{\gamma}\frac{d_i}{t_i} \quad .$$

The distance ***d_f*** and the time ***t_f*** are measured in the reference frame ***S***, while ***d_i*** and ***t_i*** are measured in the moving frame ***S’***.

Isolating the distance, we have:

$$d=\sqrt{V_G}\,t=\sqrt{\frac{1}{4\pi\gamma_0}\frac{q_G}{r}}\,t \quad \Rightarrow \quad d_f=\sqrt{V_{Gf}}\,t=\frac{1}{\gamma}\sqrt{V_{Gi}}\,t=\frac{1}{\gamma}d_i \quad \Rightarrow \quad d=\frac{1}{\gamma}d' \quad ; \quad \Delta d=\frac{1}{\gamma}\Delta d'$$

Because $1/\gamma < 1$, we see that there was a decrease in the distance ***d’***, which is the distance measured on the object in the moving system ***S’***, relative to the distance ***d***, measured in the reference frame ***S***. This corresponds to a contraction of space, that is, someone in the reference frame ***S*** perceives a decrease in the size of the object in the moving system ***S’***.

Isolating the time, we have:

$$t=\frac{d}{\sqrt{V_G}}=\frac{d}{\sqrt{\frac{1}{4\pi\gamma_0}\frac{q_G}{r}}} \quad \Rightarrow \quad t_f=\frac{d}{\sqrt{V_{Gf}}}=\gamma\frac{d}{\sqrt{V_{Gi}}}=\gamma t_i \quad \Rightarrow \quad t=\gamma t' \quad ; \quad \Delta t=\gamma\Delta t' \quad .$$

Because $\gamma > 1$, we see that there was an increase in time ***t'***, which is the proper time measured on a clock near the moving object (in ***S'***), relative to time ***t***, measured on a stationary clock in the reference frame ***S***. This corresponds to time dilation, that is, the amount of time measured in the reference frame ***S*** is greater than the amount of time measured in ***S'***. A clock traveling near the moving object counts more slowly than a clock that is in the reference frame ***S***.

As we can see, the relativistic correction, in the form of the Lorentz Factor $\gamma=1/\sqrt{1-v^2/c^2}$, is correctly used for corrections in space and time, but for gravitational charge (mass) correction we must use the squared Lorentz Factor $\gamma^2=1/(1-v^2/c^2)$.

5 Correction for Inertial and Non-Inertial Reference Frames

It is important to note that the deduction of relativistic correction developed in this work presents the currently accepted result for corrections of inertial systems; however, the equilibrium velocity of orbital systems is that which results in the so-called centrifugal force (felt in the moving frame of reference), which corresponds to the velocity tangential to the orbit $v_\theta=r\omega=r\dot{\theta}$. If the orbit is not circular, there will be radial and angular velocity and acceleration components; therefore, it is necessary to investigate what other velocities and accelerations involved in orbits may result in additional relativistic corrections.

Given that relativistic corrections are values divided by the gravitational potential of space $V_{G0}=c^2$, whose unit of measurement is [m^2 s^{-2}], these values will also be gravitational potentials. Therefore, the "fictitious" forces (which exist only in the moving frame of reference) of the orbital calculations must be converted into gravitational potentials.

The relativistic corrections for inertial and non-inertial systems discussed below apply to orbital, pendulum, and oscillatory systems; however, it is possible to create gravitational potentials in other ways not discussed in this work, whose relativistic correction will follow the same pattern described here, because they obey the same principle.

5.1 Coordinates in Inertial and Non-Inertial Frames of Reference

Let us consider an inertial coordinate system with axes $\hat{x}_1, \hat{y}_1, \hat{z}_1$ and origin *O₁*, that we call Inertial Reference Frame (with uniform movement or stationary), and another coordinate system with axes $\hat{x}, \hat{y}, \hat{z}$ and origin *O*, that we call Moving Reference Frame (with uniform movement or accelerated), in which movement is allowed in an arbitrary manner with respect to the inertial reference frame. That is, the origin *O* may undergo acceleration and the axis can rotate. These axes may be considered to be functions of the inertial reference axes. [6]

The vector from *O₁* to *O* is $\vec{R}$, the vector from *O₁* to a particle is $\vec{r}_1$, and the vector from *O* to the particle is $\vec{r}$. Then:

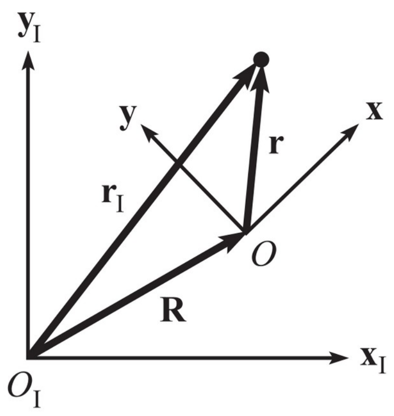

Figure 3: Coordinate axis of reference frames.

$$\vec{r}_1=\vec{R}+\vec{r}$$

The existence of these vectors is independent of any specific coordinate system, but they can be written in terms of defined coordinates. $\vec{R}$ and $\vec{r}_1$ are written in terms of the coordinates of the inertial reference frame, while $\vec{r}$ is described in terms of the coordinates of the moving reference frame:

$$\vec{r}_1=x\,\hat{x}_1+y\,\hat{y}_1+z\,\hat{z}_1 \qquad = \qquad \vec{R}=X\,\hat{x}_1+Y\,\hat{y}_1+Z\,\hat{z}_1 \qquad + \qquad \vec{r}=x\,\hat{x}+y\,\hat{y}+z\,\hat{z}$$

The first and second derivatives depend on two factors:

1. The coordinates (x, y, z) of $\vec{r}$, measured with respect to the moving reference axes, can change;
2. The axes $\hat{x}, \hat{y}, \hat{z}$ can change.

This means that even if (x, y, z) are fixed, meaning $\vec{r}$ don't change with respect to the moving reference frame, $\vec{r}$ may still change with respect to the inertial reference frame if the axes $\hat{x}, \hat{y}, \hat{z}$ are moving.

5.2 Inertial Frames of Reference

The goal is to obtain the first derivatives with respect to time of these relations and interpret the result in the form $V_G=v^2=\omega^2 r^2$. The first derivative of $\vec{r}_1$ is the velocity of the particle with respect to the inertial reference frame. The first derivative of $\vec{R}$ is the velocity of the origin *O* of the moving reference frame.

Our interest lies in obtaining $d\vec{r}/dt$ in terms of the coordinates of the moving frame of reference, and obtaining the gravitational potential $V_G = v^2$ only in terms of this frame of reference, independently of the inertial reference frame. In this way, we will obtain the "fictitious" gravitational potentials acting on the particle.

For a vector $\vec{A} = A_x\hat{x} + A_y\hat{y} + A_z\hat{z}$ in a moving frame of reference, its first derivative is obtained using the product rule because the coordinates of the moving frame of reference also move relative to the inertial frame of reference: [6]

$$\frac{d\vec{A}}{dt} = \left(\frac{dA_x}{dt}\hat{x} + \frac{dA_y}{dt}\hat{y} + \frac{dA_z}{dt}\hat{z}\right) + \left(A_x\frac{d\hat{x}}{dt} + A_y\frac{d\hat{y}}{dt} + A_z\frac{d\hat{z}}{dt}\right) \quad .$$

As we can see, although the vector $\vec{A}$ is expressed in relation to the coordinates of the moving frame of reference, its total derivative $d\vec{A}/dt$ is measured with respect to the inertial frame of reference. The first group gives us the rate of change of $\vec{A}$ with respect to the moving frame of reference, which will be denoted by:

$$\frac{\delta\vec{A}}{\delta t} = \left(\frac{dA_x}{dt}\hat{x} + \frac{dA_y}{dt}\hat{y} + \frac{dA_z}{dt}\hat{z}\right) \quad .$$

The second group indicates that the coordinate axes are moving and, because translation has already been introduced with the vector $\vec{R}$, it indicates a rotation around an axis $\vec{\omega}$ passing through the origin *O*. Considering that $\vec{\theta}$ is the angle of rotation, the angular velocity $\vec{\omega} = d\theta/dt\,\hat{\omega} = \dot{\theta}\,\hat{\omega}$ is perpendicular to the direction of rotation, that is, on the axis of rotation. Therefore, we can express the angular velocity of a vector of fixed length as its rate of change $d\vec{A}/dt = \vec{\omega}\times\vec{A}$:

$d\vec{x}/dt = \vec{\omega}\times\vec{x} \quad \Rightarrow$	$A_x(d\vec{x}/dt) = A_x(\vec{\omega}\times\hat{x}) = \vec{\omega}\times A_x\hat{x}$
$d\vec{y}/dt = \vec{\omega}\times\vec{y} \quad \Rightarrow$	$A_y(d\vec{y}/dt) = A_y(\vec{\omega}\times\hat{y}) = \vec{\omega}\times A_y\hat{y}$
$d\vec{z}/dt = \vec{\omega}\times\vec{z} \quad \Rightarrow$	$A_z(d\vec{z}/dt) = A_z(\vec{\omega}\times\hat{z}) = \vec{\omega}\times A_z\hat{z}$

Therefore, a rotation of $\vec{A}$ can be expressed by $\vec{\omega}\times\vec{A} = \vec{\omega}\times A_x\hat{x} + \vec{\omega}\times A_y\hat{y} + \vec{\omega}\times A_z\hat{z}$.

And the first total derivative can be expressed by: $\dfrac{d\vec{A}}{dt} = \dfrac{\delta\vec{A}}{\delta t} + \vec{\omega}\times\vec{A}$.

Doing $\vec{A} = \vec{r}$, we have $\dfrac{d\vec{r}}{dt} = \dfrac{\delta\vec{r}}{\delta t} + \vec{\omega}\times\vec{r} = \dot{\vec{r}} + \vec{\omega}\times\vec{r}$.

This same relation can be expressed in polar coordinates:

$$\frac{d\vec{r}}{dt} = \dot{\vec{r}} + r\dot{\vec{\theta}} = v_r\hat{r} + v_\theta\hat{\theta} = \vec{v} \quad .$$

With:

$\dot{r} = v_r$ = Radial velocity [m s^{-1}];

$\dot{\theta} = \omega$ = Angular velocity [rad s^{-1}];

$r\dot{\theta} = r\omega = v_\theta$ = Velocity in theta direction (tangential to the curve) [m s^{-1}].

5.2.1 Gravitational Potential for Inertial Frames of Reference

The gravitational potential induced by the inertial current is given by $V_G = v^2 = I_I$, and in polar coordinates, can be expressed as:

$$V_G = v^2 = (\dot{\vec{r}} + r\dot{\vec{\theta}})^2 = \dot{r}^2 + 2\dot{r}\,r\,\dot{\theta} + r^2\dot{\theta}^2 \quad ; \quad V_G = v^2 = (v_r + v_\theta)^2 = v_r^2 + 2v_r v_\theta + v_\theta^2 \quad .$$

"Fictitious" gravitational potentials, which exist only in the moving frame of reference, are:

1. Translation $V_{Gtrans} = \dot{r}^2 = v_r^2$:
   It corresponds to a gravitational potential produced by a radial velocity.
2. Centrifugal $V_{Gcent} = (\vec{\omega} \times \vec{r})^2 = r^2\dot{\theta}^2 = v_\theta^2$:
   It corresponds to a centripetal gravitational potential in the stationary frame of reference and varies as a function of the distance from the axis of rotation.
3. Coriolis $V_{Gcor} = 2\dot{r}\,r\,\dot{\theta} = 2v_r v_\theta$:
   It appears when there is relative velocity with respect to the moving frame of reference.

5.3 Non-Inertial Frames of Reference

When we are in an accelerated frame of reference relative to another inertial frame of reference (non-accelerated, that is, with constant velocity or stationary), there are forces called "fictitious" acting on us that are not present in the inertial frame of reference. To calculate these forces, we must know how the coordinates (and their derivatives) of the accelerated frame relate to those of the inertial frame of reference. [6]

The goal is to obtain the second derivatives with respect to time of these relations and interpret the result in the form $\vec{F} = q_G\vec{G} = m\vec{a}$. The second derivative of $\vec{r}_1$ is the acceleration of the particle with respect to the inertial frame of reference, so Newton's second law states that $\vec{F} = q_G\ddot{\vec{r}}_1 = m\ddot{\vec{r}}_1$. The second derivative of $\vec{R}$ is the acceleration of the origin *O* of the moving frame.

Our interest lies in obtaining $d^2\vec{r}/dt^2$ in terms of the coordinates of the accelerated frame of reference, and $\vec{F} = q_G\vec{G} = m\vec{a}$ only in terms of this frame of reference, independently of the inertial reference system, thus obtaining the "fictitious" forces acting on the particle.

The second derivative is calculated using the product rule on the first derivative:

$$\frac{d^2\vec{A}}{dt^2} = \frac{d}{dt}\left(\frac{\delta\vec{A}}{\delta t} + \vec{\omega} \times \vec{A}\right) = \frac{d}{dt}\left(\frac{\delta\vec{A}}{\delta t}\right) + \frac{d\omega}{dt} \times \vec{A} + \vec{\omega} \times \frac{d\vec{A}}{dt} = \frac{\delta}{\delta t}\left(\frac{d\vec{A}}{dt}\right) + \frac{d\omega}{dt} \times \vec{A} + \vec{\omega} \times \frac{d\vec{A}}{dt} \quad .$$

Substituting the first derivative into the second derivative, we have:

$$\frac{d^2\vec{A}}{dt^2} = \left(\frac{\delta^2\vec{A}}{\delta t^2} + \vec{\omega} \times \frac{\delta\vec{A}}{\delta t}\right) + \left(\frac{d\omega}{dt} \times \vec{A}\right) + \omega \times \left(\frac{\delta\vec{A}}{\delta t} + (\vec{\omega} \times \vec{A})\right) = \frac{\delta^2\vec{A}}{\delta t^2} + \vec{\omega} \times (\vec{\omega} \times \vec{A}) + 2\vec{\omega} \times \frac{\delta\vec{A}}{\delta t} + \frac{d\vec{\omega}}{dt} \times \vec{A}$$

Doing $\vec{A} = \vec{r}$, we have:

$$\frac{d^2\vec{r}}{dt^2} = \ddot{\vec{r}} + \vec{\omega} \times (\vec{\omega} \times \vec{r}) + 2\vec{\omega} \times \dot{\vec{r}} + \frac{d\vec{\omega}}{dt} \times \vec{r} \quad .$$

This same relationship can be expressed in polar coordinates:

$$\frac{d^2\vec{r}}{dt^2}=\ddot{r}\,\hat{r}+\dot{\theta}^2 r\,\hat{r}+2\dot{\theta}\,\dot{r}\,\hat{\theta}+\ddot{\theta}\,r\,\hat{\theta}=a_r\,\hat{r}+\omega^2 r\,\hat{r}+2\,\omega\,v_r\,\hat{\theta}+\alpha\,r\,\hat{\theta}\quad .$$

With:

$\dot{r}=v_r$ = Radial velocity [m s^{-1}];
$\dot{\theta}=\omega$ = Angular velocity [m s^{-1}];
$r\,\dot{\theta}=r\,\omega=v_\theta$ = Velocity in theta direction (tangential to the curve) [m s^{-1}];
$\ddot{r}=a_r$ = Radial acceleration [m s^{-2}];
$\ddot{\theta}=\alpha$ = Angular acceleration [m s^{-2}];
$r\,\ddot{\theta}=r\,\alpha=a_\theta$ = Acceleration in theta direction (tangential to the curve) [m s^{-2}].

Knowing that $\vec{r}_1=\vec{R}+\vec{r}$, the force acting on the particle from the origin of the inertial frame O_1 is determined by $\vec{F}_1=q_G(d^2\vec{r}_1/dt^2)=m(d^2\vec{r}_1/dt^2)$, that can be gravity, a normal force, tension, etc., then the resultant force acting on the particle relative to the accelerated frame of reference will be:

$$\vec{F}=\vec{F}_1-q_G\left(\frac{d^2\vec{R}}{dt^2}+\vec{\omega}\times(\vec{\omega}\times\vec{r})+2\,\vec{\omega}\times\dot{\vec{r}}+\frac{d\,\vec{\omega}}{dt}\times\vec{r}\right)=q_G(\vec{G}_1-\vec{G})\quad .$$

The "fictitious" forces, that exist only in the accelerated frame of reference, are:

1. Translation $\vec{F}_{trans}=-q_G\dfrac{d^2\vec{R}}{dt^2}=-m\dfrac{d^2\vec{R}}{dt^2}$:
   It corresponds to a gravitational force produced by a gravitational field equivalent to radial acceleration.
2. Centrifugal $\vec{F}_{cent}=-q_G\vec{\omega}\times(\vec{\omega}\times\vec{r})=-m\,\vec{\omega}\times(\vec{\omega}\times\vec{r})$:
   It corresponds to a centripetal force in the stationary frame of reference and varies as a function of the distance from the axis of rotation.
3. Coriolis $\vec{F}_{cor}=-2\,q_G\vec{\omega}\times\dot{\vec{r}}=-2\,m\,\vec{\omega}\times\dot{\vec{r}}$:
   It appears when there is relative velocity with respect to the accelerating frame of reference.
4. Azimuthal $\vec{F}_{az}=-q_G\dfrac{d\,\vec{\omega}}{dt}\times\vec{r}=-m\dfrac{d\,\vec{\omega}}{dt}\times\vec{r}$:
   It corresponds to a force produced by a gravitational field equivalent to tangential acceleration, and varies as a function of the distance from the axis of rotation.

It is worth noting that the gravitational acceleration generated by the tangential velocity $\vec{\omega}\times(\vec{\omega}\times\vec{r})=r\,\dot{\theta}^2\,\hat{r}$ originates a radial force, perpendicular to the motion, confirming a process of gravitational potential induction by the inertial current $V_G=v^2=I_I$.

5.3.1 Gravitational Potential for Non-Inertial Frames of Reference

The second derivative with respect to time gives us the acceleration and its equivalent gravitational field; on the other hand, the gravitational field is obtained from the gradient of the gravitational potential. To simplify the calculations, we will use polar coordinates.

$$\frac{d^2\vec{r}}{dt^2}=\vec{a}=\vec{G}=\ddot{\vec{r}}+\vec{\omega}\times(\vec{\omega}\times\vec{r})+2\,\vec{\omega}\times\dot{\vec{r}}+\frac{d\,\vec{\omega}}{dt}\times\vec{r}=\ddot{r}\,\hat{r}+\dot{\theta}^2 r\,\hat{r}+2\,\dot{\theta}\,\dot{r}\,\hat{\theta}+\ddot{\theta}\,r\,\hat{\theta}=a_r\,\hat{r}+\omega^2 r\,\hat{r}+2\,\omega\,v_r\,\hat{\theta}+\alpha\,r\,\hat{\theta}$$

$$\vec{G}=-\nabla V_G=-\left(\frac{\delta}{\delta r}V_G\hat{r}+\frac{1}{r}\frac{\delta}{\delta\theta}V_G\hat{\theta}\right) \quad .$$

Therefore, the gravitational potential V$_G$ that we need for the relativistic correction is calculated using the inverse of the acceleration gradient already calculated:

$$V_G=-\int\nabla V_G=-\int G=-\left(\int G\,dr+r\int G\,d\theta\right) \quad .$$

However, radial and angular velocities and accelerations are constant values. For example, the angular velocity $\omega=\dot{\theta}$, which determines the tangential velocity, does not change as a function of *r*, because *r* remains constant in the particle's orbit. Therefore, the calculation of gravitational potentials is done simply by multiplying the gravitational fields by *r*:

$$\ddot{r}\hat{r} \quad\Rightarrow\quad V_G=\int\ddot{r}\,dr=r\ddot{r} \quad ;$$

$$\dot{\theta}^2 r\hat{r} \quad\Rightarrow\quad V_G=\int r\dot{\theta}^2\,dr=r^2\dot{\theta}^2 \quad ;$$

$$2\dot{\theta}\dot{r}\hat{\theta} \quad\Rightarrow\quad V_G=\int 2\dot{r}\dot{\theta}\,dr=2\dot{r}r\dot{\theta} \quad ;$$

$$\ddot{\theta}r\hat{\theta} \quad\Rightarrow\quad V_G=\int r\ddot{\theta}\,dr=r^2\ddot{\theta} \quad .$$

The total gravitational potential for accelerated systems is:

$$V_G=r\ddot{r}+r^2\dot{\theta}^2+2\dot{r}r\dot{\theta}+r^2\ddot{\theta} \quad ; \quad V_G=r\,a_r+v_\theta^2+2v_r v_\theta+r\,a_\theta \quad .$$

6 Relativistic or Gravitoinertial Correction

The gravitational potentials obtained for inertial and non-inertial systems allow for the evaluation of relativistic corrections for gravitational charges (mass), distances, and times for objects moving at uniform speeds and with accelerations. These corrections aim to reconcile the differences in measurements between reference frames due to the existence of "fictitious" gravitational potentials induced in the moving frame of reference that do not exist in the reference system.

This means that gravitational charge (mass), distance, and time are relative, depending on the motion they are subjected to. Therefore, for inertial systems with uniform radial or angular velocities, it is sufficient to apply the corrections obtained for inertial systems. For non-inertial systems, which have radial or angular accelerations, it is necessary to apply the corrections for non-inertial systems.

However, it is possible to use a single correction that includes both corrections and only use the corrections appropriate to each studied system because, as we can see in the induced gravitational potential equations, the correction for non-inertial systems only lacks the square of the radial velocity.

1. Gravitational Potential for inertial systems (not accelerated):
   $$V_G=\dot{r}^2+2\dot{r}r\dot{\theta}+r^2\dot{\theta}^2 \quad .$$
2. Gravitational Potential for non-inertial systems (accelerated):
   $$V_G=r\ddot{r}+2\dot{r}r\dot{\theta}+r^2\dot{\theta}^2+r^2\ddot{\theta} \quad .$$
3. Gravitational Potential for inertial and non-inertial systems:

$$V_G = r\ddot{r} + \dot{r}^2 + 2\dot{r}\,r\,\dot{\theta} + r^2\dot{\theta}^2 + r^2\ddot{\theta} \quad .$$

The consequence of relativistic correction being related to gravitational potentials is that internal gravitational fields, which are in the moving frame of reference, and external gravitational fields, which are in the inertial frame of reference, also affect the relativistic correction. Larger gravitational potentials will cause a reduction in mass and distance, and time dilation, and vice versa. The equation for gravitational potentials induced by non-inertial systems clearly shows that one of the components refers to radial acceleration, comparable to the acceleration a body undergoes under the action of a planetary gravitational field.

A typical example is what occurs with the satellite-based Global Positioning System (GPS), because this geographic location system depends on the time count of the clock on the satellite compared to the one located on the planet's surface. This means that while the satellite's higher speed (tangential to its orbit) decreases the frequency of its clock relative to the one on the surface, the lower planetary gravitational potential to which it is subjected causes an increase in its frequency.

Therefore, practice demonstrates that special relativity, used in inertial systems, necessarily needed an adjustment to accommodate non-inertial systems, as general relativity demonstrates. But, since the relativistic correction is a gravitoinertial phenomenon, we can now complement the theory with the appropriate gravitational potentials developed here. The original equation, reproduced below, which allowed us to deduce the relativistic correction, showed that it is a consequence of the gravitational potentials induced in the moving frame of reference relative to the gravitational potential of space.

$$q_{Gi}c^2 - q_{Gi}v^2 = q_{Gi}(c^2 - v^2) = q_{Gf}c^2 \quad \Rightarrow \quad q_{Gf} = q_{Gi}\frac{c^2 - v^2}{c^2} = q_{Gi}\left(1 - \frac{v^2}{c^2}\right) \quad .$$

With:

q_{Gi} = Initial gravitational charge (mass) [kg];
q_{Gf} = Final gravitational charge (mass) [kg];
c = Light speed in vacuum = $2.998*10^8$ m s^{-1};
v = Gravitational charge (mass) velocity [m s^{-1}].

The study conducted using the time derivatives of the body's position in the moving frame of reference deduces velocities and accelerations in polar coordinates; therefore, they are decomposed into the axes $\hat{r}$ and $\hat{\theta}$ and are a consequence of the body's unique positions. Thus, the total value of the gravitational potential is calculated by summing the individual potentials. By substituting the gravitational potential $V_G = v^2$ with the set of induced gravitational potentials, we obtain the relativistic correction for both inertial and non-inertial systems:

$$q_{Gf} = q_{Gi}\left(1 - \frac{V_G}{c^2}\right) = q_{Gi}\left(1 - \frac{r\ddot{r} + \dot{r}^2 + 2\dot{r}\,r\,\dot{\theta} + r^2\dot{\theta}^2 + r^2\ddot{\theta}}{c^2}\right) \quad \Rightarrow \quad q_{Gf} = \Re^2 q_{Gi} \quad .$$

The relativistic or gravitoinertial factor $\Re$ is thus defined:

$$\Re^2 = 1 - \frac{r\ddot{r} + \dot{r}^2 + 2\dot{r}\,r\,\dot{\theta} + r^2\dot{\theta}^2 + r^2\ddot{\theta}}{c^2} \quad \Rightarrow \quad \Re = \sqrt{1 - \frac{r\ddot{r} + \dot{r}^2 + 2\dot{r}\,r\,\dot{\theta} + r^2\dot{\theta}^2 + r^2\ddot{\theta}}{c^2}} \quad .$$

With:

$\Re$ = Relativistic or gravitoinertial factor;
r = Distance to the center of mass of gravitational field [m];
$\dot{r} = v_r$ = Radial velocity [m s^{-1}];

$\dot{\theta}=\omega$ = Angular velocity [m s^{-1}];
$\ddot{r}=a_r$ = Radial acceleration [m s^{-2}];
$\ddot{\theta}=\alpha$ = Angular acceleration [m s^{-2}];
c = Light velocity in vacuum = $2.998*10^8$ m s^{-1}.

The spatial gravitational potential $V_{G0}=c^2$ of the denominator is used to calculate the fraction of increase or decrease in gravitational charge (mass), resulting in an increase or decrease in gravitational force. This occurs because the components of the numerator are also gravitational potentials corresponding to radial $v_r=\dot{r}$ and tangential $v_\theta=r\dot{\theta}$ velocities and radial $a_r=\ddot{r}$ and tangential $a_\theta=r\alpha=r\ddot{\theta}$ accelerations, in addition to the Coriolis potential $V_{Gcor}=2\dot{r}r\dot{\theta}$.

For relativistic space and time correction, the original equation, reproduced below, becomes:

$$V_G=v^2=\left(\frac{d}{t}\right)^2 \qquad \Rightarrow \qquad v=\frac{d_f}{t_f}=\sqrt{V_{Gf}}=\sqrt{V_{Gi}\Re^2}=\Re\sqrt{V_{Gi}} \quad .$$

With:

$\Re$ = Relativistic or gravitoinertial factor;
V_G = Gravitational potential [m^2 s^{-2}];
v = Velocity of gravitational charge (mass) [m s^{-1}];
d = Distance [m];
t = Time [s].

Here we use the equation for the gravitational potential perceived by the moving object; that is, the "fictitious" gravitational potentials which exist only in the moving frame of reference reduce the external gravitational potential. This means that the increase in energy of the moving frame, by decreasing its materiality, reduces the influence of external fields.

For the distance, we have:

$$d=\sqrt{V_G}\,t \qquad \Rightarrow \qquad d=\sqrt{V_{Gf}}\,t=\Re\sqrt{V_{Gi}}\,t=\Re\,d' \quad .$$

We see that there has been a decrease in the distance ***d’***, which is the distance measured on the object in the moving frame of reference ***S’***, relative to the distance ***d***, measured in the reference system ***S***. This corresponds to a contraction of space, that is, the distance measured in the direction of movement of the object will be perceived in the reference system as being smaller.

For the time, we have:

$$t=\frac{d}{\sqrt{V_G}} \qquad \Rightarrow \qquad t=\frac{d}{\sqrt{V_{Gf}}}=\frac{d}{\Re\sqrt{V_{Gi}}}=\frac{1}{\Re}t' \quad .$$

We see that there has been an increase in time ***t’***, which is the proper time measured by a clock next to the moving object in the reference frame ***S’***, relative to time ***t***, measured by a stationary clock in the reference frame ***S***. This corresponds to time dilation, that is, the clock that travels along with the moving object counts more slowly than the clock that is in the reference frame.

It is important to emphasize that the corrections for distance and time are made using the square root of the sum of the gravitational potentials induced by velocity and acceleration. Given that the correction for velocity is the same as that calculated by Lorentz and adopted by Einstein, we see that the correction for accelerations and gravitational fields differs by a square root from the currently accepted correction based on the Doppler effect.

The explanation lies in understanding the cause of the relativistic correction, which is the existence or induction of gravitational potential, which alters the materiality of whatever is under its influence. The unit of measurement for gravitational potential is [m^2 s^{-2}], therefore it corresponds to the square of the distance over time; to calculate the correction in distance or time, we must extract the square root of the gravitational potential, therefore, the correction will be under the square root.

The relativistic gravitational correction found in this work presents the acceleration (radial or angular), which corresponds to a gravitational field, multiplied by the distance to the center of mass; that is, it is the gravitational potential produced by the acceleration of the object. However, it can also represent a gravitational potential corresponding to an external or internal gravitational field, i.e., existing in the reference frame ***S*** or in the moving frame ***S'***. In these cases, the final correction for distance and time will be calculated by the square root of the resultant gravitational potential, that is, the sum of the gravitational potentials acting on the body.

$$q_G=\left(1-\frac{\sum V_G}{c^2}\right)q_G' \quad , \quad \Delta d=\sqrt{1-\frac{\sum V_G}{c^2}}\,\Delta d' \quad , \quad \Delta t=\frac{\Delta t'}{\sqrt{1-\frac{\sum V_G}{c^2}}} \quad .$$

And the relativistic or gravitoinertial factor $\Re$ is thus defined:

$$\Re^2=1-\frac{\sum V_G}{c^2} \quad \Rightarrow \quad \Re=\sqrt{1-\frac{\sum V_G}{c^2}} \quad .$$

7 Conclusion

The equation of orbital equilibrium $m g=m v^2/r$ gives us the gravitational potential neutralization $V_G=g r=v^2$ that permits any satellite maintain its orbit neutralizing its weight. This signifies that v^2 is equivalent to a gravitational potential that neutralizes that of the planet. If we take c^2 as an equivalent to a spatial gravitational potential, we can neutralize the matter of any object and transform it in energy by the relation $E=m c^2$, so that the matter transforms in energy in proportion to its velocity squared. Then, the cause of relativistic correction is the transformation of matter in energy, its immaterialization, and it is a gravitoinertial phenomenon.

Relativistic corrections for distance and time are given by the Lorentz factor $\gamma=1/\sqrt{1-v^2/c^2}$, while for mass is given by $\gamma^2=1/\left(1-v^2/c^2\right)$. The gravitational relativistic correction (for non-inertial frames) is related with gravitational potential neutralization as is done with velocity squared (for inertial frames), so it may be acquired by the complete gravitational forces that an accelerated system is submitted, because acceleration is like a gravitational field.

Then we can have a complete relativistic correction factor $\Re$, including radial and angular velocities and accelerations into one equation in the form of gravitational potentials $V_G=r\ddot{r}+\dot{r}^2+2\dot{r}r\dot{\theta}+r^2\dot{\theta}^2+r^2\ddot{\theta}$. As c^2 is equivalent to a spatial gravitational potential, dividing the sum of gravitational potentials that an object is submitted by c^2 gives us the final correction factor:

$$\Re=\sqrt{1-\frac{r\ddot{r}+\dot{r}^2+2\dot{r}r\dot{\theta}+r^2\dot{\theta}^2+r^2\ddot{\theta}}{c^2}} \quad \Rightarrow \quad \Re=\sqrt{1-\frac{\sum V_G}{c^2}} \quad .$$

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