the general science journal

The General Science Journal. March 2023.

On The Incompleteness of Special Relativity 1/3

Antonio León Sánchez

Retired Professor. Independent researcher in the foundations of science.

Abstract.-This article introduces the concept of distancing between two moving physical objects that are approaching each other, or moving away from each other. The sign of this distancing is defined as positive if both objects are moving away from each other and negative if they are moving toward each other. It is then formally proved, within the framework of special relativity, the absolute nature of the sign so defined: it is the same for all inertial reference frames. The problem is that it is also proved, within the same frame, that this sign is not always the same in different inertial reference systems. We are then faced with a duality: either the theory of relativity is incomplete and certain physical processes, such as the distancing of two moving objects, fall outside its scope of study, or else the theory is inconsistent. In the most favorable case of incompleteness, we would have to admit that special relativity cannot analyze one of the most common mechanical processes in the observable universe: the distancing between its moving objects.

Keywords: Distancing, universal distancing, absolute distancing sign, absolute space, Lorentz Transformation, local simultaneity, special relativity, formal incompleteness.

1. Introduction

The first Pythagoreans defended the idea of a discrete space formed by extensive points [37, p. 11-16], reason for which all lengths and all distances would have to be commensurable with each other, simply because all of them would be integer multiples of the minimum and indivisible extension of a point [27]. As is well known, the discovery of the non-commensurability of the side of a square with its diagonal put an end (not without some obscurantism) to this first discrete abstraction of space. Although this geometrical space did not yet have physical implications, "*apart from serving as the limiting agent between different bodies*" [19, p. 9]. The idea of physical space (*pneuma apeiron*) was still confused with the idea of matter, and it was also common to confuse the void with the air [3].

From those distant times of the early Pythagoreans to the present day, a good part of philosophers, theologians and scientists have discussed about the nature of space (and time), discussions about which there is an abundant and excellent literature (for example [19, 49, 17, 40, 36, 5, 34, 41, 35, 12, 42, 39, 57, 38]). As could not be otherwise, the controversies about the nature of space and time have always been present in the history of science, and some of them, such as the Newton-Leibniz-Huygens controversy, were and still are very well known [15, 18, 17, 51, 7], [55, p. 143-161], [56, p. 158-228], [19, p. 95-126].

Since more than a century ago, and under the absolutely dominant perspective of the special and general theories of relativity, the intensity of such discussions has considerably diminished. Indeed, today the conviction dominates that neither space nor time are physically real, but mere illusions; relational instruments useful to explain the geometrical and chronological order of physical objects and processes. Although at the same time it is assumed that space can be extended, deformed, vibrate and be the transmitting medium of its own vibrations (gravitational waves). At least in principle, this seems somewhat contradictory: how can something that does not exist expand, deform, vibrate and be the transmitting medium of its own vibrations? [30, 24, 25].

As far as I know, the physical magnitude introduced in this article has never been dealt with in the discussions on the nature of space, nor has it ever been the object of study of the theories (special and general) of relativity. As will be seen in the next section, it is a magnitude with the dimensions of a velocity because it is defined in terms of uniform changes in space with time. But it is not a velocity, it is the temporal variation of the distance between two objects moving in a certain way. The sign of that variation is universal, absolute, the same for all inertial reference frames, which does not seem to fit very well with the relational, relativistic current notion of space, now considered a simple relational and unreal entity. As we will see here, this universal observable poses serious problems for the Lorentz Transformation (LT from now on) and, consequently, for the theory of special relativity.

2. Absolute physical distancing

Although the conditions of the discussion that follows can be extended to more general cases, it will not be necessary to do so in order to achieve the end pursued in that discussion: to demonstrate the existence of absolute observables related to space, time and motion that cannot be explained by the theory of special relativity. The restriction of the conditions only simplifies the scope of the discussion, but does not alter its general objectives and conclusions.

Let, then, A and B be any two physical objects that when observed from the inertial reference frame RF_0:

1. They are separated by a distance $d_0 > 0$
2. They move with the same unknown uniform velocity along the X_0 axis of RF_0.
3. Either they move towards each other so that they will eventually collide, or they or move away from each other, so that they will never collide.
4. The direction from A to B is the direction of the increasing axis X_0 of RF_0

For simplicity, we will write distance between A and B to express the Euclidean distance between a given point of A and a given point of B. Under these conditions, let us begin by proving the following:

Proposition 1 *If A and B approach each other in RF_0, then they will also approach each other in any other inertial reference frame. And if A and B move away from each other in RF_0, they also move away from each other in any other inertial reference frame.*

Proof.-Suppose that in RF_0 the objects A and B approach each other. They will end up colliding with each other. At the instant of their collision they will be separated by a zero distance, that according to LT will be also a zero distance in al inertial reference frames, which is only possible if A and B approach each other in all reference frames. On the other hand, if A and B move away from each other they will never be at a zero distance in RF_0, so that A and B do not collide in RF_0, nor, for the same above reason, in any other inertial reference frame, which is only possible if in all of them they move away from each other. □

Comment: The successive positions of A and B before colliding are the causes of the final effect of their collision. Since the chronological order of the cause-effect relations are preserved by the theory of special relativity in all reference frames, it is evident that preservation is another proof of the above proposition.

Let us now define the distancing D between A and B as the rate of variation with time of the Euclidean distance d between A and B. So, in RF_0 the distancing D_0 will be given by:

$$D_0 = \frac{d_{02} - d_{01}}{t_{02} - t_{01}} = \frac{\Delta d_0}{\Delta t_0};\ \Delta t_0 > 0 \tag{1}$$

where d_{01} and d_{02} are the Euclidean distances between A and B measured with the appropriate sensors in RF_0 at the respective instants t_{01} and t_{02} of the RF_0 time, being $t_{01} < t_{02}$. While Δt_0 is always positive, Δd_0 can be positive (if A and B move away from each other), or negative (if A and B approach each other). Note that, though the distancing D is not a velocity, it is a physical magnitude with the same dimensions as a velocity. And unlike all known physical magnitudes defined so far, it is a physical magnitude that can vary faster than the speed of light. In fact, if A and B were two photons, their distancing would be twice the speed of light.

According to the above Proposition 1, if in the frame RF_0 the objects A and B approach each other, in any other reference frame RF_v they also approach each other; and if in RF_0 the objects A and B move away from each other, in RF_v they also move away from each other. This universal nature of distancing allows us to define its sign σ according to:

$$\sigma = \frac{D_0}{|D_0|}. \tag{2}$$

where $|D_0|$ is the absolute value of D_0. Therefore either $\sigma = +1$ or $\sigma = -1$. It is then immediate to prove the following:

Proposition 2 *Any two physical objects can move in such a way that their motion defines an absolute indicator: the sign of their distancing, whose value, either $+1$ or -1, is the same in all inertial reference frames.*

Proof.-It is an immediate consequence of Proposition 1: if $\sigma = +1$ in RF_0, then A and B are separating from each other in RF_0, and in all inertial reference frames. So, $\sigma = +1$ in all of them. For the same reason, if $\sigma = -1$ in RF_0, then A and B are approaching each other in RF_0 and in all inertial reference frames. Therefore in all of them $\sigma = -1$. □

Evidently, Proposition 2 also applies to any other pair of physical objects moving according to the above conditions (and according to many others conditions not examined here). The distancing sign σ is, therefore, an absolute indicator of how the amount of space between two moving objects is changing: if $\sigma = +1$ the space between them is increasing in all inertial reference frames; if $\sigma = -1$ the space between them is decreasing in all inertial reference frames.

Up to this point, we have defined a physical magnitude, the temporal variation of the space between two moving physical objects. A variation that can be greater than the speed of light. In addition, that magnitude also has an absolute property whose value is the same for all inertial reference frames. We have termed that magnitude distancing (D), and its absolute property its sign (σ).

3. Absolute distancing and local simultaneity

Consider again the above inertial reference frame RF_0 and the two moving physical objects A and B of the previous section, objects that, from the perspective of RF_0, we will now assume are approaching each other along the X_0 axis of RF_0 (the argument that follows would be the same if both objects were moving away, and were moving away in any other direction). According to (1) and (2), the corresponding sign of the distancing between A and B will be $\sigma = -1$. And it will be the same for all inertial reference frames (Proposition 2).

To measure the distancing (1) between two physical objects that moves along the X_0 axis of RF_0 each with the same unknown velocity, it is necessary to measure the distance between those two moving objects in two successive instants of time. Now then, for each pair of moving physical objects, such as the above A and B, and any inertial reference frame, it is evident that A is at a distance d from B at a certain instant t if, and only if, B is at the same distance d from A at the same instant t. Thus, to measure the distance between two moving objects implies the consideration of two simultaneous events occurring in two different places of the inertial reference frame in which that distance is being measured. In our case, in which both objects A and B move along the X_0 axis of RF_0 each with the same unknown uniform velocity, if when measuring the distance between A and B at the instant t_{01} of RF_0 time, (the given point P_a of) A is at the point $(x_{0a}, 0, 0, t_{01})$ and (the given point P_b of) B is at point $(x_{0b}, 0, 0, t_{01})$, the two simultaneous events in RF_0 at which the distance between A and B is measured will be:

Event e_1: P_a is at point $(x_{0a}, 0, 0, t_{01})$.

Event e_2: P_b is at point $(x_{0b}, 0, 0, t_{01})$.

Let now RF_v be an inertial reference frame that coincides with RF_0 at a certain instant and from whose perspective RF_0 moves with a uniform velocity $v = k_v c$, $0 < k_v < 1$, parallel to the X_0 axis of RF_0. According to LT, the events e_1 and e_2, that are simultaneous in RF_0, are not simultaneous in RF_v: the position of A is measured a time $\gamma d_{01} k_v / c$ before the position of B is measured (d_o is the proper distance between A and B at t_{01}, i.e. $x_{0b} - x_{0a}$). The same conclusion applies when measuring for the second time in RF_0 the distance between A and B at a posterior instant t_{02}, in order to determine the distancing (1) between A and B.

Consequently, the measurement of the distancing (1) between two moving objects made in an inertial reference frame is only valid in that inertial reference frame, and (once multiplied by γ^{-1}) in those whose relative velocity is perpendicular to the direction in which the distancing is being measured. In any other inertial reference frame, as in RF_v, the measurements carried out in RF_0 will be <u>invalid</u> because the distances between the moving A and B are observed to be measured for positions of A and B that are not simultaneous.

Therefore, to calculate the distancing between A and B in RF_v using, via LT, the measurements made in RF_0, additional calculations will have to be made. First the velocity $u_o = k_a c$, $0 < k_a \leq 1$, of A with respect to RF_o will have to be calculated and then the distances between A and B measured in RF_o will have to be corrected according to this velocity and to the lack of simultaneity in the determination of the positions of A and B. The calculations would be as follows:

The first measurement of the distance between A and B performed in RF_o is d_{01}. But from the perspective of RF_v the position of A has been measured for a time:

$$\frac{\gamma d_{01} k_v}{c} \tag{3}$$

before the position of B. During that time, A moved with a velocity $k_a c$ and traveled a distance:

$$\frac{\gamma d_{01} k_v}{c} k_a c = \gamma d_{01} k_v k_a \tag{4}$$

which will have to be subtracted from the distance $\gamma^{-1} d_{01}$ given by LT from the distance d_{01} measured in RF_o. Therefore, in RF_v the distance d_{v1} between A and B if the positions of A and B had been measured simultaneously would be:

$$d_{v1} = \gamma^{-1} d_{01} - \gamma d_{01} k_v k_a \tag{5}$$

Note that d_{v1} is not a distance converted by LT from a distance measured in RF_0, but a distance computed directly in RF_v (see caption of Figure 1, right). For the same reasons, in RF_v the second measurement of the distance d_{v2} between A and B will be:

$$d_{v2} = \gamma^{-1} d_{02} - \gamma d_{02} k_v k_a \tag{6}$$

where d_{02} is the distance between A and B measured for the second time in RF_o. According to Proposition 1, and since $d_{01} > d_{02}$, it must hold:

$$d_{v1} > d_{v2} \tag{7}$$

$$\gamma^{-1} d_{01} - \gamma d_{01} k_v k_a > \gamma^{-1} d_{02} - \gamma d_{02} k_v k_a \tag{8}$$

$$d_{01} - \gamma^2 d_{01} k_v k_a > d_{02} - \gamma^2 d_{02} k_v k_a \tag{9}$$

$$d_{01}(1 - \gamma^2 k_v k_a) > d_{02}(1 - \gamma^2 k_v k_a) \tag{10}$$

$$d_{01}(1 - \gamma^2 k_v k_a) - d_{02}(1 - \gamma^2 k_v k_a) > 0 \tag{11}$$

$$(d_{01} - d_{02})(1 - \gamma^2 k_v k_a) > 0 \tag{12}$$

And considering that $d_{01} - d_{02} > 0$, it will have to be:

$$(1 - \gamma^2 k_v k_a) > 0 \tag{13}$$

$$1 > \gamma^2 k_v k_a \tag{14}$$

$$1 > \frac{k_v k_a}{1 - k_v^2} \tag{15}$$

The problem is that, as illustrated in Figure 1, there is a large number of k_v and k_a values for which this inequality is not verified, which means that for those values the moving objects A and B would be separating from each other from the perspective of RF_v. Alternatively, if A and B are separating from each other, the same steps above (3)-(15) but with the condition $d_{01} - d_{02} < 0$, lead to the inequality:

$$1 < \frac{k_v k_a}{1 - k_v^2} \tag{16}$$

so that in RF_v the objects A and B are also separating from each other. And it happens also here that there is a large number of values for k_v and k_a that do not verify that condition, which means that for those values A and B are approaching each other according to RF_v (Figure 1 again).

Two contradictory results have thus been demonstrated: On the one hand, and according to Proposition 1, if the objects A and B are approaching each other (or moving away from each other) in RF_o, they will do the same in all inertial reference frames. On the other hand, it has just been proved that they do not do the same in all inertial reference frames. Therefore, we will have to accept either that the theory of special relativity is inconsistent, or that it is an incomplete theory and cannot analyze the distancing of two physical objects moving away from each other or approaching each other. Opting for the second alternative, the most favorable for

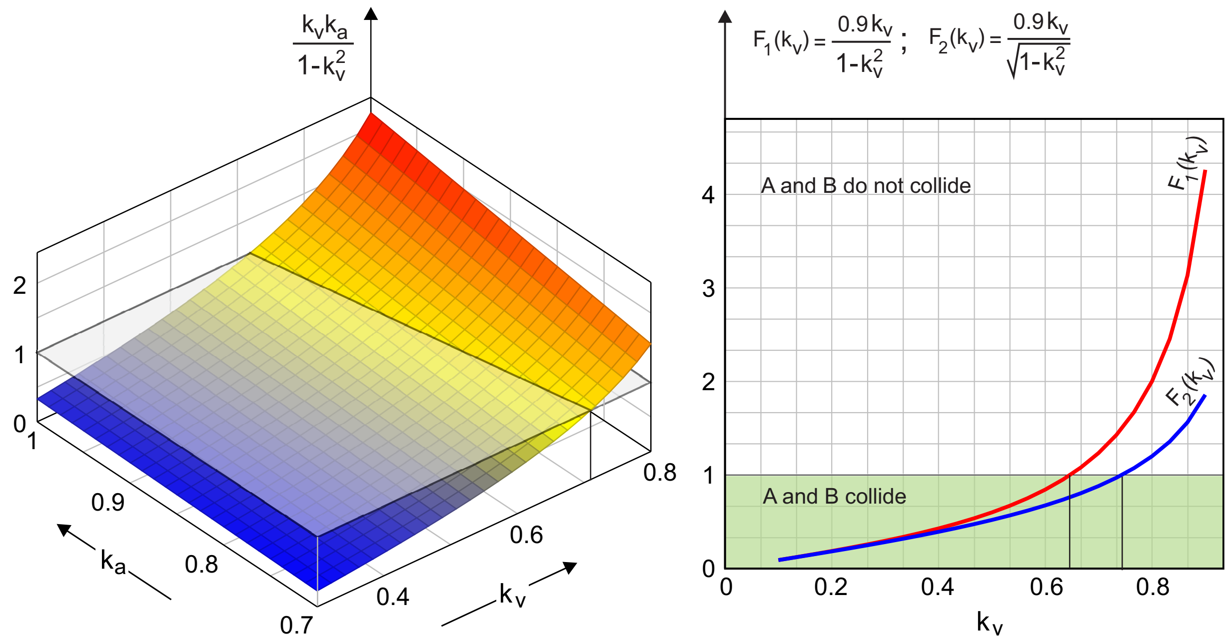

Figure 1 – Left: Surface $k_v k_a/(1-k^2)$ and plain $k_v k_a/(1-k^2) = 1$. All points above that plane represent conditions under which objects A and B would be approaching each other in RF_v, and all points below that same plane represent conditions under which those objects would be separating from each other in RF_v. Right: Functions $F_1(k_v)$ and $F_2(k_v)$ for the particular value of $k_a = 0.9$. Both functions differ in that the second one considers the contraction γ^{-1} of the correction made in RF_v on the position of A so that its measurement is simultaneous with the measurement of the position of B.

the theory, we will have to admit the following:

Theorem 1 (of the Incompleteness of the Special Relativity) *The theory of special relativity is an incomplete theory and cannot analyze the distancing of some moving physical objects.*

This theorem is anything but irrelevant: the distancing between moving physical objects is probably the most common physical process in the universe. Moreover, it is directly observable and measurable in an immense number of real moving objects. So, the magnitude defined by (1) and its sign defined by (2) are plenty of physical meaning.

On the other hand, is not the first time that an argument has been critical of special relativity, and some of them have even demonstrated other aspects of the incompleteness of that theory [20, 31]. But in our days the theories of relativity are immune to criticism because "both theories have been sufficiently confirmed by experience." The possibility that these confirmations are only of apparent deformations, or the consequence of explaining a finite and discontinuous universe with the mathematics of the infinitist continuum, is not even considered. The remaining question is, will these critical arguments ever be considered?

As suggested above, the consideration of absolute motion through absolute space suffices to formally deduced the absoluteness of the distancing sign: simultaneity would always be absolute, not local. From this perspective, all the pieces seem to fit together: absolute motion would be the only real physical motion, although it would not be detectable due to the preinertia of all physical objects [32]. Only relative motions (consequences of the different real absolute velocities) would be detectable. If, in addition, that absolute space is discrete (and if it were not discrete it would be inconsistent [28, 33]), the factor that converts between the continuous and discrete versions of Pythagoras' Theorem (a key theorem in the calculation of distances) is the relativistic Lorentz factor [26], which makes the whole experimental confirmation of the special relativity compatible with the Newtonian view of physical space.

We will end with a brief comment on the incompleteness of science and scientific theories, to place in that context the Incompleteness Theorem of Special Relativity just proved above. Long before Gödel established his famous incompleteness theorems, of which there is an overwhelming primary and secondary literature (for instance, [4, 45, 50, 16, 47, 54, 9, 52, 10, 6, 43, 44, 53, 48, 11, 8, 2, 14, 46, 13, 52]), Aristotle had already discovered the problems posed by the infinite regress of arguments [1, I.3], which can be extended to definitions and causes [29]. These three limitations are much deeper and more severe than those derived from Gödel's theorems, although usually no attention is paid to them, at least not as much as is paid to the famous Gödel's incompleteness theorems, which after all only refer to and are based on a very debatable notion: self-reference (the alleged ability of statements to refer to themselves) [22, 21], while the incompleteness based on the Aristotelian infinite regress is now founded on a hardly questionable principle [23, 28]:

> **Principle of the Directional Evolution**: The universe as a whole evolves always in the same direction of increasing its entropy.

from which an even less questionable statement is formally derived [28]:

> **Theorem of the Formal Dependence**: Concepts do not define themselves; statements do not prove themselves; and physical objects cannot be the cause of themselves.

The incompleteness of special relativity demonstrated above does not refer to a debatable and dispensable concept, such as self-reference, but to a universal, observable and measurable physical process, which, moreover, is continuously occurring in our universe: the changing distances between its objects caused by their incessant motions. It is, therefore, a fundamental process that is outside the formal competence of the theory of special relativity, a theory of spacetime that pretends to explain motion and its spatio-temporal consequences. On the other hand, the famous experimental confirmation of special relativity could also be the experimental confirmation of apparent deformations of space and time, such as refractive deformations. Or it could be the consequence of insisting on explaining the universe with the infinitist mathematics of the continuum, when the universe is essentially finite and discontinuous, i.e. of a discrete nature.

Bibliography and References

[1] Aristotle. *Posterior Analytics*. Kessinger Publishing LLC, Whitefish, MT, 2004.

[2] George Boolos. New proof of the gödel incompleteness theorem. *Notices of the American Mathematical Society*, 36:388–390, 1989.

[3] John Burnet. *Early Greek Philosophy*. A. and C. Black, London, 3 edition, 1920.

[4] Gregory Chaitin. Computational complexity and Gödel's incompleteness theorem. *Notices of the American Mathematical Society*, 17:672, 1970.

[5] Alain Connes. On the fine structure of spacetime. In *On Space and Time*, pages 196–237. Cambridge University Press, New York, 2008.

[6] John W. Dawson. Gödel and the Limits of Logic. *Scientific American*, 280(6):76–81, June 1999.

[7] Robert DiSalle. Space and Time: Inertial Frames. In Edward N. Zalta, editor, *The Stanford Encyclopedia of Philosophy*. Metaphysics Research Lab, Stanford University, Winter 2020 edition, 2020.

[8] Solomon Feferman. Kurt Gödel: Conviction and Caution. In S. G. Shanker, editor, *Gödel's Theorem*, chapter V, pages 96–114. Routledge, London, 1991.

[9] Torkel Franzén. *Gödel's Theorem. An Incomplete Guide to its Use and Abuse*. A K Peters Lts., Wellesley, MA, 2005.

[10] Harry J. Gensler. *Gödel's Theorem Simplified*. University Press of America, Lanham, MD, 1984.

[11] Rebecca Goldstein. *Incompleteness. The Proof and Paradox of Kurt Gödel*. W. W. Norton and Company, New York, 2005.

[12] F. Gómez Camacho. *Espacio y tiempo en la Escuela de Salamanca*, chapter El tratado sobre la composición del continuo de Juan de Lugo S.J., pages 41–78. Ediciones de la Universidad de Salamanca, Salamanca, 2004.

[13] Kurt Gödel. Über formal unentscheidbare Sätze der Principia Mathematica und verwandter Systeme. I. *Monatshefte für Mathematik und Physik*, 38:173–198, 1931.

[14] Kurt Gödel. *Obras completas*. Alianza, Madrid, 1989.

[15] Carl Hoefer, Nick Huggett, and James Read. Absolute and Relational Space and Motion: Classical Theories. In Edward N. Zalta and Uri Nodelman, editors, *The Stanford Encyclopedia of Philosophy*. Metaphysics Research Lab, Stanford University, Spring 2023 edition, 2023.

[16] D. R. Hofstadter. *Gödel, Escher, Bach. Un Eterno y Grácil Bucle*. Tusquets, 1987.

[17] Nick Huggett. *Space from Zeno to Einstein*. MIT Press, Cambridge, Massachusetts, 2002.

[18] Nick Huggett and Carl Hoefer. Absolute and Relational Theories of Space and Motion. In Edward N. Zalta, editor, *The Stanford Encyclopedia of Philosophy*. Stanford Universtiy, fall 2009 edition, 2009.

[19] Max Jammer. *Concepts of Space: The History of Theories of Space in Physics*. Dover Publlcations, Inc., New York, 1993.

[20] A. León Sánchez. Relativity of optical isotropy. *Fundamental Journal of Modern Physics*, 10(1):31–37, 2017.

[21] A. León Sánchez. A critique of selfreference: what Gödel theorem really proves. *The General Science Journal*, pages 1–9, 2021. PDF.

[22] A. León Sánchez. *Paradoxes and theorems*. Amazon's Kindle Direct Publishing, 2021. PDF.

[23] A. León Sánchez. *The Physical Meaning of Entropy*. Amazon's KDP, 2021. PDF.

[24] A. León Sánchez. Towards a discrete cosmology: Paper 11: Physical versus geometrical space. *The General Science Journal*, 2022.

[25] A. León Sánchez. Towards a discrete cosmology: Paper 12: On space deformations. *The General Science Journal*, 2022.

[26] A. León Sánchez. Towards a discrete cosmology: Paper 14: Relativity and discreteness. *The General Science Journal*, 2022.

[27] A. León Sánchez. Towards a discrete cosmology: Paper 4: Discrete versus continuous. *The General Science Journal*, 2022.

[28] A. León Sánchez. Towards a discrete cosmology: Paper 5: A consistent and discrete universe. *The General Science Journal*, 2022.

[29] A. León Sánchez. Towards a discrete cosmology: Paper 8: Infinite regress. *The General Science Journal*, 2022.

[30] A. León Sánchez. Towards a discrete cosmology: Paper 9: Discrete space. *The General Science Journal*, 2022.

[31] A. León Sánchez. *Apparent relativity*. Amazon's KDP, 2022. PDF.

[32] A. León Sánchez. Towards a discrete cosmology: Paper 7: Preinertia. *The General Science Journal*, 2022. PDF.

[33] A. León Sánchez. *Infinity put to the test*. Amazon's KDP, 2023 (2021). PDF.

[34] Michael Lockwood. *The Laberinth of Time*. Oxford University Press, 2007.

[35] Juan de Lugo. *Cómo se puede explicar la composición del continuo por solo indivisibles finitos, según la opinión de los filósofos actuaes*, chapter Las categorías de tiempo y espacio en el pensamiento de la Escolástica tardía, pages 84–106. Ediciones Universidad de Salamanca, Salamanca, España, 2004.

[36] Shahn Majid. Quantum space time and physical reality. In Shahn Majid, editor, *On Space and Time*, pages 56–140. Cambridge University Press, New York, 2008.

[37] Mariano Martínez. *Espacio y tiempo en la Escuela de Salamanca*, chapter Prólogo, pages 11 – 16. Ediciones Universidad de Salamanca, Salamanca, 2004.

[38] Víctor Massuh. *La flecha del tiempo*. Edhasa, Barcelona, 1990.

[39] Tim Maudlin. *Filosofía de la física I. El espacio y el tiempo*. Fondo de Cultura Económica, México, 2014.

[40] Tim Maudlin. *Philosophy of Pysics. Space and Time*. Princeton University Press, New Jersey, 2015.

[41] N. David Mermin. *It's about time. Understanding Einstein's relativty*. Princeton University Press, Princetona and Oxford, 2009.

[42] George Musser. Filosofía del tiempo. *Investigación y Ciencia (Scientifc American)*, (314):14 – 15, Noviembre 2002.

[43] Ernest Nagel and James R. Newman. Gödel Proof. *Scientific American*, pages 71–85, 1956.

[44] Ernest Nagel, James R. Newman, and Douglas R. Hofstadter. *Gödel Proof*. New York University Press, New York and London, 2001.

[45] Ernst Nagel and James R. Newman. *El teorema de Gödel*. Editorial Tecnos, Madrid, 1999.

[46] Janice Padula. The logical heart of a classic proof revisited: A guide to Gödel's'incompleteness' theorems. *Australian Senior Mathematics Journal*, 25(1):32–44, 2011.

[47] Gustavo Ernesto Piñero. *Gödel, Los teoremas de incompletitud*. Grandes Ideas de la ciencia. RBA Editores, Barcelona, 2018.

[48] Panu Raatikainen. Gödel's Incompleteness Theorems. In Edward N. Zalta, editor, *The Stanford Encyclopedia of Philosophy*. Metaphysics Research Lab, Stanford University, Spring 2021 edition, 2021.

[49] Hans Reichenbach. *The Philosophy of Space and Time*. Dover Publications Inc, New York, 1957.

[50] Barkley Rosser. Extensions of some theorems of Gödel and Church. *Journal of Symbolic Logic*, 1(3):87–91, 1936.

[51] Robert Rynasiewicz. Newton's Views on Space, Time, and Motion. In Edward N. Zalta, editor, *The Stanford Encyclopedia of Philosophy*. Metaphysics Research Lab, Stanford University, Spring 2022 edition, 2022.

[52] S. G. Shanker. Wittgenstein's Remarks on the Significance of Gödel Theorem. In S. G. Shanker, editor, *Gödel's Theorem*, chapter VIII, pages 155–256. Routledge, London, 1991.

[53] Peter Smith. *Gödel Without (To Many) Tears*. Logic Matter, 2021.

[54] Raymond M. Smullyan. *Gödel's Incompleteness Theorems*. Oxford University Press, New York, 1992.

[55] Agustín Udías Vallina. *Historia de la Física, de Arquímedes a Einstein*. Editorial Síntesis, 2004.

[56] Eugenio Villar García. *Breve Historia de la Física: sus artífices*. Ediciones Universidad de Cantabria, 2012.

[57] Jeffrey R. Weeks. *The shape of space*. Marcel Derkker INC., 2002.