Rest-Mass Energies Mapped to a General Numerical Structure

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Abstract

This work investigates whether a common numerical structure is recoverable across standard systems used to express rest mass and mass-energy. A hybrid SI and Gaussian framework is constructed in which proton, neutron, and electron mass-energies appear as dimensionless magnitudes. The construction is motivated by the dimensional asymmetry between the electric and magnetic sectors in SI units, together with the dimensional balance present in Gaussian electrodynamics where $\mathbf{E}/\mathbf{B} = 1$.

To examine whether this hybrid framework represents a more general numerical scale, an example normalisation mapping, $\mathcal{S}(x)$, is introduced. For a given input, the mapping takes a particle rest-mass expressed in units including kilograms, joules, atomic units, electron-volts, and naturalised mass ratios. Regardless of the chosen input units, the numerical output remains invariant and aligned to a corresponding hybrid magnitude. From the evidence of this particular mapping, particle rest masses expressed in different unit systems may be compared against a general numerical structure represented by the hybrid system.

Contents

1 Introduction

The present work investigates whether a common numerical structure may be recoverable across standard systems used to express mass and mass-energy. Since conventional unit systems embed different dimensional scaling conventions, recovering such a structure requires a framework capable of comparing alternative systems on a common numerical basis.

To that end, we construct a hybrid SI and Gaussian framework in which proton, neutron, and electron mass-energies appear as dimensionless magnitudes. We then introduce an example normalisation mapping, $\mathcal{S}(x)$, that rescales quantities expressed in alternative unit systems so that their transformed numerical values align with the hybrid framework.

When applied to masses or mass-energies expressed in kilograms, joules, atomic units, megaelectronvolts, and naturalised mass ratios, the mapping consistently returns the same numerical outputs. The resulting numerical structure therefore appears invariant under changes of unit system.

2 Intrinsic properties

Standard dimensional analysis already provides consistent mappings between kilograms, joules, electron-volts, and other conventional mass and mass-energy systems. The problem is therefore not whether alternative unit systems can be converted into one another. To avoid arbitrariness, a fully general structure cannot depend on any particular dimensional convention. For the same reason, the structure cannot be fixed relative to a chosen reference mass, typically the electron, against which all other masses are compared, since this choice is itself arbitrary. However, a purely abstract normalisation may remove conventional scaling factors mathematically while remaining disconnected from the physical processes associated with mass and mass-energy.

What is required instead is a physical process whose quantitative structure is not itself introduced through the choice of unit system. Elementary charge provides one possible example of such an intrinsic property. Charge occurs in discrete units, and the underlying countable structure of charge transport therefore suggests a possible route toward a more general numerical framework. Since electrical resistance relates charge flow to energy transfer, this naturally directs attention toward mass-energy expressed through electron-volts.

However, the measurement procedure must still avoid reintroducing conventional scaling dependencies. Although charge transport occurs discretely, quantities such as amperes and coulombs remain embedded within conventional definitions that determine how many charges define the ampere. What is therefore required is a way of systematically cancelling these dimensional dependencies while preserving the underlying numerical relations within a dimensionless system.

3 Dimensionless Magnitudes

If we find a system based on electrical resistance by which mass may be quantified as a dimensionless magnitude, with all conventional scaling dependencies cancelled, and where the system correctly records a physical process associated with inertia, then dimensionless mass values that are incompatible cannot both be correct. In this sense, it is the uniqueness of such a numerical structure that, if correct, allows it to provide a general scale. It is this possibility that motivates the search for a general structure based on a rescaled volt.

4 SI vs Gaussian units

Voltage appears across two distinct electrical behaviours: electrical resistance and magnetic inductance. In the case of electrical resistance, one volt drives a current of one ampere through a resistance of one ohm. This is the dissipative relation

$$V = IR,$$

where power $P = VI = I^2R$ is converted into heat and cannot be recovered.

There is also inductance L, where voltage is proportional to the rate at which the current changes,

$$V = L\frac{dI}{dt},$$

and where energy is stored in the magnetic field as

$$E = \frac{1}{2}LI^2,$$

while this energy may later be returned to the circuit and is therefore recoverable.

If voltage is to operate coherently across these two contexts, there is then an implicit demand for a common scale between the electric and magnetic sectors. However, a shared scale requires the electric and magnetic fields to share the same set of dimensions. The coulomb, however, belongs to the SI system of units, and this system does not dimensionally balance its electric and magnetic fields. The dimensions of the electric field ($\mathbf{E}$) are $(\mathrm{m} \cdot \mathrm{kg})/(\mathrm{s}^3 \cdot \mathrm{A})$, while the magnetic field ($\mathbf{B}$) has dimensions $\mathrm{kg}/(\mathrm{s}^2 \cdot \mathrm{A})$. Dimensionally,

$$\frac{\mathbf{E}}{\mathbf{B}} = \frac{\mathrm{m}}{\mathrm{s}},$$

and the two fields therefore appear at different scales.

This asymmetry is reflected in the appearance of the constant k in Coulomb's law,

$$F = k\frac{qq'}{r^2},$$

where $k = 1/(4\pi\epsilon_0)$. In the Gaussian system, by contrast, the electric and magnetic fields are expressed without the need to offset Coulomb's law with an additional constant,

$$F = \frac{qq'}{r^2}.$$

Unlike the SI system, the Gaussian fields are dimensionally balanced such that $\mathbf{E}/\mathbf{B} = 1$.[1, 2] However, the simpler algebraic form comes at the cost of the Gaussian system introducing its own conventions, foremost among them that charge becomes a derived quantity rather than a primitive unit directly comparable to the elementary charge e.

The dimensional balance of the Gaussian fields is an important property we wish to retain, while also anchoring the system to a count of primitive charges. Since the coulomb is not a derived SI unit, we may instead attempt a hybrid framework in which this quantity is allowed to float, thereby rescaling the SI system without disturbing its mechanical dimensions.

5 Hybrid system

The Gaussian analogue of the volt is the statvolt. The standard conversion between volts and statvolts is

$$1\,\mathrm{statvolt} = \frac{299{,}792{,}458}{10^6}\,\mathrm{volts}.$$

The numerator 299,792,458 corresponds to the numerical value of the speed of light in SI units. If we rescale the volt, whose unit dimensions are $(\mathrm{m}^2 \cdot \mathrm{kg})/(\mathrm{s}^3 \cdot \mathrm{A})$, then we also rescale the amperage. However, the aim is not a simple rescaling. By rescaling the volt alone, our hybrid system still retains the SI field dimensions where the electric field carries the additional dimensions m/s. We also require $\mathbf{E}/\mathbf{B} = 1$. The unit for electric potential difference must therefore first be normalised relative to the magnetic sector. The simplest way to achieve this is through the magnetic quantities H and L.

In SI, magnetic field strength satisfies

$$H = \mathrm{A/m},$$

and inductance has dimensions

$$L = \frac{\mathrm{kg} \cdot \mathrm{m}^2}{\mathrm{s}^2 \cdot \mathrm{A}^2}.$$

Their product therefore has dimensions

$$H \cdot L = \frac{\mathrm{kg} \cdot \mathrm{m}}{\mathrm{s}^2 \cdot \mathrm{A}}.$$

We then rescale electrical resistance with relative to inductance. Dividing voltage by $H \cdot L$ yields

$$\frac{V}{H \cdot L} = \frac{\mathrm{kg} \cdot \mathrm{m}^2}{\mathrm{s}^3 \cdot \mathrm{A}} \cdot \frac{\mathrm{s}^2 \cdot \mathrm{A}}{\mathrm{kg} \cdot \mathrm{m}} = \frac{\mathrm{m}}{\mathrm{s}}.$$

A rescaled hybrid electric potential difference is then

$$\mathrm{V} \times \frac{299\,792\,458\,\mathrm{V}}{10^6\,H \cdot L} = \frac{299\,792\,458}{10^6}\,\mathrm{m\,s^{-1}}.$$

For a normalised and dimensionless hybrid volt, V^0, we then divide by c:

$$V^0 \;=\; \mathrm{Volt} \times \frac{1}{10^6}.$$

The remaining 10^6 factor is a consequence of historical unit conventions. The Gaussian system is built on centimetre–gram–second (cgs) mechanical units, whereas SI is based on metre–kilogram–second (mks) units. Since

$1\,\mathrm{m} = 100\,\mathrm{cm}$, each occurrence of length in a dimensional expression introduces a factor of 10^2 when converting between the two systems. Electrical quantities are ultimately defined through mechanical quantities such as force and energy, so these powers of 10^2 propagate through the definitions, leaving a factor of 10^6 in the volt-to-statvolt conversion.

The resulting hybrid magnitudes align closely with mass-energies already familiar from MeV scaling. Rounded, the hybrid values are going to be

- Proton hybrid inertial magnitude: 938.272 089 431 508,
- Neutron hybrid inertial magnitude: 939.565 421 765 435,
- Electron hybrid inertial magnitude: 0.510 998 950 692 413.

However, these numerical values are now dimensionless.

6 Dimensionless normalisation mapping

The hybrid system becomes more plausible as a general numerical scale if there exists a normalisation map,

$$\mathcal{S}_{unit}(x; p, n, e) \;\mapsto\; x_{hybrid},$$

able to rescale quantities expressed in different unit systems to the numerical values of the hybrid magnitudes. These units could be kilograms (kg), joules (J), atomic units (u), electron-volts (eV), or mass ratios typically denominated in electron masses. The mapping $\mathcal{S}_{unit}(x)$ then rescales x, while the parameters $p_{unit}, n_{unit}, e_{unit}$ are the proton, neutron, electron mass/mass-energies given in the same units as the input. In each case the mapping yields the same invariant numerical value associated with the hybrid system.

An example of such a mapping relies on the constants $\boldsymbol{\rho}$ and $\boldsymbol{k}$,

$$\boldsymbol{\rho} = \frac{3\pi}{2},$$

$$\boldsymbol{k} = \frac{\boldsymbol{\rho}}{\boldsymbol{\rho} - 1},$$

and takes the form

$$\mathcal{S}(x) = \frac{\boldsymbol{k}x}{\dfrac{p}{\boldsymbol{\rho}} + \dfrac{p\boldsymbol{k}}{\boldsymbol{\rho}^2} + \dfrac{ne\boldsymbol{k}}{3n - 3p} - \dfrac{2e\boldsymbol{k}^2 - 6p\boldsymbol{k}^3 + 11n\boldsymbol{k}^3}{5\boldsymbol{\rho}}}.$$

When a particle mass or mass-energy is used as the input x, it does not matter whether the input values are given in kg, J, u, or eV. The resulting outputs remain invariant within the stated uncertainties when the $\mathcal{S}(x)$ input/parameters adopt the following fixed set of ratios.

Electron mass ratios scaled to hybrid magnitudes using $\mathcal{S}_{M/\mathrm{M_e}}(x)$:

	CODATA (2022)	$\mathcal{S}_{M/\mathrm{M_e}}$ input/parameters	$\mathcal{S}_{M/\mathrm{M_e}}$ output
p	1836.152 673 426 (32)	1836.152 673 425 516	938.272 089 43
n	1838.683 662 00 (74)	1838.683 661 663 699	939.565 421 77
e		1	0.510 998 950 69

Scaling factor = 0.510 998 950 692 413.

To offer partial illumination as to how this particular mapping works, the $\mathcal{S}_{M/M_e}(x)$ numerator and denominator resolve as

$$\mathcal{S}_{M/M_e}(x) = \frac{1.26936832462432x}{2.48409184188011} \Rightarrow 0.510998950692x.$$

Algebraically this corresponds to

$$\mathcal{S}_{M/M_e}(x) = \frac{\boldsymbol{k}x}{\boldsymbol{k}\left(\dfrac{n_{in} - p_{in}}{n_{\mathcal{S}} - p_{\mathcal{S}}}\right)}$$

$$= \left(\frac{n_{\mathcal{S}} - p_{\mathcal{S}}}{n_{in} - p_{in}}\right) x,$$

where n_{in} and p_{in} are the neutron and proton input values, while $n_{\mathcal{S}} = 939.565\,421\,765\,434$, and $p_{\mathcal{S}} = 938.272\,089\,431\,508$. These derived $\mathcal{S}$ numbers are themselves dimensionless. In effect $\mathcal{S}(x)$ numerically rescales the input value within its own set of units. In the case of $\mathcal{S}_{M/M_e}(x)$ the inputs are already dimensionless.

$\mathcal{S}(x)$ then works due to its denominator's complex mix of $n, p, \boldsymbol{\rho}, \boldsymbol{k}$ parameters that scale the input neutron-proton separation $(n_{in} - p_{in})$ to the separation associated with the hybrid values. In other words,

$$\frac{p_{in}}{\boldsymbol{\rho k}} + \frac{p_{in}}{\boldsymbol{\rho}^2} + \frac{n_{in} e_{in}}{3n_{in} - 3p_{in}} - \frac{2e_{in}\boldsymbol{k} - 6p_{in}\boldsymbol{k}^2 + 11n_{in}\boldsymbol{k}^2}{5\boldsymbol{\rho}} = \frac{n_{in} - p_{in}}{\boldsymbol{n_{\mathcal{S}}} - \boldsymbol{p_{\mathcal{S}}}}.$$

For alternative sets of units the input values n_{in}, p_{in}, e_{in} change accordingly but the numerical values for $\boldsymbol{n_{\mathcal{S}}}$ and $\boldsymbol{p_{\mathcal{S}}}$ are derived value and invariant, thus there is no need for manual intervention. It is only the dimensions that change in line with the input dimensions. Thus, when the input values are given in MeV the result is the physical quantity

$$\frac{\boldsymbol{n_{\mathcal{S}}} - \boldsymbol{p_{\mathcal{S}}}}{n_{in} - p_{in}} = 1\,\text{MeV}.$$

The mapping in effect tunes the input system to numerically agree with the dimensionless hybrid system, with outputs always in the dimensions of the input. Different unit systems are then operationally aligned to the hybrid system. To retrieve the dimensionless hybrid magnitude simple divide the result by 1 unit of whatever system of units is being scaled.

In the case of MeV, which should represent the simplest one-to-one alignment, we get the following results.

Million of electron-volts scaled to hybrid magnitudes using $\mathcal{S}_{\text{MeV}}(x)$:

	CODATA (2022)	$\mathcal{S}_{\text{MeV}}$ input/parameters	$\mathcal{S}_{\text{MeV}}$ output
p	938.272 089 43 (29)	938.272 089 431 508	938.272 089 43
n	939.565 421 94 (48)	939.565 421 765 434	939.565 421 77
e	0.510 998 950 69 (16)	0.510 998 950 692 413	0.510 998 950 69

Scaling factor = 1.

Here the proton and electron input/parameter values have been fine tuned in the ninth and tenth decimal to allow a precise mapping. However, the neutron output value 939.565 421 77, compared to the CODATA adjustment 939.565 421 94, is noticeably smaller at the seventh decimal. This does not quite meet with the idea of one-to-one alignment. Although this downward adjustment corresponds to less than one fifth of an electron-volt and is within standard uncertainty, it indicates this particular mapping preserves a specific numerical structure that happens to align reasonably well with observation.

This downward adjustment carries across the $\mathcal{S}$ mappings for atomic units, kilograms, and joules. In each case the input/parameter adjustment is always within 99.99999998% of the CODATA (2022) value.

Atomic units scaled to hybrid magnitudes using $\mathcal{S}_{\mathrm{u}}(x)$:

	CODATA (2022)	$\mathcal{S}_{\mathrm{u}}$ input/parameters	$\mathcal{S}_{\mathrm{u}}$ output
p	1.007 276 466 5789 (83)	1.007 276 466 578 8506	938.272 089 43
n	1.008 664 916 06 (40)	1.008 664 915 876 3447	939.565 421 77
e	0.000 548 579 909 0441 (97)	0.000 548 579 909 0441	0.510 998 950 69

Scaling factor = 931.494 103 717 4122.

Kilograms scaled to hybrid magnitudes using $\mathcal{S}_{\mathrm{kg}}(x)$:

	CODATA (2022) [$\times 10^{-27}$ kg]	$\mathcal{S}_{\mathrm{kg}}$ input/parameters	$\mathcal{S}_{\mathrm{kg}}$ output
p	1.672 621 925 95 (52)	1.672 621 925 953 63	938.272 089 43
n	1.674 927 500 56 (85)	1.674 927 500 257 33	939.565 421 77
e	0.000 910 938 371 39 (28)	0.000 910 938 371 39	0.510 998 950 69

Scaling factor = $5.609\,588\,603\,811\,70 \times 10^{29}$.

A final step of unit normalisation is needed for all mappings. In this example, the problem is acute. Without normalisation the mapping leaves the proton over 938 kilograms. Dividing the $\mathcal{S}_{\mathrm{kg}}$ output by 1 kg retrieves the dimensionless magnitude.

Joules scaled to hybrid magnitudes using $S_{\mathrm{J}}(x)$:

	CODATA (2022) [$\times 10^{-10}$ J]	$\mathcal{S}_{\mathrm{J}}$ input/parameters	$\mathcal{S}_{\mathrm{J}}$ output
p	1.503 277 618 02 (47)	1.503 277 618 025 37	938.272 089 43
n	1.505 349 765 14 (76)	1.505 349 764 870 79	939.565 421 77
e	0.000 818 710 578 80 (26)	0.000 818 710 578 80	0.510 998 950 69

Scaling factor $= 6.241\,509\,074\,444\,78 \times 10^{12}$.

7 Summary

While these results are operationally consistent across the choice of input values, without a deeper theoretical setting the equivalence on which this mapping relies, viz.

$$n_{hybrid} - p_{hybrid} = \frac{n_{in} - p_{in}}{\frac{p_{in}}{\boldsymbol{\rho}\boldsymbol{k}} + \frac{p_{in}}{\boldsymbol{\rho}^2} + \frac{n_{in}e_{in}}{3n_{in} - 3p_{in}} - \frac{2e_{in}\boldsymbol{k} - 6p_{in}\boldsymbol{k}^2 + 11n_{in}\boldsymbol{k}^2}{5\boldsymbol{\rho}}},$$

remains ad hoc. However, despite its algebraic complexity, the formulation cannot be further simplified. In particular, the mapping does not work without the constants $\boldsymbol{\rho}$ and $\boldsymbol{k}$. Moreover, the underlying neutron-proton separation,

$$\frac{n}{e} \times e - \frac{p}{e} \times e,$$

is not concealed by the complexity. This is evidenced by the mapping where $e = 1$, yet the resulting scaling factor is 0.510998950692413. The mapping therefore generates this value without requiring it as an input. Despite the simplicity of the underlying neutron-proton separation, the equivalence is non-trivial. Rather, the complexity shows that the algebraic structure is specifically tuned to reproduce $n_{hybrid} - p_{hybrid}$.

Nonetheless, the existence of $\mathcal{S}(x)$ establishes the possibility of a general mapping procedure, positioning this numerical structure as a general dimensionless system against which alternative systems of units may be directly compared.

Bibliography

[1] John D. Jackson. *Classical Electrodynamics*. Wiley, 3rd edition, 1999.

[2] David J. Griffiths. *Introduction to Electrodynamics*. Cambridge University Press, 4th edition, 2017.