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On the Koide Mystery of the Charged Lepton Mass Formula

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Abstract
This paper responds to the challenge posed in Koide's "Challenge to the Mystery of the Charged Lepton Mass Formula" and provides the exact values that enter three "improved Koide formulas". The approach is based on Ruđer Bošković's *Theory of Natural Philosophy* [1], aiming to eliminate the mystical character implied in the title.

Introduction

Let us start with the reasons why the original Koide formula should be modified in order to become ontologically and physically acceptable.

1. Yoshio Koide, in the very title of his paper [2] – ***Challenge to the Mystery of the Charged Lepton Mass Formula*** **– and in the abstract, states**: "*Why the charged lepton mass*

$$(m_e + m_\mu + m_\tau) / (m_e^{0.5} + m_\mu^{0.5} + m_\tau^{0.5})^2 = 2/3 \quad (1)$$

formula is mysterious is reviewed, and guiding principles to solve the mystery are presented. According to these principles, an example of such a mass generation mechanism is proposed, where the origin of the mass spectrum is attributed not to the structure of the Yukawa coupling constants but to a structure of vacuum expectation values of flavor-triplet scalars under Z4×S3 This acknowledges that the formula lacks an ontological foundation – it is merely "mysterious".

2. Experimental measurements and comparisons of quantities that differ enormously, such as the masses of the electron and the tau, are unproductive because they compare masses spanning an extremely wide range. On this matter, Ruđer Bošković remarked in [1, §132]:

"All our ideas, at least those concerning matter, are drawn from the evidence of our senses. Further, our senses have never been able to perceive individual elements that indeed exert forces too small to affect the nerves and thus propagate motion to the brain."

Experiment is but an extension of the senses; thus, only reason can solve Koide's mystery.

Notice that throughout the history of CODATA reports, the Koide formula tends toward the value $q_\tau = 3.1677350 * 10^{-27}$ kg

While m_τ in "Improving the Koide Formula", [4] tends toward $m_\tau = 3.1674852E\text{-}27$ **kg**

In both cases the deviations are smaller than the acceptable measurement uncertainty.

3. The original Koide formula possesses ***permutation symmetry***: it is invariant under any exchange of the electron, muon, and tau masses. On the other hand, my formula for the anomalous magnetic moments of leptons (a_e, a_μ i a_τ),

$$\frac{1}{2\pi\alpha^{-1} * a_e - 1} + \frac{2}{2\pi\alpha^{-1} * a_\mu - 1} + \frac{2}{2\pi\alpha^{-1} * a_\tau - 1} = 1 \qquad (2)$$

exhibits only ***partial permutation symmetry*** – namely between the muon and the tau. The same partial symmetry should be expected for the masses, which are the subject of this work.

4. In the general force diagram of [1, fig 1] there exists a point "***A***", (in my papers it is fundamental particle with a precisely determined mass and distance). We have established that this point is unique: its noncohesion radius equals its cohesion radius (the reduced Compton wavelength). Moreover, this point satisfies the ***allowed range*** given in [3]:

$$\frac{1}{3} < Q = \frac{x_1 + x_2 + x_3}{\left(\sqrt{x_1} + \sqrt{x_2} + \sqrt{x_3}\right)} < 1 \qquad (3)$$

In most of my papers, the value at the virtual point "***A***" is used for reduction. If we define the mass at that point as "***1***", then the lower allowed value Q^- becomes

$$Q^- = \frac{1+1+1}{\left(\sqrt{1} + \sqrt{1} + \sqrt{1}\right)^2} = \frac{3}{9} = \frac{1}{3} \qquad (4)$$

5. We have previously established that for particles of the first generation the noncohesion limit is smaller than the cohesive limit, whereas for the second and third generations the opposite holds. Even a simple formula has been derived for the noncohesion limit,

$$rn = \left(m / m_f\right)^{1/2} \qquad (5)$$

and for the cohesion limit,

$$rc = m_f / m \qquad (6)$$

6. The original Koide formula oversimplifies the situation because all square roots in the denominator represent noncohesion radii – implying that each lepton fills the entire available space. This is not only ontologically unacceptable but also fails to account for electric charge (through its classical radius) and the invariant masses of leptons during processes occurring around them.
7. Since the two limits coincide only at the virtual point "***A***" (my fundamental particle), there a tiny difference must exist between the radii and the noncohesion limit for second generation leptons (with muonium) and third generation leptons (with exotic tauonium). Hence, as we did for the electron, we may determine this difference for the muon and tau.

Original Koide formula

Using the CODATA 2018 values for the electron and muon masses, the original Koide formula gives an ideal tau lepton mass:

$$q_\tau = 3.1677350204*10^{-27}\ \text{kg}$$

We denote this Koide-satisfying mass by q_τ to distinguish it from the true mass m_τ we seek. The resulting uncertainty falls within the CODATA 2018 limits (see the table). We can verify this in an Excel spreadsheet.

Table 1 – Verification of the tau lepton mass (q_τ) using the Koide formula

	CODATA 2018
Koide mass q_τ	3.1677350204E-27
q_τ / m_μ =	16.818061
CODATA T = m_τ / m_μ 16.817(11)	16.817
Within the uncertainty limits, q_τ/m_μ - T =	0.00106
CODATA m_e =	9.1093837015E-31
CODATA m_μ	1.883531627E-28
$(m_e+m_\mu+q_\tau) / (m_e^{0.5}+m_\mu^{0.5}+q_\tau^{0.5})^2 = 2/3$	0.6666666666666

Modified Koide formula

However, for the reasons given in points 2 and 6, we nevertheless determined a more accurate value for the tau lepton mass in [4], where the dominant role is played by the constant ξ_e (related to electric charge) and the additive correction ***s*** – all of which are presented in Table 2 for clarity, easier tracking, and verifiability. From there, equation ***(9)*** yields an ideally accurate formula for the tau lepton mass $\boldsymbol{m_\tau}$ = **3.1674852349*10^{-27} kg**:

By reducing the masses in **(9)** using formula ***(5)***, we obtain a dimensionless formula in which the values in the denominator for the muon and tau are regarded as noncohesion limits, while the reduced value for the electron separates the electric charge from the rest of the electron's role – let us call it the ***Coulombic separator***. This separator, together with the proton, constitutes almost all matter in the universe (except for antimatter, muonium, and exotic tauonium).

To demonstrate this and determine the value of the ***Coulombic separator*** for the electron, we reduce the masses in ***(9)*** and denote the reduced quantities as underlined $\underline{\boldsymbol{m}}$ = ***m/mf***, thereby obtaining in Table 2 the modified Koide formula (***9b***) in dimensionless form. By definition ***(5)***, in the denominator the values for muon and tau are noncohesion radii, while for the electron it is the ***Coulombic separator*** in dimensionless form:

Table 2 – Calculation of the modified Koide formulas

	input values for	**CODATA 2018**
	$t = \log_2(2\pi)$	**2.651496129**
	$cy = e^{2\pi}$	**535.4916555**
	$m_p =$	**1.67262192369E-27**
	$\alpha^{-1} =$	**137.035999084**
	Electron mass **CODATA** $m_e =$	**9.1093837015E-31**
	Muon mass **CODATA** $m_\mu =$	**1.883531627E-28**
	Tau mass (modified) $m_\tau =$	**3.1674852349E-27**
	Derived values	
	mass of the fundamental particle $m_f =$	**1.08862171145E-28**
	radius of the fundamental particle $r_f =$	**3.23130882358E-15**
	classical radius of the electron $r_{ce} =$	**2.81794032619E-15**
	$\Delta p = 2-1/(\mu*\alpha^{-1}+2)$	**1.93506094354**
(7)	$\xi_e = (m_e/m_p*2\pi\acute{\alpha})^{0.5}*2^{(2\Delta p/3)}$	**1.14669171435**
(8)	$s=\pi*(cy/2+t-\Delta p/3)$	**847.4518552**
(9)	$(m_e+m_\mu+m_\tau) / \ (m_e/\xi_e)^{0.5}+m_\mu^{0.5}+ m_\tau^{0.5})^2-1/s = 2/3$	**0.6666666666667**
	reduced masses	
	electron $\underline{m_e}$	**0.00836781**
	muon $\underline{m_\mu}$	**1.73019848**
	tau $\underline{m_\tau}$	**29.09628939**
	reduced radii	
	electron noncohesion $\underline{n_e} = \underline{m_e}^{0.5}$	**0.091475755**
	Coulombic separator $\underline{r_{se}} = \underline{n_e} / \xi_e^{0.5}$	**0.085424543**
	muon noncohesion $\underline{n_\mu}$	**1.315370091**
	tau noncohesion $\underline{n_\tau}$	**5.394097643**
(9b)	Hence: $(\underline{m_e}+\underline{m_\mu}+\underline{m_\tau}) / (\underline{r_{se}}+\underline{n_\mu}+ \underline{n_\tau})^2-1/s$	**0.6666666666667**
	where: $\underline{r_{se}} = (m_e/m_f)^{0.5}/\xi_e^{0.5} =$	**8.5424543132E-02**

Dimensionally the ***Coulombic separator*** is:

$r_{se} = r_f*\underline{r_{se}} = r_f*(m_e/m_f)^{0.5}/\xi_e^{0.5} = \ r_f * rn_e/\xi_e^{0.5} =$	**2.7603307997E-16**
or : $r_{se}= \ r_f * rn_e/\xi_e^{0.5} =$	**2.7603307997E-16**
or : $r_{se} = r_f*\alpha^{0.5} =$	**2.7603307997E-16**
or via the classical radius of the electron: $r_{se}= \xi_e*\alpha^{0.5} *r_{ce}$	**2.7603307997E-16**

So, we conclude:

- ***r_{se}*** (***Coulombic separator of the electron***) – the radius at which the electron transitions to the classical radius, while simultaneously satisfying the modified Koide formula for the electron. It is an inner boundary.
- ***r_{ce}*** (***classical radius of the electron***) – the radius at which the electrostatic energy equals the rest mass of the electron. It is the characteristic scale of electric charge.
- **Material** (**the building of atoms**) takes place at distances larger than ***r_{ce}***. At distances smaller than ***r_{ce}***, but larger than ***r_{se}*** quantum electrodynamic effects dominate, but no stable atomic structure exists.
- At distances larger than ***r_{ce}***, the Coulomb force falls off as **$1/r^2$** and enables the binding of the electron with the proton in the hydrogen atom (Bohr radius ≈ $5.3*10^{-11}$m, far larger than ***r_{ce}***).

The above establishes a connection between the geometric separator (defined via radii) and the modified Koide formula, thereby confirming its role in the model.

Coulombic separators for muon and tau

Analogously to what was presented for the electron – but taking into account reason 3, i.e., ***partial permutation symmetry*** – we determine the Koide formulas with the same previous value for the tau lepton mass

$$m_\tau = 3.1674852349*10^{-27}\ kg$$

For the second and third generations, the coefficients multiplying the muon and tau masses will also reduce the factors, as for the electron, but in the form:

$$m*(1-1/\xi)$$

Thus constructing ***Coulombic separators*** for muon and tau, and this without the additive correction outside the bracket that has already served its purpose for the electron. From this, for the ideal validity of the "***2/3***" Koide formula for the muon, we determine (using CODATA 2018 data as an example:

$$\xi_\mu = \frac{1}{\left\{1-\left[\sqrt{3*\left(m_e+m_\mu+m_\tau\right)/2}-\sqrt{m_e}-\sqrt{m_\tau}\right]^2 \Big/ m_\mu\right\}} = 1.629985343*10^4 \qquad \textbf{\textit{(10)}}$$

Respecting the ***partial permutation symmetry*** for tau and muon:

$$\xi_\tau = \frac{1}{\left\{1-\left[\sqrt{3*\left(m_e+m_\mu+m_\tau\right)/2}-\sqrt{m_e}-\sqrt{m_\mu}\right]^2 \Big/ m_\tau\right\}} = 6.684201036*10^4 \qquad \textbf{\textit{(11)}}$$

Which, upon verification, gives in proposed form:

$$(m_e+m_\mu+m_\tau) / [m_e+m_\mu*(1-1/\xi_\mu)^{0.5}+m_\tau]^2 = 0.6666666666667 \quad \textbf{(10b)}$$

$$(m_e+m_\mu+m_\tau) / [m_e+m_\mu+m_\tau*(1-1/\xi_\tau)^{0.5}]^2 = 0.6666666666667 \quad \textbf{(11b)}$$

And from there, the square of the ratio:

$$(\xi_\tau/\xi_\mu)^2 = 16.81634498385 \quad \textbf{(12)}$$

That is, the first result, which incorporates the properties of the muon and tau into $\mathbf{m_\tau}$, corresponds to the tau/muon mass ratio for the year 2018 (**T** = 16.8170(11) [5]) within the measurement uncertainty limits, since:

$$m_\tau / m_\mu - T = -0.00026$$

$$(\xi_\tau/\xi_\mu)^2 - T = -0.00066$$

The same with the value q_τ is:

$$\xi_{q\mu} = \frac{1}{\left\{1-\left[\sqrt{3*(m_e+m_\mu+q_\tau)/2}-\sqrt{m_e}-\sqrt{q_\tau}\right]^2 \Big/ m_\mu\right\}} = 2.699999777*10^{12} \quad \textbf{(13)}$$

$$\xi_{q\tau} = \frac{1}{\left\{1-\left[\sqrt{3*(m_e+m_\mu+q_\tau)/2}-\sqrt{m_e}-\sqrt{m_\mu}\right]^2 \Big/ q_\tau\right\}} = 1.106535535*10^{13} \quad \textbf{(14)}$$

And from there, the square of the ratio for mass q_τ:

$$(\xi_{q\tau}/\xi_{q\mu})^2 = 16.79589976396 \quad \textbf{(15)}$$

So the question arises: why is the square of the ξ ratio correlated with the mass ratio of tau to muon? Naturally, because all along in the denominator we are dealing with dimensionless noncohesion radii and radii close to them, for which ***(5)*** holds, while the numerator contains the dimensionless masses.

The second one with q_τ does not match, although in the denominator of the second are the noncohesion limits (radii):

$$(\xi_{q\tau}/\xi_{q\mu})^2 - T = -0.0211$$

While from Table 1 the mass ratio corresponded, but also with a larger uncertainty:

$$q_\tau/m_\mu - T = 0.00106$$

Thus, the value m_τ = **3.1674852349*10^{-27} kg** not only yields smaller uncertainties but also underlies essential relations concerning electric charge and phenomena related to leptons, whereas the value q_τ = **3.1677350204*10^{-27} kg** provides a somewhat accurate value through the original Koide formula solely due to the wide diversity of the input masses – lacking any ontological depth.

Given the derivation of three improved Koide formulas in which the input value with the highest uncertainty is the muon mass, the uncertainty for the obtained tau lepton mass follows accordingly, namely for:

$$m_\mu = 1.883531627(42)*10^{-28}\ kg$$

the value for the tau lepton mass is determined as:

$$m_\tau = 3.167485235(73)*10^{-27}\ kg,$$

Interestingly, there exists another formula that gives approximate values for the tau lepton mass and even yields a smaller uncertainty than the value that satisfies the original Koide formula:

$$m_\tau = m_p^{3/2}*m_e^{1/2}*2^{-9/2} = 3.1675041979*10^{-27} kg \qquad (16)$$

It is tied to the proton, while neglecting the muon, thus making it incomplete.

What matters here is that, once again, by applying Bošković's general force curve and my method based on the fundamental particle, we have obtained a tau lepton mass that agrees with experimental values and at the same time linking leptons, their electric charges, and the phenomena they constitute.

Conclusion

The Koide formula has been demystified by splitting it into three formulas for the three leptons, ***(9)***, ***(10b)*** and ***(11b)*** linking it to electric charge and other phenomena.

The notion of "***partial permutation symmetry***" has been introduced, which in this paper describes the situation where the values of the muon and tau masses can be exchanged in the formulas. Here we further emphasize that this is a general property of all parameters of the second and third generations.

We have defined and applied the term "***Coulombic separator***", which separates the electric charge from the rest of the lepton's role.

We have determined the value of the tau lepton with the same precision as that of the muon.

References

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