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The Maxwell-Faraday Equation Extended for Convection

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Abstract. **To show that the curl of E = v×B is an additional convective component to the Maxwell-Faraday equation.**

The Electromagnetic Forces

I. When a charged particle is placed in a magnetic field, it experiences a force under two conditions. If the magnetic field is varying in time, the particle experiences a force,

$$\mathbf{E} = -\partial\mathbf{A}/\partial t \qquad \mathbf{(1)}$$

where **E** is the force per unit charge, and where,

$$\mathbf{B} = \nabla\times\mathbf{A} \qquad \mathbf{(2)}$$

When the charged particle is moving with velocity, **v**, it experiences a deflecting force in the form,

$$\mathbf{E} = \mathbf{v}\times\mathbf{B} \qquad \mathbf{(3)}$$

We will now take the curl of the total electromagnetic force, as in,

$$\nabla\times\mathbf{E} = \nabla\times[-\partial\mathbf{A}/\partial t + \mathbf{v}\times\mathbf{B}] \qquad \mathbf{(4)}$$

From equation **(2)**, this leads to the Maxwell-Faraday equation, but with an additional convective term, $\nabla\times(\mathbf{v}\times\mathbf{B})$, on the right-hand-side, as in,

$$\nabla\times\mathbf{E} = -\partial\mathbf{B}/\partial t + \nabla\times(\mathbf{v}\times\mathbf{B}) \qquad \mathbf{(5)}$$

If we can show that,

$$\nabla\times(\mathbf{v}\times\mathbf{B}) = -(\mathbf{v}\cdot\nabla)\mathbf{B} \qquad \mathbf{(6)}$$

then we will have shown that $\nabla\times(\mathbf{v}\times\mathbf{B})$ is the convective component of the total time derivative,

$$d/dt = \partial/\partial t + (v\cdot\nabla) \quad \textbf{(7)}$$

where the partial time derivative, $\partial/\partial t$, is known as the local time derivative, and where $(\mathbf{v}\cdot\nabla)$ is known as the convective derivative. Hence,

$$\nabla\times\mathbf{E} = -d\mathbf{B}/dt \quad \textbf{(8)}$$

Unlike in the 1982 derivation, **[1]**, this will now be demonstrated by multiplying out the components in full.

The Analysis

II. The analysis begins by multiplying out the components of $\mathbf{v}\times\mathbf{B}$. These are,

$$(v_yB_z - v_zB_y)\boldsymbol{i} \quad \textbf{(9)}$$

$$(v_zB_x - v_xB_z)\boldsymbol{j} \quad \textbf{(10)}$$

$$(v_xB_y - v_yB_x)\boldsymbol{k} \quad \textbf{(11)}$$

We then take the curl of equations **(9)**, **(10)**, and **(11)**, and at this stage it is important to bear in mind that **v** is the velocity of a charged particle and that it is not therefore a vector field. Hence, the spatial derivatives of **v** will all be zero. The result is,

$$[v_x\partial B_y/\partial y + v_x\partial B_z/\partial z]\boldsymbol{i} \quad \textbf{(12)}$$

$$[v_y\partial B_z/\partial z + v_y\partial B_x/\partial x]\boldsymbol{j} \quad \textbf{(13)}$$

$$[v_z\partial B_x/\partial x + v_z\partial B_y/\partial y]\boldsymbol{k} \quad \textbf{(14)}$$

Now we need to take account of the fact that magnetic fields are solenoidal, and that from equation **(2)**, we know that,

$$\nabla\cdot\mathbf{B} = 0 \quad \textbf{(15)}$$

and hence,

$$\partial B_x/\partial x + \partial B_y/\partial y + \partial B_z/\partial z = 0 \qquad \textbf{(16)}$$

As pointed out by Professor Halim Boutayeb, Professor at Université du Québec en Outaouais, in a private correspondence on ResearchGate, we can isolate any of the three terms on the left-hand-side of equation **(16)** and equate it with the negative of the sum of the other two terms. As such, substituting equation **(16)** into **(12)**, **(13)**, and **(14)**, we obtain,

$$-[v_x \partial B_x/\partial x]\boldsymbol{i} \qquad \textbf{(17)}$$

$$-[v_y \partial B_y/\partial y]\boldsymbol{j} \qquad \textbf{(18)}$$

$$-[v_z \partial B_z/\partial z]\boldsymbol{k} \qquad \textbf{(19)}$$

These are the equivalent of $-(\mathbf{v}\cdot\nabla)\mathbf{B}$ which is what we set out to demonstrate. $\nabla\times(\mathbf{v}\times\mathbf{B})$ is therefore the additional convective component required to make the Maxwell-Faraday equation into a total time derivative equation as per equation **(8)**.

The Galilean Transformation

III. The convective term, $\mathbf{v}\times\mathbf{B}$, in electromagnetic induction is not the consequence of a Galilean transformation. The Galilean transformation ignores the existence of the physical medium through which motion is defined and which is responsible for generating both the inertial forces and the electromagnetic forces. This medium is a dense sea of rotating electron-positron dipoles that fills all of space, **[2]**, **[3]**, **[4]**, **[5]**, **[6]**.

Conclusion

IV. The term, $\mathbf{v}\times\mathbf{B}$, which first appeared in the electromotive force equation (*equation (77)*) in Maxwell's 1861 paper, ***"On Physical Lines of Force"***, **[7]**, is the convective term which would complete the Maxwell-Faraday equation, that being the curl of equation (77). This is the deflecting force which acts on a moving charged particle in an already existing background magnetic field, and which is due to the asymmetrical contact with the field. The magnetic field is constructed from tiny rotating electron-positron dipoles, mutually aligned along their rotation axes, and so the component of the particle's motion at right-angles to the field lines, will generate a differential centrifugal pressure on either side of the particle, at right-angles to that component of its motion. This will cause the particle to deflect away from its straight-line path. Meanwhile, the moving particle itself also generates its own superimposed magnetic field which causes an additional **v**×**B** force to press inwards evenly all around it, at right-angles to its direction of motion. This latter force is also accompanied by an asymptotic factor which predicts a terminal speed and hence betrays the inertial resistance that arises due to the presence of the all-pervading electron-positron sea. In order to derive **v**×**B** in this latter context, we need to use a Lorentz transformation and we need to also involve the electrostatic force, $-\nabla\psi$, since it is involved in the construction of the magnetic field lines, **[8]**.

Finally, it's important to note that the Lorentz transformation is not actually a coordinate frame transformation at all, but rather it is an analysis which describes the manner in which a rotating dipole precesses and tends towards a terminal speed as it is linearly accelerated through the electron-positron sea, **[8]**.

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