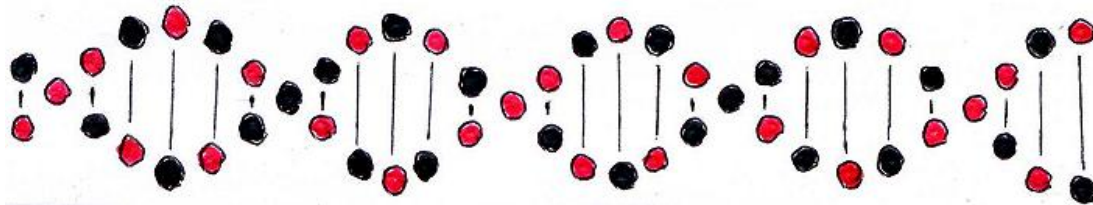


Maxwell's Displacement Current in the Two Gauges

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Abstract. *Displacement current was originally conceived by James Clerk Maxwell in 1861 in connection with linear polarization in a dielectric solid which he believed to pervade all of space. Modern textbooks, however, adopt a different approach. The official teaching today is that displacement current is a consequence of extending the original solenoidal Ampère's Circuital Law to embrace the conservation of electric charge. Yet, unless either of these two methods leads to a displacement current that is related to Faraday's Law of Induction, then it cannot serve its main purpose, which is to provide a connecting bridge between Ampère's Circuital Law and Faraday's Law, hence enabling the derivation of the electromagnetic wave equations. This matter will be investigated in both the Lorenz gauge and the Coulomb gauge.*

Ampère's Circuital Law

I. In its original form, *Ampère's Circuital Law* assumes that an electric circuit will be closed, and that there will be no accumulation of electric charge at any point along the circuital path. Since this condition does not hold in the case of a charging or discharging capacitor, then Ampère's Circuital Law, in its original solenoidal form, cannot hold in such a context. However, when we take into consideration the equation of continuity of electric charge and expand Ampère's circuital law into the extended form,

$$\nabla \times \mathbf{B} = \mu(\mathbf{J} + \varepsilon \partial \mathbf{E}_S / \partial t) \quad (1)$$

where μ is magnetic permeability and ε is electric permittivity, this will maintain consistency with the fact that the divergence of a curl is always zero. This is because the term, \mathbf{E}_S , satisfies Gauss's Law of

electrostatics, and so when we take the divergence of $\epsilon\partial\mathbf{E}_S/\partial t$, this cancels with the divergence of \mathbf{J} , since by Gauss's law of electrostatics,

$$\nabla \cdot \mathbf{E}_S = \rho/\epsilon \tag{2}$$

The fact that this term ensures that the divergence of the right-hand-side of equation (1) remains zero, is a vindication of its applicability in the context of a charging or discharging laboratory capacitor, but the textbooks seldom present a formal derivation of this equation.

Meanwhile, the extra $\epsilon\partial\mathbf{E}_S/\partial t$ term is commonly known as *Maxwell's Displacement Current*, but this is not actually the form of the displacement current that Maxwell used in his 1865 paper, "***A Dynamical Theory of the Electromagnetic Field***", [1], when he first derived the wireless EM wave equation for disturbances in the magnetic field intensity.

Modern textbooks, however, use equation (1) when deriving the EM wave equations, despite the fact that the electrostatic field, \mathbf{E}_S , is not interchangeable with the electric field in Faraday's law, yet this discrepancy has been consistently overlooked, hence creating a gaping omission in modern electromagnetic theory. As for the actual derivation of equation (1) from first principles, as opposed to simply presenting a retrospective justification, we can look to a paper written by Dr. Zhong-Cheng Liang of the Nanjing University of Posts and Telecommunications entitled, "***Dark matter and real-particle field theory***", [2]. The subject matter of this paper is actually more fundamental than electromagnetism since the speed of light has not yet entered the proceedings, but when we do connect his coefficients to the speed of light, it becomes clear that Dr. Liang has, in effect, derived equation (1) above in the ***Lorenz Gauge***. The Lorenz gauge is named after Danish physicist Ludvig Lorenz, who proposed it in 1867, [3], in an attempt to bring Coulomb's law of electrostatics into the domain of wireless EM radiation, but Maxwell was not pleased about this, claiming that Lorenz had entirely missed the point. The Lorenz gauge would however become relevant in later years as the ***Lorentz Condition***, named after the Dutch physicist Hendrik A. Lorentz, in connection with the Lorentz transformations.

Meanwhile, we will now take a closer look at the form of the displacement current that Maxwell himself actually used in his 1865 paper, this being a version which relates to time-varying EM induction, as opposed to electrostatics.

The Speed of Light

II. Although the primary justification for the displacement current term in equation (1) lies in the conservation of charge, this was not the basis upon which James Clerk Maxwell first conceived of the idea. Maxwell first conceived of displacement current in Part III of his 1861 paper “*On Physical Lines of Force*”, [4], in conjunction with an all-pervading sea of aethereal molecular vortices which doubled as a dielectric elastic solid. In 1855, Wilhelm Eduard Weber and Rudolf Kohlrausch, by discharging a Leyden Jar (a capacitor), demonstrated that the ratio of the electrostatic and electrodynamic units of charge is equal to $c\sqrt{2}$, where c is the directly measured speed of light, [5], although it wasn’t until 1857 that the numerical connection to the speed of light was first spotted by fellow German Gustav Kirchhoff, [6]. Whether Maxwell was aware of this or not, in 1861 he looked up Weber’s results in London, and on converting from electrodynamic units into electromagnetic units, he also exposed the speed of light directly, and by comparing Weber’s electric charge ratio with the ratio of the transverse elasticity to the density of his dielectric solid, Maxwell concluded that his dielectric solid is the all-pervading *luminiferous medium* that is responsible for electric, magnetic, and optical phenomena. Meanwhile, back in Part I of the same paper, Maxwell had already derived Ampère’s circuital law hydrodynamically in its original solenoidal form. Then, by employing the concept of linear polarization in his dielectric solid, he presented displacement current, \mathbf{J}_D , in the form $\varepsilon\partial\mathbf{E}_S/\partial t$, where the electric displacement, \mathbf{D} , is equal to $\varepsilon\mathbf{E}_S$, and where the electric permittivity, ε , is inversely related to the dielectric constant. Maxwell then added displacement current to Ampère’s circuital law.

Wireless Electromagnetic Radiation

III. In the preamble to Part III of his 1861 paper, Maxwell says, “*I conceived the rotating matter to be the substance of certain cells, divided from each other by cell-walls composed of particles which are very small compared with the cells, and that it is by the motions of these particles, and their tangential action on the substance in the cells, that the rotation is communicated from one cell to another.*”

Regarding the issue of transverse elasticity mentioned in the section above, Maxwell goes on to talk about rotations being communicated from vortex to vortex by the tangential action of the electric particles that surround the vortices. This was a pioneering attempt at explaining time-varying EM induction, which is central to the EM wave propagation mechanism. However, Maxwell then continues by considering the polarization effect that an externally applied electric field would have on

the electric particles if this medium were a dielectric. It was within this context that Maxwell first conceived the idea of displacement current as a polarization current in connection with transverse elasticity, and should the vortices constitute rotating dipoles, this transverse elasticity would then be tied up with fine-grained torque. Later in Part III, Maxwell went on to link the displacement current with the electrostatic force, and his equation (115) corresponds to Gauss's law of electrostatics at equation (2) in *Section I* above. Meanwhile, Maxwell's equation (112) was his extended version of Ampère's circuital law, exactly as per equation (1) above.

However, in his 1865 paper, [1], when it came to deriving the EM wave equation for disturbances in the magnetic field intensity, the electrostatic field, \mathbf{E}_S , was eliminated, indicating that the correct form of displacement current is actually based on the electric field, \mathbf{E}_K , of time-varying electromagnetic induction. Unlike the electrostatic field, \mathbf{E}_S , which is induced by dielectric polarization, this induction field, \mathbf{E}_K , would be capable of self-propagation through the sea of tiny aethereal vortices that was explained by Maxwell in Parts I and II of the 1861 paper, [4], [7], [8]. Nevertheless, it can still be shown that the EM wave-carrying medium requires a dielectric nature too, when it comes to the transfer of energy between neighbouring vortices, [9].

The Lorenz gauge, which involves the electrostatic potential, ψ , takes the form,

$$\nabla \cdot \mathbf{A} = -\mu\epsilon\partial\psi/\partial t \quad (3)$$

but in order to derive the electromagnetic wave equations, we need the divergence of \mathbf{A} to be zero. This means eliminating the electrostatic potential and approximating the meaning of \mathbf{A} to being the transverse circumferential momentum circulating around the edge of Maxwell's tiny aethereal vortices. We will refer to this fine-grained transverse momentum as, \mathbf{A}_C , and so long as we ignore the cyclical density variations that accompany electromagnetic waves, then the divergence of \mathbf{A}_C will be zero. This approximation is known as the "*the Coulomb gauge*". Maxwell operated in the Coulomb gauge, and it's badly named because it's not about the radial Coulomb force.

So, since the electric field in the displacement current needs to be interchangeable with the electric field in the time-varying component of Faraday's Law of Induction if it is to be used to derive the electromagnetic wave equations, this means that displacement current should take the mathematical form, $\epsilon\partial\mathbf{E}_K/\partial t$, such that,

$$\mathbf{E}_K = -\partial\mathbf{A}_C/\partial t \quad (4)$$

where \mathbf{B} is the vorticity of this circulating current, \mathbf{A}_C , as in,

$$\nabla \times \mathbf{A}_C = \mathbf{B} \quad (5)$$

Then further taking the curl of \mathbf{B} , this expands to,

$$\nabla \times \nabla \times \mathbf{A}_C = \nabla(\nabla \cdot \mathbf{A}_C) - \nabla^2 \mathbf{A}_C \quad (6)$$

In Dr. Liang's paper, [2], equation (B16) has the same form as equations (6), and he has shown how his $-\nabla^2 \mathcal{A}$ corresponds to $\alpha_s \mathbf{J}$, although since equation (6) is in the Coulomb gauge, where \mathbf{A}_C is purely circumferential, then,

$$\nabla(\nabla \cdot \mathbf{A}_C) = \varepsilon \partial \mathbf{E}_S / \partial t = 0 \quad (7)$$

and so, the electrostatic displacement current term, $\varepsilon \partial \mathbf{E}_S / \partial t$, which is written as $-(1/c^2) \partial G / \partial t$ in Dr. Liang's paper, vanishes. Meanwhile, if we equate α_s with magnetic permeability, μ , while equating c with the speed of light, Dr. Liang's equation (B17) becomes equivalent to equation (1), and so equations (1) and (6) both reduce to,

$$\nabla \times \mathbf{B} = \mu \mathbf{J} \quad (8)$$

which is the original solenoidal form. In the solenoidal context of the perimeter momentum of one of Maxwell's tiny molecular vortices, this results in Ampère's circuital law adopting the mathematical form,

$$\nabla \times \mathbf{B} = \mu \mathbf{A}_C \quad (9)$$

with the Coulomb gauge guaranteeing that both sides of the equation will have zero divergence.

If we consider Maxwell's vortices to be dipolar, each comprising of an aether sink (electron) and an aether source (positron), then the induction-based displacement current (in the Coulomb gauge) will be an oscillatory phenomenon tangential or axial to these tiny rotating electron-positron dipoles that fill all of space, and such that during the dynamic state when an EM wave is passing through and the vortices are precessing, pure electric fluid (aether) swirls across from the positron of one dipole into the electron of its neighbour, with this repeating again indefinitely with respect to the next neighbour along the line, [10], [11], [12].

We know from equation (4) that displacement current in this context is equal to $-\varepsilon\partial^2\mathbf{A}_C/\partial t^2$, and so from the oscillatory nature of EM waves, it follows that,

$$\mathbf{A}_C = -\varepsilon\partial^2\mathbf{A}_C/\partial t^2 \quad (10)$$

which means that displacement current is one and the same thing as the circumferential momentum, [13], but with particular relevance to the dynamic state when the electric fluid is swirling between angularly accelerating/precessing vortices, [9]. As such, if we substitute equations (4) and (10) into equation (9), we arrive at the familiar Ampère's circuital law with the displacement current added, this time fully compatible with time-varying EM induction in the form,

$$\nabla\times\mathbf{B} = \mu\varepsilon\partial\mathbf{E}_K/\partial t \quad (11)$$

Maxwell referred to the circumferential momentum as the *electromagnetic momentum* and he identified it with Faraday's *electrotonic state*, yet he never identified it with his displacement current, as he should have done. In modern textbooks, \mathbf{A}_C is referred to as the *magnetic vector potential*.

Conclusion

IV. The *Coulomb Gauge*, also known as the *Transverse Gauge*, or the *Radiation Gauge*, is the divergence formula for the magnetic vector potential, \mathbf{A} , as is used in the derivation of the electromagnetic wave equations, and which helps us to determine the deeper physical nature of wireless radiation in space. While space is densely packed with tiny dipolar vortices in an all-pervading electric fluid, the Coulomb gauge is defined through a single vortex. It is based on treating the magnetic vector potential as the momentum of the transverse perimeter circulation of the electric fluid within this vortex. In the dynamic state when radiation is passing through, these vortices are undergoing an oscillatory angular acceleration/precession, and the *electric fluid* (aether) of which the dipolar vortices are comprised, is being swirled from vortex to vortex, [10], [11], [12]. Ignoring cyclical density variations, the divergence of \mathbf{A} will therefore be zero.

In the case of the *Lorenz Gauge* however, the vector field, \mathbf{A} , is a point function for the value of \mathbf{A} at each vortex in a whole region of vortices, and the formula is time-retarded, [14]. Meanwhile, an electrostatic field is not time-retarded, but it can entrain a region of such vortices. In doing so it causes the vortices to precess about an axis

parallel to the radial electrostatic field lines. This precession increases the intrinsic centrifugal pressure of each vortex according to the field strength at that point, and it is this centrifugal pressure in the electric fluid of the vortices which defines the electrostatic potential term, ψ , for the purposes of the Lorenz gauge. When a charged body begins to linearly accelerate, the precessing vortices in the electrostatic field start to realign into a solenoidal magnetic field and this realignment propagates outwards at the speed of light in the form of electromagnetic radiation. This is what the Lorentz transformations are all about, [15].

References

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The derivation of the electromagnetic wave equation in \mathbf{H} begins on page 497 in the first link below. Note how the electrostatic component, Ψ , is eliminated after equation (68), hence leaving the elastic displacement mechanism in the wave as an effect that is connected exclusively with time-varying electromagnetic induction.

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English translation by Professor A.K.T. Assis, vol. 3, chapter 8. See page 214 regarding the connection between Weber’s constant and the speed of light.

<https://www.ifl.unicamp.br/~assis/Weber-in-English-Vol-3.pdf>

A summary by Professor A.K.T. Assis can be found on pp. 280-282 in this link,

[https://www.ifl.unicamp.br/~assis/Weber-Kohlrausch\(2003\).pdf](https://www.ifl.unicamp.br/~assis/Weber-Kohlrausch(2003).pdf)

[7] Whittaker, E.T., *“A History of the Theories of Aether and Electricity”*, Chapter 4, pages 100-102, (1910)

“All space, according to the younger Bernoulli, is permeated by a fluid aether, containing an immense number of excessively small whirlpools. The elasticity which the aether appears to possess, and in virtue of which it is able to transmit vibrations, is really due to the presence of these whirlpools; for, owing to centrifugal force, each whirlpool is continually striving to dilate, and so presses against the neighbouring whirlpools.”

[8] O’Neill, John J., *“PRODIGAL GENIUS, Biography of Nikola Tesla”*, Long Island, New York, 15th July 1944, Fourth Part, paragraph 23, quoting Tesla from his 1907 paper *“Man’s Greatest Achievement”* which was published in 1930 in the Milwaukee Sentinel,

“Long ago he (mankind) recognized that all perceptible matter comes from a primary substance, of a tenuity beyond conception and filling all space - the Akasha or luminiferous ether - which is acted upon by the life-giving Prana or creative force, calling into existence, in never ending cycles, all things and phenomena. The primary substance, thrown into infinitesimal whirls of prodigious velocity, becomes gross matter; the force subsiding, the motion ceases and matter disappears, reverting to the primary substance”.

<http://www.rastko.rs/istorija/tesla/oniell-tesla.html>

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<http://gsjournal.net/Science-Journals/Historical%20PapersMechanics%20/%20Electrodynamics/Download/4105>

See pp. 6-7 in the pdf file in the link above, beginning at the paragraph that starts with, *Possible Structure*. –, and note that while the quote suggests that the ether is incompressible, this is almost certainly not the case. The quote in question, in relation to the speed of light, reads, *“The most probable surmise or guess at present is that the ether is a perfectly incompressible continuous fluid, in a state of fine-grained vortex motion, circulating with that same enormous speed. For it has been partly, though as yet incompletely, shown that such a vortex fluid would transmit waves of the same general nature as light waves— i.e., periodic disturbances across the line of propagation—and would transmit them at a rate of the same order of magnitude as the vortex or circulation speed”*

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Appendix I (The Biot-Savart Law in the Coulomb Gauge)

“*The Double Helix Theory of the Magnetic Field*” [11], is essentially Maxwell’s sea of aethereal vortices but with the vortices replaced by rotating electron-positron dipoles. Within the context of a single rotating electron-positron dipole, the angular momentum can be written as $\mathbf{H} = \mathbf{D} \times \mathbf{v}$, where \mathbf{D} is the displacement from the centre of the dipole and \mathbf{v} is the circumferential velocity. When elastically bonded to other dipoles within the wider electron-positron sea, the displacement \mathbf{D} will be related to the transverse elasticity through Maxwell’s fifth equation, $\mathbf{D} = \varepsilon \mathbf{E}$. A full analysis can be seen in the articles “*Radiation Pressure and $E = mc^2$* ” [14], and “*The 1855 Weber-Kohlrusch Experiment*” [5]. If we substitute $\mathbf{D} = \varepsilon \mathbf{E}$ into the equation $\mathbf{H} = \mathbf{D} \times \mathbf{v}$, this leads to,

$$\mathbf{H} = -\varepsilon \mathbf{v} \times \mathbf{E}_C \quad (12)$$

See **Appendix II** regarding why the magnitude of \mathbf{v} should necessarily be equal to the speed of light. Equation (12) would appear to be equivalent to the Biot-Savart Law if \mathbf{E}_C were to correspond to the Coulomb electrostatic force. However, in the context, \mathbf{E}_C will be the centrifugal force, $\mathbf{E}_C = \mu \mathbf{v} \times \mathbf{H}$, and not the Coulomb force. If we take the curl of equation (12) we get,

$$\nabla \times \mathbf{H} = -\varepsilon [\mathbf{v}(\nabla \cdot \mathbf{E}_C) - \mathbf{E}_C(\nabla \cdot \mathbf{v}) + (\mathbf{E}_C \cdot \nabla) \mathbf{v} - (\mathbf{v} \cdot \nabla) \mathbf{E}_C] \quad (13)$$

Since \mathbf{v} is an arbitrary particle velocity and not a vector field, this reduces to,

$$\nabla \times \mathbf{H} = -\varepsilon [\mathbf{v}(\nabla \cdot \mathbf{E}_C) - (\mathbf{v} \cdot \nabla) \mathbf{E}_C] \quad (14)$$

Since \mathbf{v} and \mathbf{E}_C are perpendicular, the second term on the right-hand side of equation (14) vanishes. In a rotating dipole, the aethereal flow from positron to electron will be cut due to the vorticity, the separate flows surrounding the electron and the positron will be passing each other in opposite directions, and so the Coulomb force of attraction will be disengaged. Hence, the two particles will press against each other with centrifugal force while striving to dilate, since the aether can’t pass laterally through itself, and meanwhile the two vortex flows will be diverted up and down into the axial direction of the double helix, [11]. Despite the absence of the Coulomb force in the equatorial plane, \mathbf{E}_C is still nevertheless radial, and like the Coulomb force, as explained in **Appendix III**, it still satisfies Gauss’s Law, this time with a negative sign in the form,

$$\nabla \cdot \mathbf{E}_C = -\rho/\varepsilon \quad (15)$$

Substituting into equation (14) leaves us with,

$$\nabla \times \mathbf{H} = \rho \mathbf{v} = \mathbf{J} = \mathbf{A}_C \quad (16)$$

and hence since $\mathbf{B} = \mu\mathbf{H}$ then,

$$\nabla \times \mathbf{B} = \mu\mathbf{J} = \mu\mathbf{A}_C \quad (17)$$

which is Ampère's Circuital Law in the Coulomb gauge as per equation (9).

Appendix II (The Speed of Light)

Starting with the Biot-Savart law in the Coulomb gauge, $\mathbf{H} = -\varepsilon\mathbf{v} \times \mathbf{E}_C$, where $\mathbf{E}_C = \mu\mathbf{v} \times \mathbf{H}$, means that we can then write $\mathbf{H} = -\varepsilon\mu\mathbf{v} \times (\mathbf{v} \times \mathbf{H})$. It follows therefore that the modulus $|\mathbf{H}|$ is equal to $\varepsilon\mu v^2 H$ since \mathbf{v} , \mathbf{E}_C , and \mathbf{H} are mutually perpendicular within a rotating electron-positron dipole. Hence, from the ratio $\varepsilon\mu = 1/c^2$, it follows that the circumferential speed v must be equal to c within such a rotating dipole. In other words, the ratio $\varepsilon\mu = 1/c^2$ hinges on the fact that the circumferential speed in Maxwell's molecular vortices is equal to the speed of light.

Appendix III (Gauss's Law for Centrifugal Force)

Taking the divergence of the centrifugal force, $\mathbf{E}_C = \mu\mathbf{v} \times \mathbf{H}$, we expand as follows,

$$\nabla \cdot (\mu\mathbf{v} \times \mathbf{H}) = \mu[\mathbf{H} \cdot (\nabla \times \mathbf{v}) - \mathbf{v} \cdot (\nabla \times \mathbf{H})] \quad (18)$$

Since \mathbf{v} refers to a point particle that is in arbitrary motion, and not to a vector field, then $\nabla \times \mathbf{v} = 0$, and since $\nabla \times \mathbf{H} = \mathbf{J} = \rho\mathbf{v}$, it follows that,

$$\nabla \cdot (\mu\mathbf{v} \times \mathbf{H}) = -\mu\rho\mathbf{v} \cdot \mathbf{v} \quad (19)$$

then substituting $v = c$ as per **Appendix II**,

$$\nabla \cdot (\mu\mathbf{v} \times \mathbf{H}) = -\mu\rho c^2 \quad (20)$$

and substituting $c^2 = 1/\mu\varepsilon$, this leaves us with,

$$\nabla \cdot (\mu\mathbf{v} \times \mathbf{H}) = -\rho/\varepsilon \quad (21)$$

which is a negative version of Gauss's law for centrifugal force.