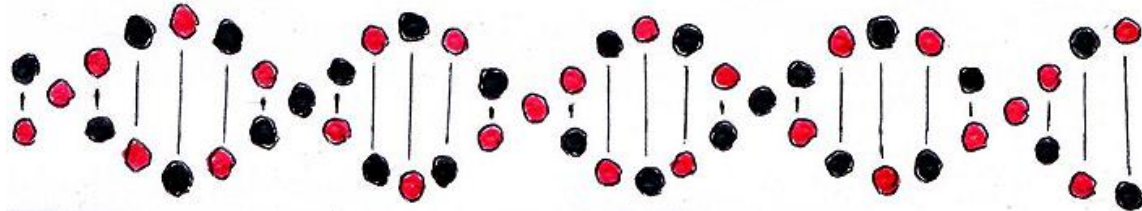


## The Physical Nature of Electromagnetic Waves

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**Abstract.** Since Scottish physicist James Clerk Maxwell wrote his *Treatise* in 1873, it has generally been believed that wireless electromagnetic radiation consists of sinusoidally oscillating electric and magnetic fields, perpendicular to each other and mutually perpendicular to the direction of propagation. The reasons as to why Maxwell concluded these mutually perpendicular orientations will now be investigated, as will the issue of the relative phase in time as between these electric and magnetic disturbances.



### The Electromagnetic Momentum

I. An analysis of the physical nature of electromagnetic waves begins in chapter **XX** of Maxwell's 1873 paper "*A Treatise on Electricity and Magnetism*" [1]. A summary, translated into the more familiar modern vector notation and using SI units, will now follow. (See the conversion chart at **Appendix I**)

At section 790, entitled '*Plane Waves*', Maxwell begins by proposing the existence of a plane wave moving with a front that is normal to the  $z$ -axis. He begins by recalling the equation of magnetic induction,  $\nabla \times \mathbf{A} = \mathbf{B}$ , which had its origins in the sea of molecular vortices that was explained in his earlier 1861 paper "*On Physical Lines of Force*" [2]. Although Maxwell was no longer overtly promoting the sea of molecular vortices at this stage, the equation of magnetic induction nevertheless still implies a rotation axis. The magnetic induction vector,  $\mathbf{B}$ , which Maxwell refers to as the *magnetic disturbance*, is an axial vector representing the vorticity density of a circulating *electromagnetic momentum*, denoted by the vector field,  $\mathbf{A}$ .

The equation of magnetic induction tells us, that as well as being mutually perpendicular,  $\mathbf{A}$  and  $\mathbf{B}$  will be in the same phase as each other in time since there is no differential time-dependence involved in their mutual relationship. Equation (13) is a projection of the three-dimensional vortex equation,  $\nabla \times \mathbf{A} = \mathbf{B}$ , unto a two-dimensional  $xy$ -plane, with  $\mathbf{A}$  being dependent only on the

propagation direction,  $z$ . The vectors  $\mathbf{A}$  and  $\mathbf{B}$  both lie in this  $xy$ -plane, this being the plane of the wave.

## Magnetic and Electric Disturbances

II. In this investigation it is important that careful attention is paid to Maxwell's terminologies, since these can be the source of much confusion. At equation (14), Maxwell introduces the reciprocal curl equation to  $\nabla \times \mathbf{A} = \mathbf{B}$  in the form of Ampère's Circuital Law,  $\mu \mathbf{J} = \nabla \times \mathbf{B} = -\nabla^2 \mathbf{A}$ , where  $\mu$  is the magnetic permeability of the luminiferous medium. Equation (14) is similarly reduced to a two-dimensional projection in the  $xy$ -plane, and the *electric disturbance*, just like the *magnetic disturbance*,  $\mathbf{B}$ , is also in the plane of the wave. By the term *electric disturbance*, Maxwell is referring to the electric current term,  $\mathbf{J}$ . It further follows from  $\mu \mathbf{J} = \nabla \times \mathbf{B}$ , that  $\mathbf{J}$  and  $\mathbf{B}$ , as well as being mutually perpendicular, will also be in the same time-phase, since they don't possess a differential time-dependent relationship with each other.

## The Wave Equation in the Electromagnetic Momentum

III. Maxwell introduces the differential time-dependent relationship  $\mathbf{J} = \partial \mathbf{D} / \partial t$  as equation (15), where  $\mathbf{D}$  is the *electric displacement* vector. This is necessary because a wave equation cannot be derived on curls alone since curls only involve spatial derivatives. The differential time-dependence involved in equation (15) ensures that if  $\mathbf{J}$  and  $\mathbf{D}$  are sinusoidal functions in time, then they will be out of time-phase with each other by ninety degrees. Hence, in such a circumstance, while the *electric disturbance*,  $\mathbf{J}$ , and the *electric displacement*,  $\mathbf{D}$ , will be out of phase with each other in time by ninety degrees, the *electric disturbance*,  $\mathbf{J}$ , will be in time-phase with the *magnetic disturbance*,  $\mathbf{B}$ . At equation (16), Maxwell writes the electric elasticity equation,  $\mathbf{D} = \epsilon \mathbf{E}_K$ , where  $\epsilon$  is the electric permittivity of the luminiferous medium. The vector fields  $\mathbf{D}$  and  $\mathbf{E}_K$  are hence both parallel and in time-phase with each other. At equation (17), Maxwell introduces the *electromotive force* equation,  $\mathbf{E}_K = -\partial \mathbf{A} / \partial t$ , where  $\mathbf{E}_K$  is the force induced by a time-changing magnetic field as per Faraday's law of induction. This is the other significant time derivative equation that is essential to the derivation of the wave equation. In this context, the *electric disturbance*,  $\mathbf{J}$ , then becomes Maxwell's *displacement current* and it becomes indistinguishable in practice from the electromagnetic momentum,  $\mathbf{A}$ , except for the fact that displacement current is only significant in the dynamic state and not in the steady state. See the article entitled, "*Displacement Current and the Electrotonic State*" [3].

Maxwell then combines these equations to derive the EM wave equation in the *electromagnetic momentum*,  $\mathbf{A}$ , at equation (19), with the speed of light emerging through the well-known relationship,  $c^2 = 1/\mu\epsilon$ . Meanwhile it is important to remember that the numerical value of  $\epsilon$  is established experimentally by discharging a capacitor, and that Maxwell relied on the 1855 Weber-Kohlrausch experiment, [4].

## The Sinusoidal Solutions

IV. At equation (20), Maxwell presents plane wave solutions to the components of the wave equation in the *electromagnetic momentum*,  $\mathbf{A}$ . Next, he considers the case of plane polarized light where  $\mathbf{A}$  is parallel to the  $x$ -axis, and he introduces the concept of *magnetic force*,  $\mathbf{H}$ , through the equation,  $\mathbf{B} = \mu\mathbf{H}$ . It follows therefore that  $\mathbf{B}$  is parallel to  $\mathbf{H}$ , and Maxwell concludes that  $\mathbf{H}$  must be parallel to the  $y$ -axis. This conclusion, however, hinges on the assumption that we are dealing with plane waves in the  $xy$ -plane, along with the fact that we already know that  $\mathbf{A}$  and  $\mathbf{B}$  are perpendicular to each other. Maxwell further states that the *electromotive force*,  $\mathbf{E}_K$ , which drives the *electric disturbance*,  $\mathbf{J}$ , must be parallel to the  $x$ -axis, but it is unclear as to how he arrived at this conclusion unless he was merely assuming that the *electric disturbance*,  $\mathbf{J}$ , which we know through Ampère's circuital law to be perpendicular to  $\mathbf{B}$ , would naturally be parallel to the *electromotive force*,  $\mathbf{E}_K$ , which drives it. This may not however be so where gyroscopics are involved.

Meanwhile we know from the equation  $\mathbf{B} = \mu\mathbf{H}$ , that  $\mathbf{B}$  and  $\mathbf{H}$  will be in phase with each other in time since there is no differential time-dependence in their mutual relationship. However, from Fig. 66, it would appear as though Maxwell believes that the *magnetic force*,  $\mathbf{H}$ , is in time-phase with the *electric displacement*,  $\mathbf{D}$ . But while the *electric disturbance*,  $\mathbf{J}$ , is in time-phase with the *magnetic disturbance*,  $\mathbf{B}$ , and hence also with the *magnetic force*,  $\mathbf{H}$ , we also know from equations (15) and (16) that it is out of phase in time, by ninety degrees, with both the *electric displacement*,  $\mathbf{D}$ , and the *electromotive force*,  $\mathbf{E}_K$ . It appears therefore that Maxwell has overlooked this fact and wrongly ascribed the same time-phase to both the *electric disturbance*,  $\mathbf{J}$ , and the *electric displacement*,  $\mathbf{D}$ , and hence wrongly ascribed the same time-phase to both the *magnetic force*,  $\mathbf{H}$ , and the *electric displacement*,  $\mathbf{D}$ . This point can be similarly argued from the electromagnetic induction equation,  $\mathbf{E}_K = -\partial\mathbf{A}/\partial t$ , in that, since  $\mathbf{E}_K$  and  $\mathbf{D}$  are in time-phase with each other, and both are ninety degrees out of phase in time with  $\mathbf{A}$ , then since  $\mathbf{A}$  is in time-phase with both  $\mathbf{B}$  and  $\mathbf{H}$ , it follows that both  $\mathbf{B}$  and  $\mathbf{H}$  will be out of phase in time by ninety degrees with both  $\mathbf{E}_K$  and  $\mathbf{D}$ . But what we don't know, is whether Maxwell's error was due to what has been stated in the above paragraph, or due, simply, to his presumption that EM waves are plane waves.

Meanwhile, the modern textbooks perpetuate Maxwell's presumption of plane wave sinusoidal solutions in  $\mathbf{B}$  and  $\mathbf{E}_K$ , and these solutions are then used to apparently prove that  $\mathbf{B}$  and  $\mathbf{E}_K$  are in time-phase with each other, as per Maxwell's Fig. 66. However, these sinusoidal solutions ignore the full three-dimensional physical interrelationships between  $\mathbf{A}$ ,  $\mathbf{B}$ , and  $\mathbf{E}_K$ , within the context of the vortices through which they were initially defined, whereby  $\mathbf{A}$  is the circumferential momentum density,  $\mathbf{B}$  is the vorticity density, and  $\mathbf{E}_K$  is the circumferential force that causes the angular acceleration. The sinusoidal solutions ignore the fact that an electromagnetic wave involves a chain reaction of precessing vortices, in which the energy is exchanged between neighbouring vortices when  $\mathbf{B}$  is pointing along the direction of wave propagation. See "*Wireless Telegraphy Beyond the Near Magnetic Field*" [5]. The textbook solutions only consider the projection of  $\mathbf{B}$  perpendicular to the direction of propagation where it appears to have reached its maximum magnitude at the same moment in time when  $\mathbf{E}_K$  reaches its maximum magnitude. In actual fact though,  $\mathbf{B}$  reaches its absolute maximum magnitude when it has rotated downwards, parallel to the direction of propagation, and so  $\mathbf{E}_K$  and  $\mathbf{B}$  are actually out of phase in time by ninety degrees. This is exactly what we would expect where  $\mathbf{E}_K$  represents the fine-grained force (torque) and  $\mathbf{B}$  represents the fine-grained (rotational) kinetic energy.

Returning to section 790 in Maxwell's treatise and reading on beyond Fig.66, Maxwell talks about a ray of plane polarized light containing both magnetic disturbances and electric disturbances that are mutually perpendicular to each other and he questions whether the plane of polarization is parallel to the magnetic disturbance or to the electric disturbance. Further down in section 797, he concludes that things best fit with Fresnel's equations if the plane of polarization is perpendicular to the electric disturbance. Some people have difficulty with the concept of a polarization filter permitting two mutually perpendicular disturbances to pass through. However, this problem is swiftly solved once we realize that these vectors refer to mechanical parameters in tiny electric dipolar fluid vortices that fill all of space [6], [7], and that the energy flow mechanism in an electromagnetic wave possesses only a singular physical nature. In radio waves, the vector fields,  $\mathbf{A}$ ,  $\mathbf{B}$ ,  $\mathbf{H}$ ,  $\mathbf{D}$ , and  $\mathbf{E}_K$ , relate to these tiny vortices. The magnetic field is only an axial vector field representing a vortex rotation axis, and in a radio wave these tiny vortices are not remotely on the scale of the wavelength of the actual radio wave itself. The magnetic fields involved in radio waves, as well as constantly changing direction, are on the picoscopic scale, and so they cannot therefore be detected by a compass in the manner of a laboratory-scale steady state magnetic field.

## Conclusion

V. In his 1864 paper “*A Dynamical Theory of the Electromagnetic Field*”, [8], Maxwell derived an electromagnetic wave equation in the *magnetic disturbance*,  $\mathbf{B}$ , while in his 1873 Treatise [1], he derived an electromagnetic wave equation in the vector field,  $\mathbf{A}$ , where  $\mathbf{A}$  denotes the *electromagnetic momentum*. In each case, Maxwell concluded that  $\mathbf{B}$ , which is equal to  $\mu\mathbf{H}$ , is in the plane of the wave. In the 1873 Treatise, as per Fig.66, he further concluded that the *magnetic force*,  $\mathbf{H}$ , will be perpendicular to the *electric displacement*,  $\mathbf{D}$ , which will also be in the plane of the wave.

It has been suggested in “*Wireless Telegraphy Beyond the Near Magnetic Field*” [5], that these vectors apply on the picoscopic scale in connection with Maxwell’s molecular vortices, and that these vortices undergo a complete 180 degrees tumble (precession), and back up again, during the transit of an electromagnetic wave pulse, with the precessional axis of the  $\mathbf{H}$  field ( *$\mathbf{H}$  can be used interchangeably with  $\mathbf{B}$  in all of these arguments*) being perpendicular to the direction of propagation, and the direction of propagation being in the plane of the precessing  $\mathbf{H}$  vector. And this will be so, no matter what the relative orientation is as between the direction of propagation and the prevailing background magnetic field lines. This same paper, [5], additionally explains how the act of precessing is crucial to the energy flow process in the EM wave, and that the orientation of  $\mathbf{E}_K$  relative to  $\mathbf{H}$  depends on the angle between the direction of propagation and the prevailing background magnetic field lines. Providing though, that we fix the parameters  $\mathbf{A}$ ,  $\mathbf{B}$ ,  $\mathbf{H}$ ,  $\mathbf{D}$ , and  $\mathbf{E}_K$  within the body of an individual vortex as it precesses, then in the context of an electromagnetic wave, Maxwell will be correct, at least with respect to the relative orientations of these parameters.  $\mathbf{B}$  and  $\mathbf{H}$  however will not in general be perpendicular to the direction of the wave propagation. The plane wave sinusoidal solutions in the textbooks truncate the full picture of the precessing vortices as they precess on an axis perpendicular to the direction of propagation, hence eliminating the  $z$ -components of  $\mathbf{B}$  and  $\mathbf{H}$ .

In the 1937 Encyclopaedia Britannica, an article entitled “*Ether (in physics)*”, [9], appeared, written by Sir Oliver Lodge. Lodge states, in relation to the speed of light, that,

*“The most probable surmise or guess at present is that the ether is a perfectly incompressible continuous fluid, in a state of fine-grained vortex motion, circulating with that same enormous speed. For it has been partly, though as yet incompletely, shown that such a vortex fluid would transmit waves of the same general nature as light waves— i.e., periodic disturbances across the line of propagation—and would transmit them at a rate of the same order of magnitude as the vortex or circulation speed”.*

It is unclear in the article as to who was promoting this idea, but the idea seems to have been shelved after the second world war. It's difficult however to believe that whoever wrote it was not on the right tracks. It would be interesting to find out whose idea this was. According to Tesla, mankind knew long ago that space was rendered into tiny whirls. In an unpublished article written in 1907, [10], Tesla states,

***“Long ago he (mankind) recognized that all perceptible matter comes from a primary substance, of a tenuity beyond conception and filling all space - the Akasha or luminiferous ether - which is acted upon by the life-giving Prana or creative force, calling into existence, in never ending cycles, all things and phenomena. The primary substance, thrown into infinitesimal whirls of prodigious velocity, becomes gross matter; the force subsiding, the motion ceases and matter disappears, reverting to the primary substance”.***

And in the 1910 E.T. Whittaker chronology of aether history, [11], it reads,

***“All space, according to the younger Bernoulli, is permeated by a fluid aether, containing an immense number of excessively small whirlpools. The elasticity which the aether appears to possess, and in virtue of which it is able to transmit vibrations, is really due to the presence of these whirlpools; for, owing to centrifugal force, each whirlpool is continually striving to dilate, and so presses against the neighbouring whirlpools.***

And so, it is concluded that wireless electromagnetic radiation is a sinusoidal fine-grained vortex-gyroscopic process, that while plane polarizable, cannot be accurately described in terms of being exclusively transverse or longitudinal.  $\mathbf{E}_K$  and  $\mathbf{H}$  are mutually perpendicular within an individual vortex while being out of phase with each other by ninety degrees in time. The situation is different however in the case of a DC transmission line pulse. This will be a non-sinusoidal pulse of electric fluid surrounded only by a magnetic field, and this magnetic field will be measurable on the laboratory scale [12].

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The derivation of the electromagnetic wave equation in  $\mathbf{H}$  begins on page 497 in the first link. Then see the note at the top of page 499 in the second link.  
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The quote below is in relation to the speed of light,  
***“The most probable surmise or guess at present is that the ether is a perfectly incompressible continuous fluid, in a state of fine-grained vortex motion, circulating with that same enormous speed. For it has been partly, though as yet incompletely, shown that such a vortex fluid would transmit waves of the same general nature as light waves— i.e., periodic disturbances across the line of propagation—and would transmit them at a rate of the same order of magnitude as the vortex or circulation speed”***  
The article then goes on to cite Lord Kelvin, “**The Vortex Theory of Ether,**” *Phil. Mag.* (1887) and *Math. and Phys. Papers*, vol. iv. and passim; also G. F. FitzGerald, *Proc. Roy. Dub. Soc.* (1899), or *Collected Papers*, pp. 154, 238, 472.  
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*infinitesimal whirls of prodigious velocity, becomes gross matter; the force subsiding, the motion ceases and matter disappears, reverting to the primary substance.”*

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*“All space, according to the younger Bernoulli, is permeated by a fluid aether, containing an immense number of excessively small whirlpools. The elasticity which the aether appears to possess, and in virtue of which it is able to transmit vibrations, is really due to the presence of these whirlpools; for, owing to centrifugal force, each whirlpool is continually striving to dilate, and so presses against the neighbouring whirlpools. It will be seen that Bernoulli is a thorough Cartesian in spirit; not only does he reject action at a distance, but he insists that even the elasticity of his aether shall be explicable in terms of matter and motion. This aggregate of small vortices, or "fine-grained turbulent motion," as it came to be called a century and a half later,\* is interspersed with solid corpuscles, whose dimensions are small compared with their distances apart. These are pushed about by the whirlpools whenever the aether is disturbed, but never travel far from their original positions. A source of light communicates to its surroundings a disturbance which condenses the nearest whirlpools; these by their condensation displace the contiguous corpuscles from their equilibrium position; and these in turn produce condensations in the whirlpools next beyond them, so that vibrations are propagated in every direction from the luminous point. It is curious that Bernoulli speaks of these vibrations as longitudinal, and actually contrasts them with those of a stretched cord, which, "when it is slightly displaced from its rectilinear form, and then let go, performs transverse vibrations in a direction at right angles to the direction of the cord." When it is remembered that the objection to longitudinal vibrations, on the score of polarization, had already been clearly stated by Newton, and that Bernoulli's aether closely resembles that which Maxwell invented in 1861-2 for the express purpose of securing transversality of vibration, one feels that perhaps no man ever so narrowly missed a great discovery. Bernoulli explained refraction by combining these ideas with those of his father. Within the pores of ponderable bodies the whirlpools are compressed, so the centrifugal force must vary in intensity from one medium to another. Thus a corpuscle situated in the interface between two media is acted on by a greater elastic force from one medium than from the other; and by applying the triangle of forces to find the- conditions of its equilibrium, the law of Snell and Descartes may be obtained. \* Cf. Lord Kelvin's vortex-sponge aether, described later in this work.”*

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## Appendix I

(Conversion Chart)

Electromagnetic Momentum,  $\mathbf{A} \rightarrow F, G, H$

Magnetic Disturbance, - - - ,  $\mathbf{B} \rightarrow a, b, c$

Magnetic Permeability, - - - ,  $\mu \rightarrow \mu$

Electric Disturbance, - - - - -,  $\mathbf{J} \rightarrow u, v, w$

Electric Displacement, - - - -,  $\mathbf{D} \rightarrow f, g, h$

Electric Permittivity, - - - - -,  $\varepsilon \rightarrow K/4\pi$

Electromotive Force, - - - - -,  $\mathbf{E}_K \rightarrow P, Q, R$

Magnetic Force, y-axis, - - -,  $H_y \rightarrow (1/\mu)dF/dz$  ( $\mathbf{B} = \mu\mathbf{H} = \nabla \times \mathbf{A}$ )