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Gravitoinertial Fields

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ABSTRACT: This article aims to demonstrate the interrelations between the gravitational and inertial fields and introduce a system of equations similar to the electromagnetic equations. We will calculate the inertial permeability of vacuum and demonstrate that the inertial field is related to the equations of motion and kinetic energies. These equations will be used to produce an equivalent to the electromagnetic Maxwell equations for these mechanical fields.

KEYWORDS: gravitational field, inertial field, inertial permeability, gravitational induction, inertial induction.

Contents

1 Symbology

In this text we will use the following symbols with its abbreviated units of measure:

N = Newton, kg = kilogram, m = meter, s = second, V = Volt, C = Coulomb, A = Ampere, Wb = Weber, rad = radian.

E = Electric field intensity [N C^{-1}] [V m^{-1}];
V_E = Electric potential [V] [Wb s^{-1}];
q_E = Electric charge [C];

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ε_0 = Electric permittivity of vacuum [$C^2\ N^{-1}\ m^{-2}$] [$C\ V^{-1}\ m^{-1}$];
μ_0 = Magnetic permeability of vacuum [$Wb^2\ N^{-1}\ m^{-2}$] [$Wb\ A^{-1}\ m^{-1}$];
G = Gravitational field intensity [$N\ kg^{-1}$] [$m\ s^{-2}$];
M = Surface density of gravitational charge [$kg\ m^{-2}$];
V_G = Gravitational potential [$N\ m\ kg^{-1}$] [$m^2\ s^{-2}$];
Φ_G = Gravitational flux [$N\ m^2\ kg^{-1}$];
q_G = Gravitational charge [kg];
I_G = Gravitational current [$kg\ s^{-1}$] [$N\ s\ m^{-1}$];
J_G = Surface density of gravitational current [$kg\ s^{-1}\ m^{-2}$];
ρ_G = Volumetric density of gravitational charge [$kg\ m^{-3}$];
γ_0 = Gravitational permeability of vacuum [$kg^2\ N^{-1}\ m^{-2}$] [$kg\ s^2\ m^{-3}$];
k_G = Gravitostatic constant = $6.6739*10^{-11}$ [$N\ m^2\ kg^{-2}$] [$m^3\ kg^{-1}\ s^{-2}$];
I = Inertial field intensity [$N\ s\ m^{-2}$] [$kg\ m^{-1}\ s^{-1}$];
O = Surface density of inertial charge [s^{-1}];
V_I = Inertial potential [$N\ s\ m^{-1}$] [$kg\ s^{-1}$];
Φ_I = Inertial flux [N s] [$kg\ m\ s^{-1}$];
q_I = Inertial charge [$m^2\ s^{-1}$];
I_I = Inertial current [$m^2\ s^{-2}$];
J_I = Surface density of inertial current [s^{-2}];
ι_0 = Inertial permeability of vacuum [$(m^2\ s^{-1})^2\ N^{-1}\ m^{-2}$] [$m^2\ N^{-1}\ s^{-2}$] [$m\ kg^{-1}$];
F = Force [N] [$kg\ m\ s^{-2}$];
a = Acceleration [$m\ s^{-2}$];
v = Velocity [$m\ s^{-1}$];
c = Speed of light in vacuum = $2.9979*10^8\ m\ s^{-1}$;
ω = Angular frequency (also called angular velocity) [$rad\ s^{-1}$];
f = Frequency [$cycle\ s^{-1}$] [Hz];
φ = Angle [rad];
r = Radial length (radius) [m];
l = Length [m];
S = Area [m^2];
t = Time [s].

2 Introduction

In the previous articles Gravitational Charge[1] and Inertial Field[2] it was introduced a new presentation form for the gravitational equations and a new field associated with the equations of motion and kinetic energies that was called inertial field. The equations of these mechanical fields followed the same presentation of the electromagnetic fields because in that format we may use the same electromagnetic calculus tools as a unified method.

There is another advantage in presenting the gravitoinertial equations in this way, that is the invariance by Lorentz transformations. With this in mind, we will seek for a link between these fields and the speed of light c and use the gravitational permeability calculated in the Gravitational Charge[1] article to calculate the inertial permeability of vacuum. With these permeability constants associated with c we may put these mechanical equations in the same format as the electromagnetic Maxwell equations.

3 Inertial Permeability of Vacuum

In the Inertial Field[2] article, we have defined a simple relation between inertial field **I** and surface density of inertial charge $\vec{O}=\iota_0\vec{I}$ with the inertial permeability of vacuum ι_0, that has unit of measure [$m\ kg^{-1}$], [$m^2\ N^{-1}\ s^{-2}$] or [$(m^2\ s^{-1})^2\ N^{-1}\ m^{-2}$]. This unit of measurement is similar to the units of measure for permeability of other fields:

- Its electrical counterpart (called permittivity) ε_0 has [C^2 N^{-1} m^{-2}];
- Its magnetic counterpart μ_0 has [Wb^2 N^{-1} m^{-2}];
- Its gravitational counterpart γ_0 has [kg^2 N^{-1} m^{-2}] [kg s^2 m^{-3}].

We will calculate the value of the inertial permeability by dimensional analysis and by the gravitational potential of the vacuum.

3.1 Calculus by Dimensional Analysis

With the unit of measure of ι_0 being [m kg^{-1}] and of γ_0 being [kg s^2 m^{-3}], we may see that the product of gravitational and inertial permeabilities has the inverse of velocity squared as unit of measure $\gamma_0 \iota_0 = 1/v^2 [s^2 m^{-2}]$.

With the electromagnetic relation $\varepsilon_0 \mu_0 = 1/c^2$, we may estimate, by analogy and dimensional analysis, a first approximation to the inertial permeability assuming that the relation between the mechanical permeabilities and speed of light is the same as the electromagnetic permeabilities, $\gamma_0 \iota_0 = 1/c^2$, so isolating the inertial permeability of the vacuum:

$$\iota_0 = \frac{1}{c^2 \gamma_0} = \frac{1}{(2{,}998 \cdot 10^8)^2 1{,}1924 \cdot 10^9} = 9{,}337 \cdot 10^{-27} m kg^{-1}$$

The inertial permeability has the inverse of the linear density of gravitational charge (mass) as unit of measure.

3.2 Calculus by Gravitational Potential of the Vacuum

This same value may be obtained in relating the gravitational potential with the inertial current $V_G = G r = v^2 = (\omega r)^2 = I_I$, like it was done in studying satellite orbit in the Inertial Field[2] article. Considering that the orbital velocity **v** of the satellite is in equilibrium with the gravitational potential $\mathbf{V_G}$ produced by the Earth gravitational charge $\mathbf{q_G}$ at a distance **r** from the center of it, we have:[2]

$$V_G = G r = k_G \frac{q_G}{r} = \frac{1}{4\pi \gamma_0} \frac{q_G}{r} = v^2 \quad , \quad G = k_G \frac{q_G}{r^2}$$

The factor 4π is the solid angle Ω of the spherical shell around the gravitational charge:

$$\Omega = \int d\Omega = \int_0^{2\pi} d\varphi \int_0^{\pi} sen\theta \, d\theta = 4\pi \, rad^2$$

This solid angle has dimension of area (radian squared) because it is the total spherical angular surface centered at the gravitational charge q_G. This way, we may consider that the gravitational potential is taken on only one point direction, so $q_G/(4\pi r)$ may be considered as a linear density of gravitational charge (mass) α_G, and the gravitational potential at a distance r from the Earth may be calculated by:

$$\alpha_G = \frac{q_G}{4\pi r} \implies V_G = v^2 = \frac{1}{4\pi \gamma_0} \frac{q_G}{r} = \frac{\alpha_G}{\gamma_0}$$

2 Here G is the gravitational field and k_G is the Universal Gravitational Constant.

We may extend this analogy to the vacuum considering that the speed of light is an inertial current equivalent to a gravitational potential of vacuum, as if we would be at a distance r from a gravitational charge q_G whose point of equilibrium occurs at $V_{G0}=c^2=I_{I0}$. This way, the gravitational potential of the vacuum may be related with the linear density of gravitational charge (mass) of the vacuum:

$$V_{G0}=c^2=\frac{1}{4\pi\gamma_0}\frac{q_{G0}}{r}=\frac{\alpha_{G0}}{\gamma_0}$$

With this equation, we get the linear density of gravitational charge (mass) of the vacuum:

$$\alpha_{G0}=c^2\gamma_0=(2,9979*10^8)^2*1,1924*10^9=1,0716*10^{26}\,kg\,m^{-1}$$

The relation $\alpha_{G0}=c^2\gamma_0$ lead us to consider α_{G0} as being the inverse of the inertial permeability of vacuum $\alpha_{G0}=1/\iota_0$, and we have the relation $\gamma_0\iota_0=1/c^2$, as before. The inertial permeability of vacuum may be calculated by:

$$\iota_0=\frac{1}{c^2\gamma_0}=\frac{1}{(2.998*10^8)^2 1.1924*10^9}=9.337*10^{-27}\,m\,kg^{-1}$$

4 Gravitational and Inertial Energies

Actually, we relate potential energy with the configuration of a system in which a conservative force acts. A force is conservative if the net work it does on a particle moving between two points does not depend on the path taken by the particle or, equivalently, if the net work this force does on a particle moving around any closed path, from an initial point and then back to that point, is zero. The gravitational force exerted on an object by the gravitational field of the Earth is conservative.[3]

When the conservative force does work W_G on an object within a system consisting of Earth and a nearby object, the change ΔU_G in the gravitational potential energy of the system, considering that $\phi=180°$ is the angle between the directions of $\vec{F}$ and $d\vec{r}$, is:

$$\Delta U_G=-W_G=-\int_{h_i}^{h_f}\vec{F}\cdot d\vec{r}=-\int_{h_i}^{h_f}F\,dr\cos\phi=\int_{h_i}^{h_f}F\,dr \qquad F=k_G\frac{q_G Q_G}{r^2}=q_G G$$

$$\Delta U_G=\int_{h_i}^{h_f}q_G G\,dr=q_G G[r]_{h_i}^{h_f}=q_G G(h_f-h_i)=q_G(V_{Gf}-V_{Gi})=U_{Gf}-U_{Gi} \qquad V_G=G\,h \qquad U_G=q_G V_G$$

In the Gravitational Charge[1] article, we put the gravitational equations in the same format as the electromagnetic equations, so the potential energy associated with a gravitational charge inside a gravitational field may be calculated by the product of the gravitational charge (mass) by the gravitational potential, as the potential energy associated with an electric charge inside an electric field may be calculated by the product of the electric charge by the electric potential. The two equations are put below for comparison:

$$U_E=q_E V_E=q_E E\,r[N\,m][J] \quad \text{and} \quad U_G=q_G V_G=q_G G\,r[kg\,m^2 s^{-2}][N\,m][J]$$

This gravitational potential energy, when considering two bodies with gravitational charges (masses) q_G and Q_G separated by a distance r, is the negative of the work that would be done by the gravitational force of either particle acting on the other if the separation between the particles were changed from infinity to r: $\Delta U = U_{G\infty} - U_G = -W_G$.[3] Considering that k_G is the gravitostatic constant (Universal Gravitational Constant), the gravitational potential energy is defined by:

$$U_G = W_G = \int_R^\infty \vec{F}(r) \cdot d\vec{r} = -\int_R^\infty k_G \frac{q_G Q_G}{r^2} dr = k_G q_G Q_G \left[\frac{1}{r} \right]_R^\infty = -k_G \frac{q_G Q_G}{R}$$

In the Inertial Field[2] article we put the inertial equations in the same format as the electromagnetic and gravitational equations, so the energy associated with an inertial charge inside an inertial field may be calculated, too, by the product of the inertial charge $q_I [m^2 s^{-1}]$ by the inertial potential $V_I [kg s^{-1}]$:

$$U_I = q_I V_I = q_I I r [kg\, m^2 s^{-2}][N\, m][J]$$

We see that we may consider an inertial energy, that is linked with motion, like a potential energy, but kinetic energy is linked with motion too. Let’s find out what its relation is.

4.1 Inertial Field and Equations of Motion

The position of a particle in space is defined by its radius vector $\vec{r}$, whose components in Cartesian coordinates are x, y, z. The derivative of $\vec{r}$ with respect to time t is called velocity of the particle $\vec{v} = d\vec{r}/dt$, and the second derivative $\vec{a} = d^2\vec{r}/dt^2$ is its acceleration.[4]

If all the coordinates and velocities are simultaneously specified, the experience shows that the state of the system is completely determined and that its subsequent motion can, in principle, be calculated. This means that, knowing all coordinates and velocities at some instant, the acceleration at that instant is uniquely defined.

The relations between acceleration, velocities and coordinates are called the equations of motion. They are second-order differential equations for the coordinate's functions of time, and their integration makes possible the determination of the path of the system. Considering only the x-coordinate, with the initial position x_0, initial velocity v_{x0} and acceleration a_x, we have the equation of motion defined by:

$$x(t) = x_0 + v_{x0} t + \frac{a_x t^2}{2}$$

Clearly this equation does not involve any electric charge q_E [C] or magnetic charge q_M [Wb], neither the gravitational charge q_G [kg], but it does involve the acceleration a_x, that is a gravitational field, and the velocity, that may be expressed with the surface density of inertial charge $\vec{O} = q_I / \vec{S} = \vec{\omega}$ [rad s^{-1}].

So, mechanics are related with two mechanical fields: gravitational (potential) and inertial (kinetic). The Newtonian, Lagrangian and Hamiltonian equations for the mechanical systems may be defined considering these two gravitoinertial fields.

4.2 Kinetic and Inertial Energies

Kinetic energy is energy associated with the state of motion of an object; the faster the object moves, the greater is its kinetic energy. When the object is stationary, its kinetic energy is zero. When we accelerate an object to a greater speed by applying a force on it, the kinetic energy of the object is increased by the work realized on it, so energy is transferred to the object. The change ΔK in the kinetic energy of the object is calculated considering the speed variation:

$$\Delta K = W = \int_{v_i}^{v_f} p(v) dv = \int_{v_i}^{v_f} q_G v\, dv = q_G \left[\frac{v^2}{2}\right]_{v_i}^{v_f} = \frac{1}{2} q_G (v_f^2 - v_i^2) = K_f - K_i$$

If the object was in a stationary initial state ($v_i = 0$), we have $\Delta K = q_G v_f^2 / 2 = K_f$ and this we call kinetic energy of the object: it is the energy furnished by the system to the object. Considering that $U_I = q_I V_I = q_G v^2 [kg\, m^2 s^{-2}][N\, m][J]$ we see that inertial energy is not what we actually call kinetic energy because the former is the intrinsic energy of the object that stand at a velocity and the last is the energy spent to put an object at a velocity, and corresponds to the mean variation of the inertial energy.

$$\Delta K = K_f - K_i = \frac{1}{2} q_G (v_f^2 - v_i^2) = \frac{1}{2}(U_{If} - U_{Ii}) = \frac{1}{2} \Delta U_I$$

This explains why the equilibrium point of a satellite in orbit is not obtained with the kinetic energy of the satellite, but with its inertial energy $U_G = q_G G r = q_G V_G = q_G v^2 = q_I V_I = U_I$ [3], that must be equivalent to its gravitational (potential) energy.

5 Gravitational and Inertial Potentials Induction

With electromagnetism we have two forms of induction, that we have defined in its complete form (conduction and displacement) in the Magnetic Charge[5] article:

1. Faraday's induction law (electric potential induction)
   Relates the induced electric potential with a magnetic current (conduction and displacement).
2. Ampère-Maxwell's induction law (magnetic potential induction)
   Relates the induced magnetic potential with an electric current (conduction and displacement).

With gravitoinertialism[4] we will define two forms of induction in its complete form (conduction and displacement):

1. Gravitational potential induction
   Relates the induced gravitational potential with an inertial current (conduction and displacement).
2. Inertial potential induction
   Relates the induced inertial potential with a gravitational current (conduction and displacement).

5.1 Gravitational Potential Induction

In studying satellites orbit in the Inertial Field[2] article, we have seen that in a stable orbit the satellite velocity squared is equal to the gravitational potential of the Earth with the equation $V_G = G r = v^2 = (\omega r)^2 = I_I$. This equation is derived from the known formula of equilibrium between the gravitational force and centripetal force $F = m g = m v^2 / r$ or, with our notation, $F = q_G G = q_G v^2 / r = q_I I$.

3 Please see Inertial Field article in the bibliography.
4 Gravitoinertialism relates to gravitoinertial fields like electromagnetism relates to electromagnetic fields.

In that article we deduced that the inertial current $I_I = dq_I/dt = v^2$ was neutralizing the gravitational potential of the satellite $V_G = G r$ in a form of induction because the gravitational force was perpendicular to the velocity of the satellite, so the result is $V_G = I_I$.

Considering this, we may establish an equivalent to the Faraday's induction law for the gravitational potential: the integral of the gravitational field around a closed line that encircle an area passed through by an inertial current is equal to the inertial current enclosed by the line. Then, the gravitational potential induction law may include two forms of inertial current: conduction and displacement. The complete equations in integral and differential forms are:

$$\oint_L \vec{G} \cdot d\vec{l} = I_I + \iota_0 \frac{d\Phi_I}{dt} \quad , \quad \nabla \times \vec{G} = \vec{J}_I + \iota_0 \frac{\partial \vec{I}}{\partial t} = \vec{J}_I + \frac{\partial \vec{O}}{\partial t} \quad , \quad V_G = -\oint_L \vec{G} \cdot d\vec{l} \quad , \quad \vec{G} = -\nabla V_G$$

5.2 Inertial Potential Induction

We may establish an equivalent to the Ampère-Maxwell's induction law for the inertial potential: the integral of the inertial field around a closed line that encircle an area passed through by a gravitational current is equal to the gravitational current enclosed by the line. The inertial potential induction law may include two forms of gravitational current: conduction and displacement. The complete equations in integral and differential forms were already presented in the Inertial Field[2] article, and are put here for convenience:

$$\oint_L \vec{I} \cdot d\vec{l} = I_G + \gamma_0 \frac{d\Phi_G}{dt} \quad , \quad \nabla \times \vec{I} = \vec{J}_G + \gamma_0 \frac{\partial \vec{G}}{\partial t} = \vec{J}_G + \frac{\partial \vec{M}}{\partial t} \quad , \quad V_I = -\oint_L \vec{I} \cdot d\vec{l} \quad , \quad \vec{I} = -\nabla V_I$$

6 Gravitational and Inertial Hall Effect and Lorentz Force

The surface density of charge **O** is defined as inertial charge [m^2 s^{-1}] per area [m^2], and this lead us to a unit of measurement [s^{-1}], that is a frequency. This frequency can be of two types, cyclic f and angular ω, related by $\omega = 2\pi f$ because a cycle (the perimeter of a circle) has 2π radians. The unit of measure of cyclic frequency is Hertz [Hz], that means cycles per second [cycle s^{-1}]; the unit of measure of angular frequency is radians per second [rad s^{-1}].

The angular frequency is defined by $\omega = d\varphi/dt$, that is the rate of angular variation linked to rotations, and the linear velocity tangent to the curve is defined by:

$$\vec{v} = \vec{\omega} \times \vec{r} = \frac{d\vec{\varphi}}{dt} \times \vec{r} = \frac{d\vec{l}}{dt}$$

This vector product can be worked out in order to obtain the gravitational equivalent of the Hall effect, resulting from a mechanical equivalent of the Lorentz's force. In cylindrical polar coordinates, obeying the right-hand rule, the components of this equation are in the following axes:

1. Speed tangent to the curve on the angular axis $\hat{\varphi}$;
2. Angular frequency (or angular velocity) at the axis $\hat{z}$;
3. Radius of the curve or circumference on the radial axis $\hat{r}$.

However, we want to find an acceleration in the radial axis, consequent of the derivative of the velocity. Using the right-hand rule with cylindrical polar coordinates, we have:

$$\vec{a} = d\vec{v}/dt = a\hat{r} \quad \text{and} \quad \vec{v} = \vec{r} \times \vec{\omega} \quad \Longrightarrow \quad v\hat{r} = r\hat{\varphi} \times \omega\hat{z}$$

Knowing that $\vec{a} = \vec{G}$ and $\vec{O} = \vec{\omega}$ we have:

$$\frac{d}{dt}\vec{v}=\vec{a}=\vec{G}=\frac{d}{dt}(\vec{r}\times\vec{\omega})=\frac{d\vec{r}}{dt}\times\vec{\omega}+\vec{r}\times\frac{d\vec{\omega}}{dt}=\vec{v}\times\vec{\omega}+\cancel{\vec{r}\times\vec{\alpha}}=\vec{v}\times\vec{O}\quad\Longrightarrow$$

$$\vec{a}=\vec{v}\times\vec{\omega}=\frac{v^2}{r}\hat{r}=\omega^2\vec{r}\quad\text{and}\quad\vec{G}=\vec{v}\times\vec{O}\quad\Longrightarrow\quad\vec{F}=q_G\vec{G}=q_G(\vec{v}\times\vec{O})$$

A gravitational charge traveling inside an inertial field (created by a surface distribution of inertial charges) suffers a force perpendicular to the field and velocity that prints out a centripetal acceleration $a=v^2/r=\omega^2 r$ consequence of the radial gravitational field acting on it. The component of the angular acceleration α tangential to the curve is zero because the angular velocity ω is constant.

The above equations give us the gravitational equivalent of the force that establishes the orbit of electric charges when they are moving inside a magnetic field. The equations are given below for comparison:

$$\vec{F}=q_E\vec{E}=q_E(\vec{v}\times\vec{B})=q_G\frac{v^2}{r}\hat{r}\quad\text{and}\quad\vec{F}=q_G\vec{G}=q_G(\vec{v}\times\vec{O})=q_G\frac{v^2}{r}\hat{r}$$

Being this relation mathematically correct, we can deduce that there is also the inertial Hall effect and inertial Lorentz’s force, that is, produced by inertial charges moving within gravitational fields produced by surface distribution of gravitational charges:

$$\vec{I}=\vec{v}\times\vec{M}\quad\text{e}\quad\vec{F}=q_I\vec{I}=q_I(\vec{v}\times\vec{M})$$

7 Gravitational and Inertial Field Equations

The majority of equations that relates gravitational and inertial fields are presented in previous articles and complemented with the equations presented in this article. They are put in a table below for comparison with the electromagnetic equations presented in the Magnetic Charge[5] article.

	Gravitational		**Inertial**	
Greatness	**Equation**	**Unit**	**Equation**	**Unit**
Field from a surface charge	$G(r)=\frac{1}{4\pi\gamma_0}\int_S\frac{\vec{M}\cdot d\vec{S}}{r^2}$	[N kg^{-1}] [m s^{-2}]	$I(r)=\frac{1}{4\pi\iota_0}\int_S\frac{\vec{O}\cdot d\vec{S}}{r^2}$	[N s m^{-2}] [kg m^{-1} s^{-1}]
Field from a scalar potential	$\vec{G}=-\nabla V_G$	[N kg^{-1}] [m s^{-2}]	$\vec{I}=-\nabla V_I$	[N s m^{-2}] [kg m^{-1} s^{-1}]
Surface charge density	$\vec{M}=\gamma_0\vec{H}$	[kg m^{-2}]	$\vec{O}=\iota_0\vec{I}$	[s^{-1}]
Flux	$\Phi_G=\int_S\vec{G}\cdot d\vec{S}$	[N m^2 kg^{-1}] [m^3 s^{-2}]	$\Phi_I=\int_S\vec{I}\cdot d\vec{S}$	[N s] [kg m s^{-1}]
Charge	$q_G=\int_S\vec{M}\cdot d\vec{S}=\gamma_0\Phi_G$	[kg]	$q_I=\int_S\vec{O}\cdot d\vec{S}=\iota_0\Phi_I$	[m^2 s^{-1}]
Current	$I_G=\frac{dq_G}{dt}=\gamma_0\frac{d\Phi_G}{dt}$	[kg s^{-1}] [N s m^{-1}]	$I_I=\frac{dq_I}{dt}=\iota_0\frac{d\Phi_I}{dt}$	[m^2 s^{-2}]
Scalar potential	$V_G=-\int_L\vec{G}\cdot d\vec{l}$	[N m kg^{-1}] [m^2 s^{-2}]	$V_I=-\int_L\vec{I}\cdot d\vec{l}$	[N s m^{-1}] [kg s^{-1}]

The gravitoinertial equations similar to the electromagnetic Maxwell equations are put in a table below. Evidently, it is not easy to experimentally prove its correctness by now. Specially the inertial charge monopole is so difficult to find as the magnetic charge monopole. But we understand that not experimentally proving that the magnetic or inertial monopoles exist is not the proof of its nonexistence, so it is wise to have the benefit of the doubt.

Gravitoinertial (Maxwell) Equations	
Gravitational Gauss's Law or Charge Law: Relates the gravitational flux to gravitational charges enclosed by a Gaussian surface.	$\gamma\Phi_G=\gamma\oint\vec{G}\cdot d\vec{S}=\oint\vec{M}\cdot d\vec{S}=q_G$
Inertial Gauss's Law or Charge Law: Relates the inertial flux to inertial charges enclosed by a Gaussian surface.	$\iota\Phi_I=\iota\oint\vec{I}\cdot d\vec{S}=\oint\vec{O}\cdot d\vec{S}=q_I$
Gravitational Faraday's Law or Induction Law: Relates the gravitational field induced by a time variation of inertial flux and inertial current.	$\oint\vec{G}\cdot d\vec{l}=I_I+\iota\frac{d\Phi_I}{dt}$
Inertial Ampère-Maxwell's Law or Induction Law: Relates the inertial field induced by a time variation of gravitational flux and gravitational current.	$\oint\vec{I}\cdot d\vec{l}=I_G+\gamma\frac{d\Phi_G}{dt}$

The integral form of the above equations may be converted to differential form applying two theorems:

1. Divergence theorem $\oint\vec{A}\cdot d\vec{S}=\int(\nabla\cdot\vec{A})dV$
   Relates the closed surface integral of a vector $\vec{A}$ with the volume integral of the vector's divergent.
2. Rotational theorem $\oint\vec{A}\cdot d\vec{l}=\int(\nabla\times\vec{A})\cdot d\vec{S}$
   Relates the closed line integral of a vector $\vec{A}$ with the surface integral of the vector's rotational.

Applying the divergence theorem on the Gravitational Charge Law:

$$\gamma\Phi_G=\gamma\oint\vec{G}\cdot d\vec{S}=\gamma\int(\nabla\cdot\vec{G})dV=\int\rho_G dV=q_G \text{ , and } \gamma\nabla\cdot\vec{G}=\rho_G \text{ .}$$

With $\vec{G}=\nabla V_G$ we have $\gamma\nabla\cdot\vec{G}=\gamma(\nabla\cdot\nabla)V_G=\rho_G$ and $\gamma\nabla^2 V_G=\rho_G$.

Applying the divergence theorem on the Inertial Charge Law:

$$\iota\Phi_I=\iota\oint\vec{I}\cdot d\vec{S}=\iota\int(\nabla\cdot\vec{I})dV=\int\rho_I dV=q_I \text{ , and } \iota\nabla\cdot\vec{I}=\rho_I \text{ .}$$

With $\vec{I}=\nabla V_I$ we have $\iota\nabla\cdot\vec{I}=\iota(\nabla\cdot\nabla)V_I=\rho_I$ and $\iota\nabla^2 V_I=\rho_I$.

Applying the rotational theorem on the Gravitational Induction Law:

$$\oint\vec{G}\cdot d\vec{l}=\int(\nabla\times\vec{G})\cdot d\vec{S}=\int\left(\vec{J}_I+\frac{d\vec{O}}{dt}\right)\cdot d\vec{S} \text{ and } \nabla\times\vec{G}=\vec{J}_I+\iota\frac{d\vec{I}}{dt}=\vec{J}_I+\frac{d\vec{O}}{dt}$$

Applying the rotational theorem on the Inertial Induction Law:

$$\oint \vec{I}\cdot d\vec{l}=\int(\nabla\times\vec{I})\cdot d\vec{S}=\int\left(\vec{J}_G+\frac{d\vec{M}}{dt}\right)\cdot d\vec{S} \quad \text{and} \quad \nabla\times\vec{I}=\vec{J}_G+\gamma\frac{d\vec{G}}{dt}=\vec{J}_G+\frac{d\vec{M}}{dt}$$

Gravitoinertial (Maxwell) Equations	
Gravitational Gauss's Law or Charge Law: Relates the gravitational field to volumetric density of gravitational charges enclosed by a Gaussian surface.	$\gamma\nabla\cdot\vec{G}=\nabla\cdot\vec{M}=\rho_G$
Inertial Gauss's Law or Charge Law: Relates the inertial field to volumetric density of inertial charges enclosed by a Gaussian surface.	$\iota\nabla\cdot\vec{I}=\nabla\cdot\vec{O}=\rho_I$
Gravitational Faraday's Law or Induction Law: Relates the gravitational field induced by a time variation of inertial field and surface density of inertial current.	$\nabla\times\vec{G}=\vec{J}_I+\iota\frac{d\vec{I}}{dt}=\vec{J}_I+\frac{d\vec{O}}{dt}$
Inertial Ampère-Maxwell's Law or Induction Law: Relates the inertial field induced by a time variation of gravitational field and surface density of gravitational current.	$\nabla\times\vec{I}=\vec{J}_G+\gamma\frac{d\vec{G}}{dt}=\vec{J}_G+\frac{d\vec{M}}{dt}$

8 Poisson and Laplace Gravitoinertial Equations

The gravitational Poisson's equation can be obtained by substituting $\vec{G}=-\nabla V_G$ in the differential form of the Gravitational Gauss's Law obtained in the table above:

$$\gamma\nabla\cdot\vec{G}=-\gamma(\nabla\cdot\nabla)V_G=-\gamma\nabla^2 V_G=\rho_G$$

In regions of space that have no density of gravitational charges, the scalar gravitational potential satisfies the Laplace's equation $\nabla^2 V_G=0$.

The inertial Poisson's equation can be obtained by substituting $\vec{I}=-\nabla V_I$ in the differential form of the Inertial Gauss's Law obtained in the table above:

$$\iota\nabla\cdot\vec{I}=-\iota(\nabla\cdot\nabla)V_I=-\iota\nabla^2 V_I=\rho_I$$

In regions of space that have no density of inertial charges, the scalar inertial potential satisfies the Laplace's equation $\nabla^2 V_I=0$.

9 Gravitoinertial Continuity Equations

Applying the divergent in the differential form of the Inertial Induction Law, we have:

$$\nabla\cdot\nabla\times\vec{I}=0=\nabla\cdot\left(\vec{J}_G+\frac{d\vec{M}}{dt}\right) \quad \Longrightarrow \quad 0=\nabla\cdot\vec{J}_G+\frac{d}{dt}\nabla\cdot\vec{M}$$

Because the divergent of the rotational is always zero, the above result can be expressed by substituting the Gravitational Gauss's Law in differential form $\gamma\nabla\cdot\vec{G}=\nabla\cdot\vec{M}=\rho_G$:

$$\nabla\cdot\vec{J}_G=-\frac{d}{dt}\nabla\cdot\vec{M}=-\frac{\partial\rho_G}{\partial t} \quad \Rightarrow \quad \nabla\cdot\vec{J}_G+\frac{\partial\rho_G}{\partial t}=0$$

This equation is called "gravitational continuity equation". When $\partial\rho_G/\partial t=0$ we get $\nabla\cdot\vec{J}_G=0$. This means that the vector flux, that is, the gravitational conduction current is conservative. In other words, the gravitational current that enters a certain volume is the same that exits the volume. When this does not occur it is because a certain amount of gravitational charge is being accumulated or extracted from the volume.

Applying the divergent in the differential form of the Gravitational Induction Law, we have:

$$\nabla\cdot\nabla\times\vec{G}=0=\nabla\cdot\left(\vec{J}_I+\frac{d\vec{O}}{dt}\right) \quad \Rightarrow \quad 0=\nabla\cdot\vec{J}_I+\frac{d}{dt}\nabla\cdot\vec{O}$$

Because the divergent of the rotational is always zero, the above result can be expressed by substituting the Inertial Gauss's Law in differential form $\iota\nabla\cdot\vec{I}=\nabla\cdot\vec{O}=\rho_I$.

$$\nabla\cdot\vec{J}_I=-\frac{d}{dt}\nabla\cdot\vec{O}=-\frac{\partial\rho_I}{\partial t} \quad \Rightarrow \quad \nabla\cdot\vec{J}_I+\frac{\partial\rho_I}{\partial t}=0$$

This equation is called "inertial continuity equation". When $\partial\rho_I/\partial t=0$ we get $\nabla\cdot\vec{J}_I=0$. This means that the vector flux, that is, the inertial conduction current is conservative. In other words, the inertial current that enters a certain volume is the same that exits the volume. When this does not occur it is because a certain amount of inertial charge is being accumulated or extracted from the volume.

10 Conclusion

It was presented a mathematical approach to the numerical value of the inertial permeability of the vacuum $\iota_0=9,337\cdot10^{-27}m\,kg^{-1}$ and its relation with the gravitational permeability of the vacuum and light speed $c^2=1/(\gamma_0\iota_0)$. This relation is similar to the electromagnetic relation $c^2=1/(\varepsilon_0\mu_0)$ and show us that the equations for the gravitational field need other field, that we call inertial, to maintain the same symmetries.

This inertial field is related with the equations of motion: the kinetic energy of a moving object is the mean variation of the inertial energy of this object. The energy furnished to or extracted from the object is kinetic energy, while inertial energy is the intrinsic energy of the object that stay in a certain velocity. This explains why an object maintain its orbit with the equilibrium of gravitational energy and inertial energy $U_G=q_G G r=q_G V_G=q_G v^2=q_I V_I=U_I$, and has nothing to do with kinetic energy.

This orbital equilibrium lead us to the principle of gravitational and inertial induction, like its electromagnetic counterpart, so an inertial current induces a gravitational potential and a gravitational current induces an inertial potential. Then we may produce, for the gravitoinertial fields, similar Maxwell's equations in its integral and differential forms.

We may attempt to the fact that these mathematical deductions are not yet tested experimentally. They are the establishment of a new mathematical model for calculating gravitoinertial quantities that are symmetric with electromagnetic equation. It is expected that this new mathematical model is far superior to the actual non symmetric method and will permits us to correctly deduce laws until now misunderstood.

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