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Electromagnetic and Gravitoinertial Fields Unification

Julio C. Gobbi[1]
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ABSTRACT: This article aims to integrate the electromagnetic and gravitoinertial fields equations into a symmetric form of presentation, together with an extended form of Maxwell equations for the gravitoinertial fields. It will be produced a unique equation for the classical physics from which all other equations may be derived, in its electromagnetic and gravitoinertial forms.

KEYWORDS: electromagnetic fields, gravitoinertial fields, unified fields theory, extended Maxwell equations.

Contents

1 Symbology

In this text we will use the following symbols with its abbreviated units of measure:

N = Newton, kg = kilogram, m = meter, s = second, V = Volt, C = Coulomb, A = Ampere, Wb = Weber.

E = Electric field intensity [$N\ C^{-1}$] [$V\ m^{-1}$];
D = Surface density of electric charge [$C\ m^{-2}$];
V_E = Electric potential [V] [$Wb\ s^{-1}$];
Φ_E = Electric flux [$N\ m^2\ C^{-1}$] [V m];

1 E-mail: solismagnus@gmail.com

q_E = Electric charge [C];
I_E = Electric current [C s^{-1}] [A];
ε_0 = Electric permittivity of vacuum [C^2 N^{-1} m^{-2}] [C V^{-1} m^{-1}];
k_E = Electrostatic constant [N m^2 C^{-2}] [m^3 C^{-1} s^{-2}];
H = Magnetic field intensity [N Wb^{-1}] [A m^{-1}];
B = Surface density of magnetic charge [Wb m^{-2}];
V_M = Magnetic potential [A] [C s^{-1}];
Φ_M = Magnetic flux [N m^2 Wb^{-1}] [A m];
q_M = Magnetic charge [Wb];
I_M = Magnetic current [Wb s^{-1}] [V];
μ_0 = Magnetic permeability of vacuum [Wb^2 N^{-1} m^{-2}] [Wb A^{-1} m^{-1}];
G = Gravitational field intensity [N kg^{-1}] [m s^{-2}];
M = Surface density of gravitational charge [kg m^{-2}];
V_G = Gravitational potential [N m kg^{-1}] [m^2 s^{-2}];
Φ_G = Gravitational flux [N m^2 kg^{-1}];
q_G = Gravitational charge [kg];
I_G = Gravitational current [kg s^{-1}] [N s m^{-1}];
γ_0 = Gravitational permeability of vacuum [kg^2 N^{-1} m^{-2}] [kg s^2 m^{-3}];
k_G = Gravitostatic constant = $6.6739*10^{-11}$ [N m^2 kg^{-2}] [m^3 kg^{-1} s^{-2}];
I = Inertial field intensity [N s m^{-2}] [kg m^{-1} s^{-1}];
O = Surface density of inertial charge [s^{-1}];
V_I = Inertial potential [N s m^{-1}] [kg s^{-1}];
Φ_I = Inertial flux [N s] [kg m s^{-1}];
q_I = Inertial charge [m^2 s^{-1}];
I_I = Inertial current [m^2 s^{-2}];
ι_0 = Inertial permeability of vacuum [m^2 N^{-1} s^{-2}] [m kg^{-1}];
F = Force [N] [kg m s^{-2}];
v = Velocity [m s^{-1}];
r = Radial length (radius) [m];
l = Length [m];
S = Area [m^2];
t = Time [s].

2 Introduction

Before undergoing the journey of unifying the electromagnetic and gravitoinertial equations, we need to expose some relations that we already have developed along four published articles: Magnetic Charge[1], Gravitational Charge[2], Inertial Field[3] and Gravitoinertial Fields[4].

The symmetry of these relations permits us to anticipate some new experiments but, to investigate if we can reproduce the equations that relates electromagnetic fields with gravitoinertial fields starting with only one equation, we need to find the similarities between some known equations. If there is a starting equation that reproduces all the others, then these others may be converted between themselves with mathematical manipulations.

3 EM and GI Fields Equations

Here we present the extended electromagnetic Maxwell equations in vacuum and matter, and the new gravitoinertial equations with a similar group of equations.

3.1 EM and GI Fields Equations in Vacuum

In the vacuum, there is no free charges nor conduction current, so the electromagnetic Maxwell equations assume the form bellow:

Electromagnetic Equations in Vacuum	
Electric Flux Law (Gauss' Law for electricity): Relates the electric flux to electric charges enclosed by a Gaussian surface.	$\varepsilon_0 \Phi_E = \varepsilon_0 \oint \vec{E} \cdot d\vec{S} = \oint \vec{D} \cdot d\vec{S} = 0$ $\Phi_E = \oint \vec{E} \cdot d\vec{S} = 0$
Magnetic Flux Law (Gauss' Law for magnetism): Relates the magnetic flux to magnetic charges enclosed by a Gaussian surface.	$\mu_0 \Phi_M = \mu_0 \oint \vec{H} \cdot d\vec{S} = \oint \vec{B} \cdot d\vec{S} = 0$ $\Phi_M = \oint \vec{H} \cdot d\vec{S} = 0$
Electric Induction Law (Faraday's Law): Relates the electric field induced by a time variation of magnetic flux.	$\oint \vec{E} \cdot d\vec{l} = \mu_0 \frac{d\Phi_M}{dt}$ $\oint \vec{D} \cdot d\vec{l} = \varepsilon_0 \mu_0 \frac{d\Phi_M}{dt}$
Magnetic Induction Law (Maxwell's Law): Relates the magnetic field induced by a time variation of electric flux.	$\oint \vec{H} \cdot d\vec{l} = \varepsilon_0 \frac{d\Phi_E}{dt}$ $\oint \vec{B} \cdot d\vec{l} = \varepsilon_0 \mu_0 \frac{d\Phi_E}{dt}$

In the vacuum, the gravitoinertial equations assume the form bellow:

Gravitoinertial Equations in Vacuum	
Gravitational Flux Law: Relates the gravitational flux to gravitational charges enclosed by a Gaussian surface.	$\gamma_0 \Phi_G = \gamma_0 \oint \vec{G} \cdot d\vec{S} = \oint \vec{M} \cdot d\vec{S} = 0$ $\oint \vec{G} \cdot d\vec{S} = 0$
Inertial Flux Law: Relates the inertial flux to inertial charges enclosed by a Gaussian surface.	$\iota_0 \Phi_I = \iota_0 \oint \vec{I} \cdot d\vec{S} = \oint \vec{O} \cdot d\vec{S} = 0$ $\oint \vec{I} \cdot d\vec{S} = 0$
Gravitational Induction Law: Relates the gravitational field induced by a time variation of inertial flux.	$\oint \vec{G} \cdot d\vec{l} = \iota_0 \frac{d\Phi_I}{dt}$ or $\oint \vec{M} \cdot d\vec{l} = \gamma_0 \iota_0 \frac{d\Phi_I}{dt}$
Inertial Induction Law: Relates the inertial field induced by a time variation of gravitational flux.	$\oint \vec{I} \cdot d\vec{l} = \gamma_0 \frac{d\Phi_G}{dt}$ or $\oint \vec{O} \cdot d\vec{l} = \gamma_0 \iota_0 \frac{d\Phi_G}{dt}$

3.2 EM and GI Fields Equations in Matter

In any substance where there are free charges or may conduct current (the medium presents a finite resistance to the passage of direct current), the electromagnetic Maxwell equations assume the form bellow:

Electromagnetic Equations in Matter	
Electric Flux Law (Gauss' Law for electricity): Relates the electric flux to electric charges enclosed by a Gaussian surface.	$\varepsilon \Phi_E = \varepsilon \oint \vec{E} \cdot d\vec{S} = \oint \vec{D} \cdot d\vec{S} = q_E$

Electromagnetic Equations in Matter	
Magnetic Flux Law (Gauss' Law for magnetism): Relates the magnetic flux to magnetic charges enclosed by a Gaussian surface.	$\mu\Phi_M = \mu\oint \vec{H}\cdot d\vec{S} = \oint \vec{B}\cdot d\vec{S} = q_M$
Electric Induction Law (Faraday's Law): Relates the electric field induced by a time variation of magnetic flux.	$\oint \vec{E}\cdot d\vec{l} = I_M + \mu\frac{d\Phi_M}{dt}$ $\oint \vec{D}\cdot d\vec{l} = \varepsilon I_M + \varepsilon\mu\frac{d\Phi_M}{dt}$
Magnetic Induction Law (Ampère-Maxwell's Law): Relates the magnetic field induced by a time variation of electric flux.	$\oint \vec{H}\cdot d\vec{l} = I_E + \varepsilon\frac{d\Phi_E}{dt}$ $\oint \vec{B}\cdot d\vec{l} = \mu I_E + \varepsilon\mu\frac{d\Phi_E}{dt}$

In these same substances, the gravitoinertial equations assume the form bellow:

Gravitoinertial Equations in Matter	
Gravitational Flux Law: Relates the gravitational flux to gravitational charges enclosed by a Gaussian surface.	$\gamma\Phi_G = \gamma\oint \vec{G}\cdot d\vec{S} = \oint \vec{M}\cdot d\vec{S} = q_G$
Inertial Flux Law: Relates the inertial flux to inertial charges enclosed by a Gaussian surface.	$\iota\Phi_I = \iota\oint \vec{I}\cdot d\vec{S} = \oint \vec{O}\cdot d\vec{S} = q_I$
Gravitational Induction Law: Relates the gravitational field induced by a time variation of inertial flux.	$\oint \vec{G}\cdot d\vec{l} = I_I + \iota\frac{d\Phi_I}{dt}$ $\oint \vec{M}\cdot d\vec{l} = \gamma I_I + \gamma\iota\frac{d\Phi_I}{dt}$
Inertial Induction Law: Relates the inertial field induced by a time variation of gravitational flux.	$\oint \vec{I}\cdot d\vec{l} = I_G + \gamma\frac{d\Phi_G}{dt}$ $\oint \vec{O}\cdot d\vec{l} = \iota I_G + \gamma\iota\frac{d\Phi_G}{dt}$

It may be noted that q_M in the Magnetic Flux Law and q_I in the Inertial Flux Law are put here to maintain the symmetry of equations. Until now it was not proved that magnetic and inertial monopoles exist but, if we put zero in these equations, what is the proof that these monopoles do not exist in nature or can't be artificially created?

4 Equivalences between Equations

The objective of this section is to show that some equations are only algebraic variations of another; they are mathematically equivalent although they refer to different experiments. So, the multitude of equations actually used in electromagnetism, forces (linked with acceleration/gravitational field) and velocities (linked with inertial field) may be reduced to a few ones that will derive all.

4.1 Hall Effect and Electric Induction Law Equivalence

The Hall effect is the deflection of drifting conduction charges by a magnetic field in any electric conductor or semi-conductor material. The effect is the same as the deflection of a beam of electrons in an evacuated tube by a magnetic field and is determined by $\vec{F}=q_E\vec{E}=q_E(\vec{v}\times\vec{B})$ equation, considering that v is the drift speed of the electric carriers.

The vector product show us that E, v and B produces the maximum force when the vectors are perpendicular. If we consider that the electric charge is drifting in the x-coordinate and the magnetic field is in the z-coordinate direction, lets put each term in its own coordinates:

$$\vec{F}=F\,\hat{y} \qquad \vec{v}=\frac{d\vec{x}}{dt} \qquad \vec{B}=B\hat{z} \implies \vec{F}=q_E(\vec{v}\times\vec{B})=q_E\left(\frac{d\vec{x}}{dt}\times\vec{B}\right)=\frac{dq_E}{dt}\vec{x}\times\vec{B}=I_E\vec{x}\times\vec{B}$$

So, this is the expression of a force due to an electric current passing in a section of length x of a wire inside a magnetic field $\vec{F}=I_E\vec{l}\times\vec{B}$. And this show us that algebraic manipulations of the equations does not lead us to wrong results, because these are experimental results, but can lead us to new experiments. We can continue with these algebraic manipulations to produce the Electric Induction Law.

Taking the electric field produced by the deflected carriers, we have $\vec{E}=\vec{v}\times\vec{B}$. If we consider the same directions:

$$E\,\hat{y}=v\,\hat{x}\times B\,\hat{z}=\left(\frac{dx}{dt}\hat{x}\times\frac{q_M}{S_{xy}}\hat{z}\right)=\frac{dq_M}{dt}\frac{x}{S_{xy}}\hat{x}\times\hat{z} \quad \text{with} \quad B=\frac{q_M}{S_{xy}}$$

Integrating both sides in relation to the y-coordinate, we have the Electric Induction Law with a magnetic conduction current[2]:

$$V_E=-\int\vec{E}\cdot d\vec{l}=-\int_{y1}^{y2}E\,dy=-\int_{y1}^{y2}\frac{dq_M}{dt}\frac{x}{S_{xy}}dy=-\frac{dq_M}{dt}\frac{x\,y}{S_{xy}}=-\frac{dq_m}{dt}=-I_M \quad \text{with} \quad \int_{y1}^{y2}x\,dy=x\,y=S_{xy}$$

So, the Hall effect is a direct consequence of the Magnetic Induction Law: the induced magnetic field produced by an electric charge in motion interacts with the external magnetic field and produces a force on the electric charge.

In other point of view, we may use the relation $\vec{E}=\vec{v}\times\vec{B}=\mu_0\vec{v}\times\vec{H}$, considering the same directions:

$$E\,\hat{y}=\mu_0 v\,\hat{x}\times H\,\hat{z}=\mu_0\left(\frac{dx}{dt}\hat{x}\times H\,\hat{z}\right)=\mu_0\frac{dH}{dt}x\,\hat{x}\times\hat{z}$$

Integrating both sides in relation to the y-coordinate, we have the Electric Induction Law with a time variation of the magnetic flux:

$$V_E=-\int\vec{E}\cdot d\vec{l}=-\int_{y1}^{y2}E\,dy=-\mu_0\int_{y1}^{y2}\frac{dH}{dt}x\,dy=-\mu_0\frac{d\Phi_M}{dt} \quad \text{with} \quad \int_{y1}^{y2}H\,x\,dy=H\,S_{xy}=\Phi_M$$

2 Please read the Magnetic Charge article where it is demonstrated that the Faraday’s Induction Law is consequence of a magnetic current.

If we consider that the force over the electric charge is $\vec{F}=q_E\vec{E}$, then we may create this same force with a stationary electric charge and a time varying magnetic field, whose variation of flux induces an electric potential that interacts with the electric charge.

4.2 Charge Equivalence

We know that electric charges interact with electric fields and, if they are stationary, they do not interact with magnetic fields. But electric charges in motion interact with magnetic fields because its motion (electric current) induces a magnetic field, and this induced field interact with the external one. This way, we may think that an electric charge in motion is equivalent to a magnetic charge.

The Magnetic Induction Law (Ampère's Law) can help us to get the equivalence between an electric charge in motion and a magnetic charge. It is the magnetic field induced by an electric current. It is considered that the velocity $v=dx/dt$ of the electric charges is on the x-coordinate; the closed path l is on the y-coordinate and z-coordinate; the surface $S=x\,l$ of the magnetic charge distribution is on the x-coordinate and path l:

$$\oint\vec{B}\cdot d\vec{l}=\mu_0\oint\vec{H}\cdot d\vec{l}=\mu_0 I_E \;\Rightarrow\; \oint\frac{q_M}{\vec{S}}\cdot d\vec{l}=\mu_0\frac{d\,q_E}{dt} \;\Rightarrow\; \oint q_M\,dl=\mu_0\frac{d\,q_E}{dt}S=\mu_0\frac{dx}{dt}\oint q_E\,dl$$

$$\Rightarrow\; \oint q_M\,dl=\mu_0 v\oint q_E\,dl \;\Rightarrow\; q_M=\mu_0 v\,q_E$$

The equivalence between a magnetic charge in motion and an electric charge may be obtained with the Electric Induction Law (Faraday's Law):

$$\oint\vec{D}\cdot d\vec{l}=\varepsilon_0\oint\vec{E}\cdot d\vec{l}=\varepsilon_0 I_M \;\Rightarrow\; \oint\frac{q_E}{\vec{S}}\cdot d\vec{l}=\varepsilon_0\frac{d\,q_M}{dt} \;\Rightarrow\; \oint q_E\,dl=\varepsilon_0\frac{d\,q_M}{dt}S=\varepsilon_0\frac{dx}{dt}\oint q_M\,dl$$

$$\Rightarrow\; \oint q_E\,dl=\varepsilon_0 v\oint q_M\,dl \;\Rightarrow\; q_E=\varepsilon_0 v\,q_M$$

If this seems obscure, lets take the Hall effect $\vec{F}=q_E\vec{E}=q_E(\vec{v}\times\vec{B})$, whose deflection of the carriers in a strip of electric conducting material submitted to a magnetic field produces an electric field. We may define the magnetic field module in terms of the force and confirm that the force of a magnetic field on an electric charge in motion is equivalent to a force on a stationary magnetic charge $\vec{F}=q_M\vec{H}$:

$$F=|q_E|v\,B \;\Rightarrow\; B=\mu_0 H=\frac{F}{|q_E|v} \;\Rightarrow\; H=\frac{F}{\mu_0|q_E|v}=\frac{F}{q_M} \;\Rightarrow\; q_M=\mu_0 v\,q_E$$

If we use the magnetic equivalent of the Hall effect $\vec{F}=q_M\vec{H}=q_M(\vec{v}\times\vec{D})$, we may define the electric field module in terms of the force and confirm that the force of an electric field on a magnetic charge in motion is equivalent to a force on a stationary electric charge $\vec{F}=q_E\vec{E}$:

$$F=|q_M|v\,D \;\Rightarrow\; D=\varepsilon_0 E=\frac{F}{|q_M|v} \;\Rightarrow\; E=\frac{F}{\varepsilon_0|q_M|v}=\frac{F}{q_E} \;\Rightarrow\; q_E=\varepsilon_0 v\,q_M$$

An other method to obtain the same relations is considering the electric field developed in the Hall effect $\vec{E}=\vec{v}\times\vec{B}$ and the magnetic field developed in the magnetic equivalent of the Hall effect and substituting the following relations:

$$\vec{D}=\varepsilon_0\vec{E}=\frac{q_E}{\vec{S}} \implies \vec{E}=\frac{q_E}{\varepsilon_0\vec{S}} \quad \text{and} \quad \vec{B}=\mu_0\vec{H}=\frac{q_M}{\vec{S}} \implies \vec{H}=\frac{q_M}{\mu_0\vec{S}}$$

With the Hall effect:

$$\vec{E}=\vec{v}\times\vec{B} \implies \frac{q_E}{\varepsilon_0\vec{S}}=\vec{v}\times\frac{q_M}{\vec{S}} \implies q_E=\varepsilon_0 v q_M$$

With the magnetic equivalent of the Hall effect:

$$\vec{H}=\vec{v}\times\vec{D} \implies \frac{q_M}{\varepsilon_0\vec{S}}=\vec{v}\times\frac{q_E}{\vec{S}} \implies q_M=\mu_0 v q_E$$

In what concerns to charges in motion, an electric charge may be mathematically and experimentally considered a magnetic charge and vice-versa.

4.3 Biot-Savart Law and Magnetic Field Equivalence

The Biot-Savart Law asserts that the contribution $d\vec{B}$ to the magnetic field produced by a current-length element $I d\vec{l}$ at a point P located at a distance r from the current element may be equated by:[5]

$$d\vec{B}=\frac{\mu_0}{4\pi}\frac{I_E d\vec{l}}{r^2}\times\hat{r} \qquad dB=\frac{\mu_0}{4\pi}\frac{I_E dl}{r^2}\sin\theta$$

The magnitude of the differential magnetic field is on the right. The vector product indicates that the direction of the magnetic field produced is perpendicular to the motion direction of the electric charges, so θ is the angle between the directions of $d\vec{l}$ and $\hat{r}$, a unit vector that points from dl towards P.

Assuming that the unit vector that points from dl towards P is perpendicular to the vector $d\vec{l}$ and with $B=\mu_0 H$, $I=dq_E/dt$ and $q_M=\mu_0 v q_E$, we may reduce this equation to:

$$d\vec{H}=\frac{1}{4\pi}\frac{I_E d\vec{l}}{r^2}\times\hat{r}=\frac{1}{4\pi}\frac{dq_E}{r^2}\frac{d\vec{l}}{dt}\times\hat{r}=\frac{1}{4\pi\mu_0}\frac{\mu_0 dq_E\vec{v}}{r^2}\times\hat{r}=\frac{1}{4\pi\mu_0}\frac{dq_M}{r^2}\hat{l}\times\hat{r} \qquad dH=\frac{1}{4\pi\mu_0}\frac{dq_M}{r^2}$$

Integrating this relation along a little section l:

$$H=\int_{l1}^{l2}\frac{1}{4\pi}\frac{I_E dl}{r^2}=\frac{1}{4\pi\mu_0}\frac{1}{r^2}\int\mu_0 v\,dq_E=\frac{1}{4\pi\mu_0}\frac{1}{r^2}\int dq_M=\frac{1}{4\pi\mu_0}\frac{q_M}{r^2} \qquad \vec{H}=\frac{1}{4\pi\mu_0}\frac{q_M}{r^2}\hat{l}\times\hat{r}$$

So, the Biot-Savart Law is the magnetic field equation calculated at a point P that is at a distance r from a magnetic charge q_M, considering the equivalence of electric and magnetic charges.

4.4 Forces Over Charges Equivalence

The force exerted over a mobile peace of ferromagnetic material (like a relay) when submitted to a magnetic field is given by $F=\mu_0 H^2 S$, with the magnetic field H calculated by the electric current I_E and the number of coils turns n by the equation $Hl=nI_E$. The magnetic field is created inside the volume of the air gap, determined by its length l and the surface S, so both faces of the gap have a surface distribution of magnetic charge $B=q_M/S=\mu_0 H$, and the total surface magnetic charge is $q_M=\mu_0 H S$.

Then, the force between the magnetized part and the magnetic field may be equated by $F=\mu_0 H^2 S=q_M H$. This equation is similar to its electric counterpart $F=q_E E$, that is the force between an electric particle inside an electric field. Because the magnetic force equation only considers magnetic fields, we may consider any magnetic field like being created by a surface distribution of magnetic charge and calculate the force in the analogous electric way.

We may go further to create a magnetic equivalent for the Lorentz's Force $\vec{F}=q_E(\vec{E}+\vec{v}\times\vec{B})$ that is given by $\vec{F}=q_M(\vec{H}+\vec{v}\times\vec{D})$ and consider that there is a magnetic equivalent to the Hall effect $\vec{F}=q_M(\vec{v}\times\vec{D})$. This time, magnetic charges moving inside an electric field suffer a force whose deflection produces a magnetic field $\vec{H}=\vec{v}\times\vec{D}$.

4.5 Hall Effect and Wave Equation Equivalence

Lets consider the one-dimensional traveling wave that propagates with speed v in the x-coordinate whose amplitude of oscillation is in the y-coordinate. It is mathematically expressed by:

$$y(x,t)=f(z) \quad \text{with} \quad z=x\pm vt$$

The positive or negative sign indicates that the wave propagates to the left or right direction. The equation of motion associated with the wave propagation may be calculated with the velocity and acceleration at a point x. The velocity and acceleration that the point x moves in the y-coordinate at instant t is:

$$v=\frac{\partial}{\partial t}y(x,t) \quad \text{and} \quad a=\frac{\partial^2}{\partial t^2}y(x,t)$$

Here, $y(x,t)=f(z)$ depends on t through $z=x\pm vt$ (function of function), so we use the chain rule to calculate the first and second derivatives:

$$\frac{\partial z}{\partial t}=\frac{\partial}{\partial t}(x-vt)=-v \qquad \frac{\partial y}{\partial t}=\frac{df}{dz}\frac{\partial z}{\partial t}=-v\frac{df}{dz} \qquad \frac{\partial^2 y}{\partial t^2}=-v\frac{\partial}{\partial t}\left(\frac{df}{dz}\right)=-v\frac{d}{dz}\left(\frac{df}{dz}\right)\frac{\partial z}{\partial t}=v^2\frac{d^2 f}{dz^2}$$

$$\frac{\partial z}{\partial x}=\frac{\partial}{\partial x}(x-vt)=1 \qquad \frac{\partial y}{\partial x}=\frac{df}{dz}\frac{\partial z}{\partial x}=\frac{df}{dz} \qquad \frac{\partial^2 y}{\partial x^2}=\frac{\partial}{dx}\left(\frac{df}{dz}\right)=\frac{d}{dz}\left(\frac{df}{dz}\right)\frac{\partial z}{\partial x}=\frac{d^2 f}{dz^2}$$

With the first derivatives we obtain:

$$\frac{\partial y}{\partial t}=-v\frac{df}{dz} \qquad \frac{\partial y}{\partial x}=\frac{df}{dz} \quad \Rightarrow \quad \frac{\partial y}{\partial t}=-v\frac{dy}{dx}$$

With the second derivatives we obtain:

$$\frac{\partial^2 y}{\partial t^2}=v^2\frac{d^2 f}{dz^2} \qquad \frac{\partial^2 y}{\partial x^2}=\frac{d^2 f}{dz^2} \quad \Rightarrow \quad \frac{\partial^2 y}{\partial t^2}=v^2\frac{d^2 y}{dx^2}$$

This is the one-dimensional traveling wave equation; it is a linear partial derivative equation of second order and rules electromagnetic and gravitoinertial waves. But its partial first order derivatives gives us another relation, considering that the amplitude variation of the field in the y-coordinate is given by the motion of charges. Substituting y by magnetic charge q_M:

$$\frac{\partial q_M}{\partial t}=-v\frac{dq_M}{dx}=-v\int\vec{B}\cdot d\vec{l} \quad \text{and} \quad I_M=\frac{\partial q_M}{\partial t}=-V_E=\int\vec{E}\cdot d\vec{l} \quad \Longrightarrow \quad \int\vec{E}\cdot d\vec{l}=-v\int\vec{B}\cdot d\vec{l}$$

Derivation of both sides in relation to l and considering the product of vectors, we have $\vec{E}=\vec{v}\times\vec{B}$, the equation that defines the Hall effect. If we substitute y by electric charge q_E:

$$\frac{\partial q_E}{\partial t}=-v\frac{dq_E}{dx}=-v\int\vec{D}\cdot d\vec{l} \quad \text{and} \quad I_E=\frac{\partial q_E}{\partial t}=-V_M=\int\vec{H}\cdot d\vec{l} \quad \Longrightarrow \quad \int\vec{H}\cdot d\vec{l}=-v\int\vec{D}\cdot d\vec{l}$$

Derivation of both sides in relation to l and considering the product of vectors, we have $\vec{H}=\vec{v}\times\vec{D}$, the magnetic counterpart of the Hall effect.

Here we see that the Hall effect equation is another form of the wave equation.

5 Unification of EM and GI Fields

The equivalence of the equations relating electromagnetic and gravitoinertial fields seen above lead us to a unifying equation that can reproduce all these equations.

Starting from the Hall effect and integrating both sides in relation to distance:

$$\vec{E}=\vec{v}\times\vec{B} \quad \Longrightarrow \quad \int\vec{E}\cdot d\vec{l}=\vec{v}\times\int\vec{B}\cdot d\vec{l}$$

And using $v=\frac{dl}{dt}=\int\vec{O}\cdot d\vec{l} \quad \Longrightarrow \quad \int\vec{E}\cdot d\vec{l}=\int\vec{O}\cdot d\vec{l}\times\int\vec{B}\cdot d\vec{l}$

Multiplying both sides by v and knowing that the inertial current induces a gravitational potential $I_I=v^2=V_G=\int\vec{G}\cdot d\vec{l}$:

$$v\int\vec{E}\cdot d\vec{l}=v^2\times\int\vec{B}\cdot d\vec{l} \quad \Longrightarrow \quad \int\vec{O}\cdot d\vec{l}\int\vec{E}\cdot d\vec{l}=\int\vec{G}\cdot d\vec{l}\times\int\vec{B}\cdot d\vec{l}$$

Starting from the magnetic equivalent to the Hall effect and integrating both sides in relation to distance:

$$\vec{H}=\vec{v}\times\vec{D} \quad \Longrightarrow \quad \int\vec{H}\cdot d\vec{l}=\vec{v}\times\int\vec{D}\cdot d\vec{l}$$

And using $\vec{v}=\frac{d\vec{l}}{dt}=\int\vec{O}\cdot d\vec{l} \quad \Longrightarrow \quad \int\vec{H}\cdot d\vec{l}=\int\vec{O}\cdot d\vec{l}\times\int\vec{D}\cdot d\vec{l}$.

Multiplying both sides by v and knowing that $v^2=V_G=\int\vec{G}\cdot d\vec{l}$:

$$v\int\vec{H}\cdot d\vec{l}=v^2\times\int\vec{D}\cdot d\vec{l} \quad \Longrightarrow \quad \int\vec{O}\cdot d\vec{l}\int\vec{H}\cdot d\vec{l}=\int\vec{G}\cdot d\vec{l}\times\int\vec{D}\cdot d\vec{l}$$

These same equations may be deduced starting from the Electric Induction Law and knowing that $1/\varepsilon_0\mu_0=c^2=V_G=\int\vec{G}\cdot d\vec{l}$:

$$\oint\vec{D}\cdot d\vec{l}=\varepsilon_0\mu_0\frac{d\Phi_M}{dt} \quad \Longrightarrow \quad \frac{1}{\varepsilon_0\mu_0}\oint\vec{D}\cdot d\vec{l}=\frac{d\Phi_M}{dt}=\oint\vec{G}\cdot d\vec{l}\oint\vec{D}\cdot d\vec{l}$$

And from the Magnetic Induction Law:

$$\oint \vec{B}\cdot d\vec{l}=\varepsilon_0\mu_0\frac{d\Phi_E}{dt} \implies \frac{1}{\varepsilon_0\mu_0}\oint \vec{B}\cdot d\vec{l}=\frac{d\Phi_E}{dt}=\oint \vec{G}\cdot d\vec{l}\oint \vec{B}\cdot d\vec{l}$$

The temporal derivative of the magnetic flux may be expressed by:

$$\frac{d\Phi_M}{dt}=O\Phi_M=O\oint \vec{H}\cdot d\vec{S}=\oint \vec{O}\cdot d\vec{l}\oint \vec{H}\cdot d\vec{l}$$

The temporal derivative of the electric flux may be expressed by:

$$\frac{d\Phi_E}{dt}=O\Phi_E=O\oint \vec{E}\cdot d\vec{S}=\oint \vec{O}\cdot d\vec{l}\oint \vec{E}\cdot d\vec{l}$$

The relation between electromagnetic and gravitoinertial fields, without considering the vector product, may be equated by:

$$\oint \vec{G}\cdot d\vec{l}\oint \vec{D}\cdot d\vec{l}=\oint \vec{O}\cdot d\vec{l}\oint \vec{H}\cdot d\vec{l} \quad \text{and} \quad \oint \vec{G}\cdot d\vec{l}\oint \vec{B}\cdot d\vec{l}=\oint \vec{O}\cdot d\vec{l}\oint \vec{E}\cdot d\vec{l}$$

Applying the rotational theorem on the equations above, we have their differential form:

$$\int(\nabla\times\vec{G})\cdot d\vec{S}\int(\nabla\times\vec{D})\cdot d\vec{S}=\int(\nabla\times\vec{O})\cdot d\vec{S}\int(\nabla\times\vec{H})\cdot d\vec{S} \implies \nabla\times\vec{G}\,\nabla\times\vec{D}=\nabla\times\vec{O}\,\nabla\times\vec{H}$$

$$\int(\nabla\times\vec{G})\cdot d\vec{S}\int(\nabla\times\vec{B})\cdot d\vec{S}=\int(\nabla\times\vec{O})\cdot d\vec{S}\int(\nabla\times\vec{E})\cdot d\vec{S} \implies \nabla\times\vec{G}\,\nabla\times\vec{B}=\nabla\times\vec{O}\,\nabla\times\vec{E}$$

6 Conclusion

Now we have reached our goal, the unification of electromagnetic and gravitoinertial equations into a unique equation. We see that, until now, what prevented us from achieving this objective was the inconsistency and the lack of equations about the fields as a whole.

With the correction to the inconsistent magnetic flux equation we have reached to completely symmetrical electromagnetic equations. The inclusion of the inertial field, that interacts with the gravitational field by an induction relation, permitted us to discover the connection of the mechanical fields (gravitational and inertial) with the speed of light and, as consequence, have created extended Maxwell equations that include mechanical fields.

It was established that the inertial field is directly related with velocity and kinetic energy, and the mechanical fields may be invariant by Lorentz transformation, thanks to the speed of light link, so the Newtonian, Lagrangean and Hamiltonian formulations may include a relativistic correction inside of it for the mechanical fields, because the inertial field is the cause of the relativistic correction.

We may attempt to the fact that some of these mathematical deductions are not yet tested experimentally. They are the establishment of a new symmetric mathematical model for calculating electromagnetic and gravitoinertial quantities. It is expected that this new mathematical model for classical physics is far superior to the actual incomplete and non symmetric classical model, and will permits us to correctly deduce laws until now misunderstood.

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