

## THE PROBLEM OF PHOTON GAS: HOW TO SOLVE IT CORRECTLY?

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**Abstract:** The correct theoretical analysis of the generally accepted foundations of the theory of photon (quantum) gas are proposed. The principle of the unity of formal logic and of rational dialectics is the correct methodological basis of the analysis. The new results – the correct quantum-statistical foundations – obtained within the framework of the formulated master equation taking into consideration both the quantum states of the radiating molecule and the quantum states of the photon gas in the isolated macroscopic systems “molecule + molecular gas + monochromatic photon gas” are as follows: (a) Planck’s, Einstein’s, and Bose’s works on the theory of photon (quantum) gas contain logical errors; (b) photon (quantum) gas being born by radiating molecule obeys “Gibbs statistics”: equilibrium photon (quantum) gas is described by Gibbs quantum canonical distribution; (c) Planck function (“Bose’s distribution”) is an consequence of Gibbs quantum canonical distribution; (d) Einstein coefficients (i.e. the coefficients of spontaneous emission, induced emission and absorption) are equal to each other.

**Keywords:** quantum theory, statistical physics, heat radiation, Planck function, photon gas

**PACS numbers:** 02.50.-r, 03.65.Ta, 05.30.-d

### Introduction

Science is the sphere of human activity which manifests clearly the unity of two aspects of cognition: the faith in the possibility of knowing the world and the doubt about the truth of knowing. The way of cognition is characterized, for example, by A. Einstein’s words: (a) *there has been formed a view that the foundations of physics were finally established and the work of a theoretical physicist should be to bring a theory in correspondence with all the time increasing abundance of the investigated phenomena. Nobody thought that a need for radical rebuilding of the basis of all physics could arise;* (b) *but the progress of science will cause revolution in its foundations. Our notions of physical reality never can be final ones. We should be always ready to change axiomatic basis of physics to substantiate facts of perception in logically most perfect form.* It follows from these words that “the progress in science is the underlining of difficulties” (N. Bohr).

As is known, the generally accepted statistical theory of photon (quantum) gas is a result of research of the problem of heat radiation. The foundations of this theory were proposed by M. Planck [1, 2], A. Einstein [3], and S.N. Bose [4] and represent the fundamental part of modern theoretical physics. Up to now, nobody cast doubt on this part of physics since one assumed that the theory of photon gas is in accordance with experimental data. There has been formed the point of view that the works of classics of physics were finally understood and a need for critical analysis these works cannot arise. However, it was recently shown [5-11] for the first time that the generally accepted foundations of statistical physics contain logical errors. Consequently, there is the problem of truth of Planck’s, Einstein’s, and Bose’s works in theoretical physics, and

the critical analysis of the theory of photon gas is the urgent problem of our time. The purpose of the present work is to give critical analysis of Planck's, Einstein's, and Bose's works on the theory of heat radiation and to propose the correct foundations of the statistical theory of photon gas. The obtained results are based on the key idea that the problem of correct quantum-statistical description of photon gas should be solved within the framework of master equation taking into consideration both the quantum states of the radiating molecule and the quantum states of the photon gas in the isolated macroscopic systems "molecule + molecular gas + monochromatic photon gas". From this point of view, the following connected problems are considered: (a) the new foundations of the theory of quantum particle; (b) the theory of equilibrium ideal gas of nonradiating molecules; (c) the theory of non-equilibrium ideal gas of radiating molecules; (d) the theory of photon (quantum) gas being born by radiating molecule.

### **1. Methodological basis for a theoretical analysis**

Modern theoretical physics consists of the set of theories but does not contain criterion of the truth of physical theories. In my opinion, lack of the criterion of the truth of theories in theoretical physics is explained by the fact that the system of physical (i.e. special scientific) concepts and laws is incomplete: it does not include many universal (i.e. the general scientific) concepts and laws. The complete system – the system of physical concepts and the laws, supplemented with the system of universal concepts and laws – would represent not only basis of physics but also methodological basis for the deductive analysis of physics. From this point of view, the unified criterion of the truth of physical theory should be formulated as follows: a physical (i.e. special scientific) theory must not contradict the system of the universal (i.e. general scientific) concepts and laws. The system of the universal concepts and laws represents a unity of formal logic and of rational dialectics. And this unity is a science of most general laws of development of the Nature, human society, and correct thinking. Consequently, this system is a methodological basis for a critical analysis of physical theories. The main dialectics principle is the principle of objectivity of human knowledge. It is formulated as follows: objective laws and truth must be invariant under choice of means and of methods of cognition, i.e. under change of properties of system of reference (in particular, objective laws and truth must not contain references to devices, procedure and accuracy of measurement or of calculation). Thus, the principle of the unity of formal logic and of rational dialectics is the correct methodological basis for any theoretical analysis.

### **2. The critical analysis of Planck's, Einstein's, and Bose's theory of photon gas**

As is known, the generally accepted foundations of the theory of photon gas were proposed by Planck [1, 2], Einstein [3], and Bose [4]. Below, stages of development and the basic representations of this theory are considered.

The first stage is a stage of research of the law of heat radiation. The law of heat radiation was formulated by Planck in work [1, 2]. The derivation of the radiation formula, proposed by Planck, consists of three parts: the classical-electrodynamic calculation, the statistical-thermodynamic approach and the quantum-statistical interpretation.

A starting point of the classical-electrodynamic calculation is "the understanding of the phenomena of emission and absorption of heat radiation as electromagnetic processes. Emission of heat beams is stipulated by emission of electromagnetic waves from certain elementary oscillators; absorption of heat beams is obliged to resonance phenomenon at which named oscillators not only radiate waves, but also are set in oscillatory motion by waves falling on them" (Planck). The equation of radiative balance between oscillator and radiation represents Planck's formula (i.e. formulation of Planck's radiation law):

$$u_{\nu}^{(radiation)} d\nu = \frac{8\pi\nu^2}{c^3} U_{\nu}^{(oscillator)} d\nu$$

where  $\nu$  is the frequency of the resting linear oscillator;  $U_{\nu}^{(oscillator)}$  is the energy of the radiating oscillator;  $u_{\nu}^{(radiation)}$  is the spectral energy density of radiation;  $c$  is the speed of light in vacuo. In order to find  $u_{\nu}^{(radiation)}$ , it is necessary to define  $U_{\nu}^{(oscillator)}$ .

The statistical-thermodynamic approach to definition of  $U_{\nu}^{(oscillator)}$  is characterized by introduction of two hypotheses. Planck assumed that, firstly, the energy of the resting oscillator is a random quantity. And, secondly, “energy of oscillator must be the whole multiple of element of energy”. These hypotheses and the definition

$$\frac{1}{T} = \frac{dS^{(oscillator)}}{dU_{\nu}^{(oscillator)}} \equiv \frac{1}{E_1} \ln \left( \frac{E_1}{U_{\nu}^{(oscillator)}} + 1 \right)$$

of the temperature of the oscillator lead to the following expression:

$$U_{\nu}^{(oscillator)} = E_1 [\exp(E_1/T) - 1]^{-1}.$$

The quantum-statistical interpretation of quantities  $U_{\nu}^{(oscillator)}$  and  $E_1$  represents the following procedure:

a) an identification of the quantity  $U_{\nu}^{(oscillator)}$  with the statistical-average energy  $E$  of the harmonic oscillator, i.e.

$$U_{\nu}^{(oscillator)} \equiv E, \quad E = \sum_{n=0}^{\infty} E_n f_n^{\circ}, \quad E_n = E_1 n, \quad f_n^{\circ} = f_0^{\circ} \exp(-E_n/T)$$

where  $f_n^{\circ}$  is Gibbs quantum canonical distribution;

b) an identification of the energy quantum  $E_1$  of the harmonic oscillator with the energy quantum  $h\nu_1^{(radiation)} \equiv E_{n+1} - E_n$  of the radiation, i.e.

$$E_1 \equiv h\nu_1^{(radiation)};$$

c) an identification of the quantity  $\nu_1^{(radiation)}$  with the continuous frequency  $\nu$  of the electromagnetic radiation, i.e.

$$\nu_1^{(radiation)} \equiv \nu.$$

As a result of such interpretation (i.e. substitution of the energy  $h\nu$  of photon into the expression for average energy  $U_{\nu}^{(oscillator)}$  of the oscillator), the oscillator transmutes into non-identical object – photon gas:

$$U_{\nu} \equiv h\nu\rho_{\nu}^{\circ}, \quad \rho_{\nu}^{\circ} \equiv [\exp(h\nu/T) - 1]^{-1}$$

where  $\rho_\nu^\circ$  is Planck function (i.e. the average number of the  $\nu$ -monochromatic photons being born by the oscillator),  $T$  is the temperature of the oscillator. This interpretation is the reason why the expression  $U_\nu \equiv h\nu\rho_\nu^\circ$  does not satisfy to the formal-logic law of identity. Really, according to the law of identity, the left and right parts of a mathematical (quantitative) relationship must belong to one and the same qualitative determinacy of physical object:

$$\begin{aligned} &(\text{oscillator}) = (\text{oscillator}) \text{ or } (\text{photon gas}) = (\text{photon gas}) \\ &\text{or } (\text{oscillator} + \text{photon gas}) = (\text{oscillator} + \text{photon gas}). \end{aligned}$$

However, the left and right parts of the expression  $U_\nu \equiv h\nu\rho_\nu^\circ$  belong to different qualitative determinacy: the left part belongs to the oscillator and the right part belongs to the photon gas. Consequently, this interpretation represents a logical error.

The achievement of the first stage was formulated by Planck as follows: “Measurements confirmed validity of the formula for radiation. However, it has only formal sense of lucky guessed law”. “One can give to it its true physical sense if two circumstances will be taken into consideration”:

1) “Heat radiation concluded in pure vacuum limited by reflecting walls keeps for ever initial spectral distribution of energy. The fundamental discrepancy between the theory of radiation and the kinetic theory of gases becomes apparent here. The cause of this distinction is that gas molecules collide with each other whereas beams pass through each other. Results of the collisions can be found only with the help of probabilistic laws” (Planck).

2) “Introduction of a minimum quantity of absorbing and of emitting substance leads to gradual change of distribution and its transition into stationary state corresponding black body radiation. Therefore, from the described point of view, calculation of probability of the certain value of energy is impossible in principle without taking into consideration the act of appearance of this energy. And it forces to consider more in detail the processes occurring at emission and absorption of heat radiation” (Planck).

The second stage is characterized by tendency for “giving such form of the theory that it was based upon non-contradictory premises” (Einstein). These premises were formulated by Einstein in work [3]. Einstein considered isolated ideal gas of molecules being in statistical equilibrium with heat radiation. He showed that Planck formula is a consequence of quantum-statistical properties of substance and hypothesis about three processes: spontaneous emission, induced emission and absorption. He proposed the following form of master equation, expressing a condition of statistical equilibrium in isolated system:

$$(A_{nm} + B_{nm} \rho^\circ) f_m^\circ = B_{mn} \rho^\circ f_n^\circ,$$

where  $A_{nm}$ ,  $B_{nm}$ ,  $B_{mn}$  are coefficients of spontaneous emission, induced emission and induced absorption, respectively;  $\rho^\circ$  and  $f_n^\circ$  are Planck function and Gibbs quantum canonical distribution, respectively. Einstein coefficients depend only on quantum states of molecule. In order to find the solution in the form of Planck’s law, Einstein introduced the following assumptions:

- a) Gibbs quantum canonical distribution is valid at  $T \rightarrow \infty$  ;
- b) coefficients of induced emission and of induced absorption are always equal to each other, i.e.

$$B_{nm} = B_{mn},$$

because they are equal in the case of  $T \rightarrow \infty$ ;

c) the ratio of coefficients of spontaneous emission and of induced emission is not equal to one; this ratio expresses Wien displacement law.

The solution of the master equation at these assumptions represents Planck formula. However, Einstein's arguments are open to objections. The main objections are as follows: (a) Gibbs quantum canonical distribution  $f_n^\circ$  loses probabilistic meaning at  $T \rightarrow \infty$  because the set

$$\sum_{n=0}^{\infty} f_n^\circ$$

is diverged at  $T \rightarrow \infty$ ; (b) the relationship  $B_{nm} = B_{mn}$  obtained under condition  $T \rightarrow \infty$  is

incorrect; (c) Einstein formulation of the master equation does not contain any information about the quantum states of the photon gas, i.e. Einstein coefficients do not depend on the quantum states of the photon gas.

Thus, Einstein's work [3] contains logical errors. The main logical error is that coefficients of the master equation does not take into consideration emission and absorption of photons by a molecule.

The third stage is connected with the method of derivation of Planck formula, proposed by Bose [4]. The method is characterized by the peculiarity that the quantum-statistical description of heat radiation is reached without consideration of interaction between radiation and substance: “the hypothesis of light quanta in a combination with statistical mechanics (in the form in what it has been adapted by Planck for needs of the quantum theory) is sufficient ground for derivation of the law independently of the classical theory” (Bose). Bose’s method can be interpreted as follows. One considers the isolated gas of photons putted in volume. The phase space of one photon is divided into  $s$ -layers (i.e. “elementary regions of energy” – the monochromatic layers defined by the relationship  $|E_m - E_n|$  where  $E_m$  and  $E_n$  are values of energy of the molecule emitting and absorbing photons),  $0 \leq s < \infty$ . And each  $s$ -layer is divided into phase space cells – states of photon gas. One postulates that: (a) existence of phase space cells (states of photon gas) of the layer  $|E_m - E_n|$  does not depend on existence of photon gas; (b) “empty phase space cell” (i.e. “vacuum state of photon gas”) not containing any photon can exist; (c) energy of monochromatic photon gas is a discrete random quantity; (d) energy state of photon gas in a cell is characterized by quantum number – number of monochromatic photons (i.e. there is “secondary quantization” of energy of gas); (e) this quantum number takes values from 0 up to  $\infty$ ; (f) the space of permissible states of photon contains the “empty phase space cell”. Quantum-statistical task is to find dependence of probability of state on energy of monochromatic photon gas under condition of conservation of full energy of photon gas.

This task is solved with the help combinatory method and Stirling’s formula. The found solution represents Gibbs quantum canonical distribution. As a result of calculation of average energy of the monochromatic photon gas in cell, one obtains Planck function, i.e. “Bose’s distribution”. Product of average energy of the monochromatic photon gas in cell and number of cells in  $s$ -layer gives Planck's formula. Einstein characterized Bose’s work as follows: “Bose’s derivation is elegant but its essence remains foggy”. In my opinion, the essence of this method is foggy because Bose’s reasoning contains logical errors. The main logic errors are as follows. (1) One considers the isolated gas of photons. In this case, energy of monochromatic photon gas

cannot be random quantity. (2) Quantum-statistical description of heat radiation is made without taking into consideration the probability of quantum states of the molecule emitting and absorbing radiation. Therefore, the parameter  $T$  of Bose's distribution is treated as temperature of photon gas. (3) The method is based on concept "empty phase space cell". In accordance with definition, the phase space (set of phase space cells) of physical object (for example, photon) represents set of available (accessible, permissible) states of this object. Since the physical object (photon) cannot be in unavailable (inaccessible, impermissible) state, this state represents the "empty phase space cell". If the "phase space of one photon" is interpreted as space of elementary events at the quantum-statistical description of photon gas, the concept of probability that photon gas is in the "empty phase space cell" loses sense.

Bose's logical error is that he included "empty phase space cell" (i.e. inaccessible, impermissible state) in space of elementary events (i.e. set of accessible, permissible states). Obviously, such inclusion is equivalent to replacement of the isolated system "photon gas" by the isolated system "molecule + molecular gas + photon gas" since cells (states) of  $s$ -layer are born by molecule and are defined by the relationship  $|E_m - E_n|$ . In this case, the concept of probability that photon gas is in the "empty phase space cell" (i.e. there are no photons in the state  $|E_m - E_n|$ ) has sense. Owing to it, Bose's logical error was happy: Bose derived Planck formula.

Thus, Planck's, Einstein's, and Bose's reasonings concerning the theory of photon gas are open to objections because these reasonings contain formal-logical errors. Till now, these errors exist in the modern theoretical physics and are manifested in concepts "Bose-Einstein statistics", "Fermi-Dirac statistics", "Dirac physical vacuum", etc. Therefore, elimination of these errors is possible only by the way of construction of the correct statistical theory of photon (quantum) gas. The correct theory should evidently consider interrelation between statistics of photon gas and statistics of radiating molecule of gas. Hence, the purpose of the sections 4-6 of the present work is to propose the correct theory of photon (quantum) gas within the framework of master equations for gas molecule and photon gas. The sections 4-6 are based on the new foundations of the theory of quantum particle [13-15].

### 3. The new foundations of the theory of quantum particle

As is known, the result of the correct critical analysis of the generally accepted foundations of quantum mechanics is as follows [13-15]: (a) the generally accepted foundations (i.e., the interpretation of the experimental data on diffraction of quantum particles; the conception of wave-corpuscule dualism; the probabilistic interpretation of the psi-function) represent logical errors; (b) the pseudo-informational meaning is the true meaning of the psi-function. Consequently, quantum mechanics is not a physical, objective theory but a pseudo-informational one. It should be replaced by a physical, objective quantum theory.

The new foundations of the objective theory of quantum particle are represented by the following set of heuristic principles [13-15].

(1) The principle of motion of quantum particle. It reads as follows: the motion is the form of existence of quantum particle; the motion represents unity of both internal and external (i.e. translatory) motions.

(2) The principle of energy of quantum particle. It reads as follows: the energy  $E_n$ ,  $n = 0, 1, 2, \dots$  (where  $n$  is the energetic quantum number) is inalienable property of a quantum particle. Energy levels  $n$  of the quantum particle arise and disappear only as a result of absorption and of emission of other quantum particles, respectively. (Consequently, the problem

of quantization of energy is not the Schrödinger problem of eigenvalues). The set of all possible (permissible) values of energy  $E_n$  forms the ordered sequence  $E_n < E_{n+1}$ . The particle is identical to itself at all permissible values of energy.

(3) The principle of equivalence of energy  $E_n$  and frequency  $\nu_n$  of quantum particle. It reads as follows: energy  $E_n$  is related to frequency  $\nu_n$  by the formula  $E_n \equiv h\nu_n$  where  $h$  and  $\nu_n$  are the Planck constant (i.e. quantum of action) and the frequency of the periodic process of mutual transformation of the internal and of external motions, respectively. The concepts of energy  $E_n$  and  $h\nu_n$  are identical ones. Multiplication of the quantities  $h$  and  $\nu_n$  is permitted by logic law of identity if  $h$  is an oscillating quantity.

(4) The principle of speed of translatory motion of quantum particle. It reads as follows: the speed  $v_n$  is defined by the formula  $v_n \equiv \lambda_n \nu_n$  where  $\lambda_n$  is the size (the diameter) of the particle. The  $\lambda_n$  equals the distance traveled the particle for the oscillation period  $\tau_n \equiv 1/\nu_n$ . This translatory motion is a result of contraction and extension of the size (diameter) of the particle. Therefore, the translatory motion of the quantum particle relative to a reference system is an absolute one. The absolute motion is invariant under choice of a reference system. This statement means that the velocity addition theorem for quantum particle is not valid.

(5) The principle of acceleration and of deceleration of quantum particle. It reads as follows: acceleration and deceleration of particle are results of absorption and of emission of other quantum particles, respectively. The acceleration  $w_{n+1,n}$  of the quantum particle under the transition  $n \rightarrow (n+1)$  which is due to absorption of other quantum particle (photon) is defined by the formula  $w_{n+1,n} \equiv (\nu_{n+1} - \nu_n)(V_{n+1} - V_n) \equiv \nu_{n+1,n} V_{n+1,n}$ .

(6) The principle of mass and momentum of quantum particle. It reads as follows: the mass  $m_n$  and the momentum  $p_n$  are defined by the formula  $E_n \equiv (E_n/\nu_n^2)\nu_n^2 \equiv m_n \nu_n^2 \equiv p_n \nu_n$ . The concept of mass  $m_n$  and the concept of energy  $E_n$  are not identical ones. Therefore, the formula  $E_n \equiv m_n \nu_n^2$  does not express the principle of equivalency of mass and energy.

(7) The principle of equivalency of mass and energy of quantum particle. It reads as follows: the energy  $E_n$  is related to the mass  $M_n$  by the formula  $E_n \equiv kM_n$  where the concepts of the energy  $E_n$  and of the mass  $M_n$  are identical ones,  $k$  is a universal constant,  $[k] = \text{erg/g}$ .

These heuristic principles are basis for the correct description of ideal gas of quantum particles.

#### 4. The theory of equilibrium ideal (normal) gas of nonradiating molecules

The quantum-statistical theory of isolated macroscopic system – equilibrium ideal (normal) gas of nonradiating molecules – represents the following assertions (premises).

(1) Macroscopic set of free quantum particles is called ideal gas. Ideal gas is called normal (non-quantum) gas if particles “interact only by the way of mutual collisions” (Einstein). Ideal gas is called abnormal (quantum) gas if particles of gas do not interact with each other.

(2) Ideal (normal) gas represents the system “molecule + molecular gas” where “molecule” is a subsystem and “molecular gas” is an surroundings.

(3) A molecule of isolated ideal (normal) gas is individual quantum particle. The energy of the molecule represents discrete random quantity since molecules collide with each other in a random way.

(4) The random quantity takes on the values  $E_n$ ,  $n=0, 1, 2, \dots$  where  $E_0=0$  is origin of counting of the random quantity;

(5)  $f_n$  is probability that molecule is in energetic quantum state  $n$  and has energy  $E_n$ . The probabilities  $f_n$ ,  $n = 0, 1, 2, \dots$  give complete quantum-statistical description of the ideal gas of molecules.

(6) Rule of addition of probabilities has the following form:

$$\sum_{n=0}^{\infty} f_n = 1$$

where

$$0 < f_n < 1, \quad \lim_{n \rightarrow \infty} (f_{n+1}/f_n) < 1.$$

(7) Rule of combination (multiplication) of probabilities for independent random events has the following form:

$$f_{n,m} = f_n f_m$$

where  $f_{n,m}$  is combined probability that two molecules have energy  $E_n + E_m$ . In this case,  $f_n$  is the exponential function (A. Cauchy, 1821):

$$f_n = f_0 \exp(-\beta E_n)$$

where  $1/\beta$  is a statistical parameter of molecule. The parameter  $1/\beta$  is introduced for mathematical reasons: quantity  $\beta E_n$  must be dimensionless one. This parameter is consequence of existence of energy spectrum of quantum particle (atom, molecule) and does not depend on structure of energy spectrum. If the parameter was dependent on  $n$ , it would represent value of some (indefinable) random quantity.

(8) The parameter

$$1/\beta = -E_n / \ln(f_n/f_0)$$

or (in other form)

$$1/\beta = -(E_{n+1} - E_n) / \ln(f_{n+1}/f_n)$$

represents physical-statistical property of molecules of gas and has both mathematical and physical meaning. From mathematical point of view,  $1/\beta$  is a continuous and limited variable. From physical point of view,  $1/\beta$  is the physical quantity which has energy dimension. Zero is origin of counting of this physical-statistical quantity and is the same for molecule of any kind. Range of existence of this parameter is defined by the relationship

$$0 < 1/\beta < E_{\infty}.$$

Quantum-statistical description of ideal gas of molecules loses statistical meaning outside this range: (a) if  $1/\beta = 0$ , then the energy of the molecule is not a random quantity; (b) if  $1/\beta = E_\infty$ , then the set  $\sum_{n=0}^{\infty} f_n$  is diverged.

(9) The parameter  $1/\beta$  has the same value for any (every) molecule of the system. Consequently,  $1/\beta$  is the universal statistical parameter (i.e. statistical potential) of the system. This parameter has essential property of temperature. As is empirically known, this property is that temperature has the same value for every part (subsystem) of system if the system is in a state of heat equilibrium. Therefore, the identity

$$1/\beta \equiv T_{(\text{statistical})}$$

is the conjecture, the postulate. Owing to this postulate, the function  $f_n$  is called Gibbs quantum canonical distribution, and the temperature  $T_{(\text{statistical})}$  is called absolute temperature. The absolute temperature is temperature in the sense of the concept ‘‘Gibbs quantum canonical distribution’’. The existence of the absolute temperature does not depend on the existence of a thermometer (device).

From the above, it follows that the exponential function

$$f_n = f_0 \exp(-E_n/T_{(\text{statistical})})$$

is Gibbs quantum canonical distribution. It has objective meaning because  $E_n$  and  $T_{(\text{statistical})}$  are independent of existence of a thermometer. It gives complete quantum-statistical description of the equilibrium ideal (normal) gas of molecules.

## 5. The theory of non-equilibrium ideal (normal) gas of nonradiating molecules

The quantum-statistical theory of isolated macroscopic system – non-equilibrium ideal (normal) gas of nonradiating molecules – include the theory of equilibrium ideal (normal) gas and represents the following assertions (premises).

- 1) The theory of equilibrium ideal (normal) gas of nonradiating molecules is valid.
- 2) A molecule of non-equilibrium ideal (normal) gas is individual quantum particle. Gas represents system ‘‘molecule + molecular gas’’ where ‘‘molecule’’ is a subsystem and ‘‘molecular gas’’ is an surroundings.
- 3) Molecules collide with each other in a random way. The energy of the molecule represents discrete random quantity since change of energy of a molecule occurs in a random way as a result of the great number of collisions.
- 4) The random quantity takes on the values  $E_n$ ,  $n = 0, 1, 2, \dots$  where  $E_0 = 0$  is origin of counting of the random quantity.
- 5) Every element  $E_n$  of the set of possible values of energy is in unambiguous (one-to-one) correspondence with the probability  $f_n(t)$  of energetic state of molecule. Distribution of probability  $f_n(t)$  gives the complete quantum-statistical description of molecule of gas.
- 6) Statistical ensemble of identical systems ‘‘molecule in  $n$ th quantum state + molecular gas’’ defines probability  $f_n(t)$  that the molecule is in the quantum state  $n$  with energy  $E_n$ :

$$f_n(t) \equiv \lim_{N \rightarrow \infty} \frac{N_n(t)}{N}, \quad N = \sum_{n=0}^{\infty} N_n(t), \quad 0 \leq t < \infty,$$

where  $N_n(t)$  is number of systems “molecule in  $n$  th quantum state + molecular gas” in moment of time  $t$ ;  $N$  is full number of systems “molecule + molecular gas” in ensemble (this number does not depend on time).

7) Probability  $f_n(t)$  characterizes ensemble of systems and satisfies to conditions of normalization (and conservation of normalization)

$$\sum_{n=0}^{\infty} f_n(t) = 1, \quad 0 < f_n(t) < 1,$$

of unambiguity and of uniqueness.

8) Stochastic process of change of quantum state of molecule represents Markovian process with numerable states. Change of  $f_n(t)$  in time  $t$  is described by the master equation

$$\frac{df_n}{dt} = P(f_n), \quad P(f_n) \equiv \sum_{\substack{m=0 \\ m \neq n}}^{\infty} [P_{nm}(t) f_m(t) - P_{mn}(t) f_n(t)], \quad n = 0, 1, 2, \dots$$

where  $P_{nm}(t)$  is rate of transition  $m \rightarrow n$  (i.e. the probability of transition  $m \rightarrow n$  per unit time) in the moment of time  $t$ . The coefficients  $P_{nm}(t)$  and  $P_{mn}(t)$  are smooth functions of time  $t$  and do not characterize separate (elementary) acts of collisions. These coefficients characterize statistical process of transitions consisting of great number of separate (elementary) acts.

9) There exist limit of quantity  $f_n(t)$  at  $t \rightarrow \infty$ . The limit characterizes stationary state. If the state of statistical equilibrium is exists, then the stationary state in the system “molecule + molecular gas” is the state of statistical equilibrium:

$$f_n(\infty) \equiv f_n^\circ, \quad f_n^\circ = f_0^\circ \exp(-E_n/T)$$

where  $f_n^\circ$  is Gibbs quantum canonical distribution,  $T$  is statistical temperature of molecule.

10) The principle of detailed balance reads as follows: the rates of the processes  $m \rightarrow n$  and  $n \rightarrow m$  are equal to each other in statistical equilibrium. The formulation of this principle is as follows:

$$P_{nm}(\infty) f_m^\circ = P_{mn}(\infty) f_n^\circ, \quad \text{i.e.}$$

$$P_{nm}(\infty) \exp(-E_m/T) = P_{mn}(\infty) \exp(-E_n/T)$$

10) The formulation of the principle of detailed balance represents a necessary condition of existence of statistical equilibrium:

$$\frac{P_{nm}(\infty)}{P_{mn}(\infty)} = \exp\left(-\frac{E_n - E_m}{T}\right) \neq 1.$$

This condition means that coefficients  $P_{nm}(t)$  и  $P_{mn}(t)$  do not obey the principle microscopic reversibility of transition processes.

11) Statistical-average energy  $E(t)$  of molecule is defined by the relationship

$$E(t) \equiv \sum_{n=0}^{\infty} E_n f_n(t).$$

12) Time  $\tau(t)$  of relaxation of energy  $E(t)$  is defined by the relationship

$$\frac{dE}{dt} = - \frac{E - E^\circ}{\tau(t)}$$

The following assertions result from the above. If the isolated ideal gas tends to statistical equilibrium and reaches statistical equilibrium at  $t \rightarrow \infty$ , then coefficients  $P_{nm}(t)$  and  $P_{mn}(t)$  of the master equation depend on time and do not obey the principle microscopic reversibility of transition processes. If these coefficients obeyed principle of microscopic reversibility of transition processes, i.e. if

$$P_{nm} = P_{mn},$$

then the stationary state of gas would not be a state of statistical equilibrium.

## 6. The theory of photon (quantum) gas being born by radiating molecule

The quantum-statistical theory of photon (quantum) gas – isolated macroscopic system “molecule + molecular gas + photon gas” where the subsystem “photon gas” being born by the subsystem “molecule” in the process of quantum transitions not stipulated by acts of collisions – includes the theory of non-equilibrium ideal (normal) gas of nonradiating molecules and represents the following assertions (premises).

- 1) The theory of non-equilibrium ideal (normal) gas of nonradiating molecules is valid.
- 2) Process of change of number of photons in the system is stipulated by acts of emission and of absorption of photons by molecules of ideal (normal) gas. This is stochastic process because the process of change of energetic quantum state of the colliding molecule is stochastic process.
- 3) The statistical ensemble of identical macroscopic systems “molecule in  $n$ th quantum state + molecular gas + photon gas” defines probability  $f_n(t)$  that molecule is in  $n$ th quantum state:

$$f_n(t) \equiv \lim_{N \rightarrow \infty} \frac{N_n(t)}{N}, \quad N = \sum_{n=0}^{\infty} N_n(t), \quad 0 \leq t < \infty,$$

where  $N_n(t)$  is number of systems “molecule in  $n$ th quantum state + molecular gas + photon gas” in moment of time  $t$ ;  $N$  is full number of systems “molecule + molecular gas + photon gas” in the ensemble (this number does not depend on time).

4) Stochastic process of change of quantum state of molecule of the system “molecule + molecular gas + photon gas” represents Markovian process with numerable states and is described by the master equation

$$\frac{df_n}{dt} = P(f_n), \quad P(f_n) \equiv \sum_{\substack{m=0 \\ m \neq n}}^{\infty} [P_{nm}(t) f_m(t) - P_{mn}(t) f_n(t)], \quad n=0, 1, 2, \dots$$

5) The set  $k$  of identical (i.e. monochromatic) photons being emitted (born) with the energy  $h\nu_{nm} \equiv |E_m - E_n|$  by molecule in the process  $m \rightarrow n$  ( $m > n$ ) represents  $\nu_{nm}$  - monochromatic photon gas. The number  $k$  of identical photons takes on values from 0 to  $\infty$  since there is no physical prohibition on number of photons being emitted by colliding molecule. (In other words, there exist “secondary quantization” of gas energy).

6) The energy of the monochromatic photon (quantum) gas is a discrete random quantity. Every value of energy  $h\nu_{nm} k$  – element of numerable set  $\{h\nu_{nm} k\}$ ,  $k=0, 1, 2, \dots$  – is in unambiguous (one-to-one) correspondence with the probability of energetic state of the monochromatic photon gas of system “molecule + molecular gas +  $\nu_{nm}$  -monochromatic photon gas”.

7) The statistical ensemble of identical macroscopic systems “molecule + molecular gas +  $\nu_{nm}$  -monochromatic photon gas” defines the probability  $q_k(\nu_{nm}; t)$  that  $\nu_{nm}$  -monochromatic photon gas is in  $k$  th quantum state in moment of time  $t$ :

$$q_k(\nu_{nm}; t) \equiv \lim_{M(\nu_{nm}) \rightarrow \infty} \frac{M_k(\nu_{nm}; t)}{M(\nu_{nm})}, \quad \sum_{k=0}^{\infty} q_k(\nu_{nm}; t) = 1, \quad M(\nu_{nm}) = \sum_{k=0}^{\infty} M_k(\nu_{nm}; t)$$

where  $M_k(\nu_{nm}; t)$  is number of the systems “molecule + molecular gas +  $\nu_{nm}$  -monochromatic photon gas in  $k$  th quantum state” in moment of time  $t$ ;  $M(\nu_{nm})$  - full number of the systems “molecule + molecular gas +  $\nu_{nm}$  -monochromatic photon gas” in the ensemble.

In view of these assertions, stochastic process of change of states of  $\nu_{nm}$  -monochromatic photon gas represents Markovian process with numerable states. It is described by the master equation ( $m > n$ )

$$\frac{dq_k}{dt} = W_{mn}^{k, k+1} f_n q_{k+1} - (W_{nm}^{k+1, k} f_m + W_{mn}^{k-1, k} f_n) q_k + W_{nm}^{k, k-1} f_m q_{k-1}$$

where  $W_{mn}^{k, k+1}$  is probability of transition  $(n, k+1) \rightarrow (m, k)$  per unit of time. In accordance with the principle microscopic reversibility of transition processes, the relationship

$$W_{mn}^{k, k+1} = W_{nm}^{k+1, k}$$

is valid. The coefficients  $W_{mn}^{k, k+1}$  and  $W_{nm}^{k+1, k}$  characterize the transition processes  $(n, k+1) \leftrightarrow (m, k)$  consisting of a great number of separate (elementary) acts. This master equation at  $t \rightarrow \infty$  represents the equation of detailed balance

$$f_n^\circ q_{k+1}^\circ = f_m^\circ q_k^\circ, \text{ i.e. } q_{k+1}^\circ = q_k^\circ \exp(-h\nu_{nm}/T),$$

where  $T$  is the statistical temperature of molecule. Obviously, unique solution  $q_k^\circ$  of this functional equation is Gibbs quantum canonical distribution

$$q_k^\circ = q_0^\circ \exp(-h\nu_{nm} k/T), \quad q_0^\circ = 1 / \sum_{k=0}^{\infty} \exp(-h\nu_{nm} k/T) = 1 - \exp(-h\nu_{nm}/T).$$

This distribution is in accordance with the logic law of identity:

*(information on system "molecule + molecular gas +  $\nu_{nm}$  -monochromatic photon gas") = (information on system "molecule + molecular gas +  $\nu_{nm}$  -monochromatic photon gas").*

Substituting well-known quantum-mechanical relationship

$$W_{mn}^{k,k+1} = (k+1)W_{mn}^{01}$$

into the master equation and taking into consideration the definition of statistical-average energy

$$h\nu_{nm} \rho(\nu_{nm}; t) \equiv h\nu_{nm} \sum_{k=0}^{\infty} k q_k(\nu_{nm}; t)$$

of  $\nu_{nm}$  -monochromatic photon gas, one can reduce the master equation to form

$$\frac{d\rho}{dt} = W_{nm}^{10} [(\rho+1)f_m - \rho f_n]$$

where  $\rho$  is statistical-average number of monochromatic photons being born by molecule. In the stationary case (i.e. at  $t \rightarrow \infty$ ), this equation takes on the form

$$(\rho^\circ + 1)f_m^\circ = \rho^\circ f_n^\circ.$$

The solution of this equation is Planck function (i.e. Bose's distribution):

$$\rho^\circ = [\exp(h\nu_{nm}/T) - 1]^{-1}.$$

This correct result permits to compare the master equation in the  $\rho$  with Einstein's equation. The comparison leads to the unique correct relationship for Einstein coefficients:

$$A_{nm} \equiv B_{nm} \equiv B_{mn} \equiv W_{nm}^{10}.$$

Thus, the correct theory of photon gas is based on statistics of radiating molecule of the normal gas. The equilibrium photon (quantum) gas obeys "Gibbs statistics": photon gas in the isolated equilibrium system "molecule + molecular gas + photon gas" is described by Gibbs quantum canonical distribution. Planck function (Bose's distribution) is consequence of Gibbs distribution. The temperature  $T$  in Planck function represents the temperature of radiating

molecule of the normal gas. Einstein coefficients  $A_{nm}$ ,  $B_{nm}$ ,  $B_{mn}$  are equal to each other and, consequently, loss the generally accepted sense.

## 7. Discussion

As is known, the generally accepted science paradigm stipulates an inductive way of knowledge of the world. A scientific truth (as a system of experimental facts and theories) obeys the principle of dialectical development. The dialectical development (i.e., quantitative and qualitative changes in the direction of ascension from simple forms to complicated ones) of the truth includes the “birth and extermination” of some theories, transformation and unification of others. Selection of theories is made on the base of criteria of validity. According to Einstein, there exist two criteria: the “external justification” criterion (i.e. agreement with experimental data) and the “internal perfection” criterion (i.e. accordance with logic laws, with sense of harmony and beauty). “When experience is in accordance with theory, it means “it may be so” for the theory” (Einstein). The theories which do not satisfy these two criteria are obviously incorrect. But they are not unavailing ones: they plays important role in development of knowledge of the world. Incorrect theories and errors are psychological means of broadening of scientists’ consciousness, and broadened consciousness promotes deductive revision of foundations of science. In my point of view, the errors are explained by the global cause: the errors are a collateral and inevitable result of inductive method of knowledge of the Nature. One of such erroneous theories of the 20th century is the generally accepted theory of quantum (photon) gas.

If the principle of the unity of formal logic and of rational dialectics is a correct methodological basis of science, then the concept of random quantity must be a starting-point of any physical-statistical theory. In this case, distribution of probabilities gives correct and complete physical-statistical description of the physical system. “The insufficient understanding of this circumstance is a root of those difficulties which one should overcome now” (Einstein). As is known, in the case of statistical theory of heat phenomena, energy is a random quantity. However, Planck’s, Einstein’s, Bose’s works on the theory photon (quantum) gas and the generally accepted Boltzmann distribution (used by Planck, Einstein, and Bose) are not in accordance with this argument. For example, Boltzmann distribution

$$b_i = b g_i Z^{-1} \exp(-E_i/T)$$

(where  $b$  is the total density of the molecules,  $b_i$  is the density of molecules in state  $i$ ,  $T$  is the temperature,  $g_i$  is the degeneracy of state  $i$ , and  $Z$  is the partition function) has no any probabilistic meaning because: (a) the energy of the molecule is not a random quantity and  $b_i$  is not probability; (b)  $b_i/b \neq f_i^\circ$ ; (c) from the probabilistic point of view, the relationship  $b_i/b = f_i^\circ$  is essential error. Therefore, Planck’s, Einstein’s, and Bose’s arguments concerning to the theory of photon (quantum) gas are open to objection: these arguments contain formal-logical errors. The formal-logical errors in physics could not be realized and comprehended by the classics of physics and by the physicists followed them. As is known from the autobiographies of classics of physics, classics of physics did not sense hard psychological shock when they detected some inconsistencies (obstacles) in physics because they take on trust the possibility of knowing the world. They tried to get round an obstacle and did not try to destroy it. That is why scientific achievements of classics of physics are an effect of long reflection and of momentary-lucid consciousness. The momentary-lucid consciousness cannot propose correct and complete solution of problem. And correct knowledge is effect of both hard psychological shock at detection of inconsistency in science and constantly lucid consciousness. The constantly lucid

consciousness promotes comprehension of the following main statements [21-23]: (1) the problem of scientific truth is the most urgent problem of our time. This problem can be solved only with help of a new theory of knowledge since “science without the theory of knowledge becomes primitive and muddled” (A. Einstein); (2) the correct theory of knowledge cannot be built if there is no solution of the problem of existence of God (Creator, Ruler) in science; (3) scientific achievements depend on the moral qualities of man: in ancient Greek philosopher Socrates’ opinion, the existence of objective truth is consequence of the existence of objective moral principles. Therefore, “the moral qualities of the prominent person are, probably, of great importance for the given generation and all course of history than purely intellectual achievements. The lasts depend on greatness of spirit to an greater degree than it is usually accepted to consider” (A. Einstein).

Many yeas later, Bose recollected: “I did not imagine that I did something new. I not so understood statistics to understand how much my approach differed from the approach which Boltzmann could have proposed on the basis of his statistics. Instead of imagining light quanta in the form of particles, I spoke about these states” [12]. Einstein characterized Bose’s work as follows: “Bose’s derivation is elegant but its essence remains foggy”. Bose’s idea and method rendered essential influence on Einstein’s, Fermi’s, and Dirac’s works: “The derivation of Planck formula, proposed by Bose, is a great achievement. The method used by him gives also the quantum theory of ideal gas... since light quantum in essence differs from one-atom molecule only in the respect that the rest mass of quantum is vanishing small. The analogy between gas of quanta and gas of molecules should be full” (Einstein). This analogy and “foggy essence of Bose’s method” resulted in theories of molecular quantum gas, “Bose-Einstein statistics”, “Fermi-Dirac statistics”, and the concepts “chemical potential”, “secondary quantization”, “physical vacuum” [16-20]. Today only a genius can be possessed of both intuition and courage to cast doubt on traditional theories and concepts because a genius goes in his own, independent way.

## Conclusion

Thus, the correct theoretical analysis of the generally accepted foundations of theory of photon (quantum) gas leads to the following main propositions.

(1) Planck’s, Einstein’s, and Bose’s works on the theory of photon (quantum) gas contain logical errors.

(2) The correct foundations of the statistical theory of photon (quantum) gas are based on the key idea that the problem of correct quantum-statistical description of photon gas must be solved within the framework of master equation taking into consideration both the quantum states of the radiating molecule and the quantum states of the photon gas in the isolated macroscopic systems “molecule + molecular gas + monochromatic photon gas”.

(3) The formulated master equation describing photon gas in the isolated macroscopic systems “molecule + molecular gas + monochromatic photon gas” gives the correct and complete quantum-statistical description.

(4) The main results of the quantum-statistical description of photon gas are as follows:

(a) photon (quantum) gas being born by radiating molecule obeys “Gibbs statistics”: equilibrium photon (quantum) gas is described by Gibbs quantum canonical distribution; the statistical temperature in Gibbs quantum canonical distribution represents the temperature of the radiating molecule;

(b) Planck function (“Bose’s distribution”) is a consequence of Gibbs quantum canonical distribution.

(c) Einstein coefficients (i.e. the coefficients of spontaneous emission, induced emission and absorption) are equal to each other.

## Acknowledgment

The books “Surprises in Theoretical Physics” (1979) and “More Surprises in Theoretical Physics” (1991) by Sir Rudolf Peierls stimulated my work.

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