THEORY OF COMPLEX NUMBERS: GROSS ERROR IN MATHEMATICS AND PHYSICS

Temur Z. Kalanov
Home of Physical Problems, Yozuvchilar (Pisatelskaya) 6a,
100128 Tashkent, Uzbekistan
tzk_uz@yahoo.com, t.z.kalanov@mail.ru, t.z.kalanov@rambler.ru

Abstract. The critical analysis of the starting point of the theory of complex numbers is proposed. The unity of formal logic and rational dialectics is methodological basis of the analysis. The analysis leads to the following main results: (1) the definition of a complex number contradicts to the laws of formal logic, because this definition is the union of two contradictory concepts: the concept of a real number and the concept of a non-real (imaginary) number - an image. The concepts of a real number and a non-real (imaginary) number are in logical relation of contradiction: the essential feature of one concept completely negates the essential feature of another concept. These concepts have no common feature (i.e. these concepts have nothing in common with each other), therefore one cannot compare these concepts with each other. Consequently, the concepts of a real number and a non-real (imaginary) number cannot be united and contained in the definition of a complex number. The concept of a complex number is a gross formal-logical error; (2) the real part of a complex number is the result of a measurement. But the non-real (imaginary) part of a complex number is not the result of a measurement. The non-real (imaginary) part is a meaningless symbol, because the mathematical (quantitative) operation of multiplication of a real number by a meaningless symbol is a meaningless operation. This means that the theory of complex number is not a correct method of calculation. Consequently, mathematical (quantitative) operations on meaningless symbols are a gross formal-logical error; (3) a complex number cannot be represented (interpreted) in the Cartesian geometric coordinate system, because the Cartesian coordinate system is a system of two identical scales (rulers). The standard geometric representation (interpretation) of a complex number leads to the logical contradictions if the scales (rulers) are not identical. This means that the scale of non-real (imaginary) numbers cannot exist in the Cartesian geometric coordinate system.

Consequently, the theory of complex numbers and the use of the theory of complex numbers in mathematics and physics (electromagnetism and electrical engineering, fluid dynamics, quantum mechanics, relativity) represent a gross methodological error and lead to gross errors in mathematics and physics.

Keywords: general mathematics, complex numbers, geometry, methodology of mathematics, mathematical physics, physics, special relativity, electromagnetism, quantum mechanics, general relativity, engineering, formal logic, dialectics, philosophy of mathematics, philosophy of science, education.

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Introduction

As is known, the theory of complex numbers is a branch of mathematics [1-11] and an important part of the mathematical formalism of theoretical physics [12]. “Many mathematicians contributed to the development of complex numbers: Gerolamo Cardano, Rafael Bombelli, William Rowan Hamilton, Niccolò Fontana Tartaglia, René Descartes, Abraham de Moivre, Leonhard Euler, Caspar Wessel, Jean-Robert Argand, Carl Friedrich Gauss, Buée, Mourey , Warren, Français, Bellavitis, G.H. Hardy, Niels Henrik Abel, Carl Gustav Jacob Jacobi, Augustin Louis Cauchy, Bernhard Riemann. Later classical writers on the general theory include Richard Dedekind, Otto Hölder, Felix Klein, Henri Poincaré, Hermann Schwarz, Karl Weierstrass and many others. Important work (including a systematization) in complex multivariate calculus has been started at the beginning of the 20th century. Important results have been achieved by Wilhelm Wirtinger in 1927” (Wikipedia). Complex numbers are used in physics: electromagnetism and electrical engineering, fluid dynamics, quantum mechanics, relativity. But complex numbers are not the result of measurements. Moreover, complex numbers are not contained in the final results of mathematical and physical theories. This means that the use of complex numbers is a way of calculation.

Until now, the theory of complex numbers has not been questioned [1-11]. It was believed that the names of famous scientists who contributed to the development of the theory of complex numbers are a guarantee of truth. But famous scientists could not find the correct criterion of truth of mathematical and physical theories. Famous scientists ignored the correct methodological basis of science: the unity of formal logic and rational dialectics. Until now, the works of mathematicians and theoretical physicists [1-11] do not satisfy the correct criterion of truth. Therefore, the purpose of the present work is to propose the critical analysis of the starting point of complex number theory within the framework of the correct methodological basis: the unity of formal logic and rational dialectics. This way of analysis gives an opportunity to understand the erroneous essence (erroneous concepts) of complex number theory.

1. Analytical aspect of the theory of complex numbers. Arithmetic and algebra of complex numbers

1) As is known [1-11], the expression

$$a + bi$$

is called a complex number. In this expression, a and b are any real numbers; the symbol $i \equiv \sqrt{-1}$ is called the imaginary unit; $i^2 \equiv -1$; the number a is the real part of the complex number; bi is the imaginary part of the complex number; the number b is the coefficient of the imaginary unity. Expression

$$a - bi$$

is called the conjugate complex number. Complex numbers (similar to real numbers) obey all standard arithmetic and algebraic operations. For example,

(a) the operation of addition (subtraction) of complex numbers is:

$$(a_1 + b_1 i) \pm (a_2 + b_2 i) = (a_1 \pm a_2) + (b_1 \pm b_2) i \, ;$$

(b) the operation of multiplication of complex numbers is:

$$(a_1 + b_1 i) \cdot (a_2 + b_2 i) = (a_1 a_2 - b_1 b_2) + (a_1 b_2 + a_2 b_1) i \, ;$$

(c) the operation of division of complex numbers is:

$$\frac{a_2 + b_2 i}{a_1 + b_1 i} = \frac{a_1 a_2 + b_1 b_2}{a_1^2 + b_1^2} + \frac{a_1 b_2 - a_2 b_1}{a_1^2 + b_1^2} i\,;$$

(d) the modulation operation of complex number is:

$$|a + bi| = \sqrt{(a + bi)(a - bi)} = \sqrt{a^2 + b^2}\,, \quad \text{where} \quad abi - abi = 0\,;$$

(e) the identity operation (condition) is:

$$(a_1 + b_1 i) = (a_2 + b_2 i) \text{ under } a_1 = a_2,\ b_1 = b_2;$$
$$a + bi = 0 \text{ under } a = 0,\ b = 0;$$
$$a + 0i = a \text{ under } 0i = 0\,;$$

(f) the trigonometric form of the complex number $\alpha = a + bi$ is $\alpha = r(\cos\varphi + i\sin\varphi)$, where the quantity r is the magnitude of the complex number, the quantity of the angle φ is the argument of the complex number.

2) As is known [1-11], the quantity $z = x + yi$ is called a complex variable, where x and y are real variables (in particular, $x = a$, $y = b$). The trigonometric form of the complex quantity is $z = r(\cos\varphi + i\sin\varphi)$, where r is the magnitude of the complex variable. A complex variable z (similar to a real variable) obeys all standard algebraic and differential operations.

2. The geometric aspect of the theory of complex numbers

As is known [1-11], the standard geometric representation (interpretation) of complex numbers is that each complex number is associated with a vector (or a point on the plane) in the Cartesian coordinate system XOY, where the scale OX is called the scale of real numbers, and the scale OY is called the scale imaginary numbers (Figure 1).

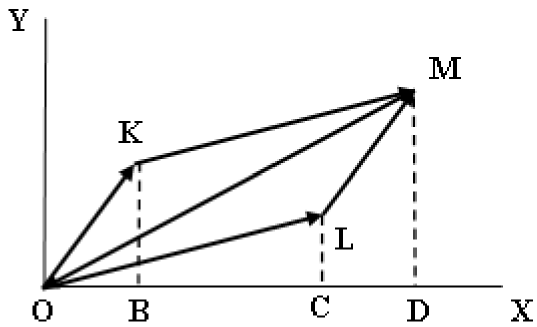

Figure 1. Vector diagram in the Cartesian coordinate system XOY. The vector $\overrightarrow{OM}$ is the sum of the vectors $\overrightarrow{OK}$ and $\overrightarrow{OL}$. Vector addition is performed according to the parallelogram rule. The relation between vectors and complex numbers is the following: $\overrightarrow{OM} \equiv a + bi$, $\overrightarrow{OK} \equiv a_1 + b_1 i$, $\overrightarrow{OL} \equiv a_2 + b_2 i$.

The term “correspondence” means that each vector (or point on the plane) represents a complex number: $\overrightarrow{OM} \equiv a+bi$, $\overrightarrow{OK} \equiv a_1+b_1 i$, $\overrightarrow{OL} \equiv a_2+b_2 i$, etc. The numbers a and bi are the quantities of the projections of the vector $\overrightarrow{OM}$ onto the coordinate scales. The complex number $\alpha = a+bi$ is called the affix of a point in the plane. Geometric operations on vectors mean algebraic operations on complex numbers.

3. Objections

(1) The definition of a complex number contradicts to formal logic and the fundamental dialectical concept (category) of measure. Really, measure is a philosophical category denoting (designating) the unity of the qualitative and quantitative determinacy of a material object. Pure mathematics ignores the qualitative determinacy of the object and considers only the quantitative (numerical) determinacy of the object. This is fundamental and gross error in pure mathematics.

By definition, mathematics is the science of operations on quantitative determinacy. Quantitative determinacy represents real numbers as a result of measurements. But the symbol $i \equiv \sqrt{-1}$ is not a number as a result of measurement. In other words, the symbol $i \equiv \sqrt{-1}$ has no quantitative determinacy; the symbol $i \equiv \sqrt{-1}$ is not quantifiable. Consequently, the expression $a \pm i$ is an inadmissible (impermissible) quantitative operation. In addition, the expressions $\sqrt{-1}$, $i \equiv \sqrt{-1}$ $i=i$, i^2, $0i=0$, bi, $bi/i=b$, $bi/b=i$, $a+bi$, etc. are impermissible (inadmissible) quantitative operations, because the symbols $i \equiv \sqrt{-1}$, i^2, bi, bi/i, etc. do not represent the quantitative determinacy (i.e., numbers); expressions $i+i=2i$, $i-i=0$, $abi-abi = 0$, $bi/i=b$, etc. are impermissible (inadmissible) quantitative operations, because the symbol i is not a number.

Consequently, all expressions that contain the symbol i represent dialectical and formal-logical errors. The expressions that contain the symbol i cannot contain symbols of mathematical (quantitative) operations. These expressions are not mathematical relationships.

There is a standard statement [13] that “the sign (+) in the expression $\alpha = a+bi$ is not a sign of the mathematical operation. This expression should be considered as a single symbol for the complex number $\alpha = \mathrm{Re}(\alpha) + \mathrm{Im}(\alpha)$”. However, this statement contradicts to the laws of formal logic. Really, if a is a real number, and bi is not a real number, then $\alpha = a+bi$ is a union of contradictory definitions (concepts) in one mathematical expression: the number $\alpha = a+bi$ is both a real number and a non-real number. But the union of contradictory definitions (concepts) in one mathematical expression is prohibited by the formal-logical law of lack of contradiction and the law of excluded middle. Consequently, the expression $\alpha = a+bi$ is a gross logical error.

Thus, all initial definitions (positions) of the theory of complex numbers, arithmetic and algebra of complex numbers are gross methodological errors.

(2) The complex number $\alpha = a+bi$ cannot be represented (interpreted) on the geometric scale OX of the Cartesian coordinate system XOY (Figure 2).

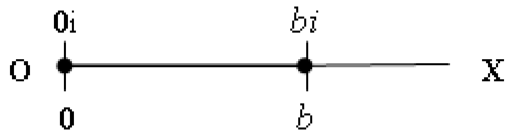

Figure 2. Representation (interpretation) of the complex number $\alpha = a + bi$ on the geometric scale OX of the Cartesian coordinate system XOY.

If one represented (interpreted) the numbers bi and b on the scale OX of the Cartesian coordinate system XOY, then the following contradiction would arise: $bi = b$, $i = 1$. Consequently, the complex number $\alpha = a + bi$ cannot exist on the scale OX of the Cartesian coordinate system XOY.

(3) The complex number $\alpha = a + bi$ cannot be represented (interpreted) in the Cartesian coordinate system XOY (Figure 3).

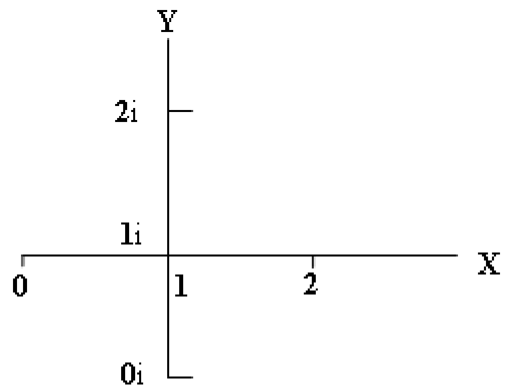

Figure 3. The intersection of the scale OX of real numbers and the scale OY of imaginary numbers.

The scale of real numbers (OX) and the scale of imaginary numbers (OY) cannot have a common point of intersection. If the scales OX and OY intersected each other, then the following contradiction would arise: $0i = 0$, $1i = 1$, $i = 1$. Therefore, the imaginary number scale OY cannot exist in the Cartesian coordinate system XOY.

(4) The ordinate bi of the point B does not exist in the material Cartesian coordinate system XOY (Figure 4).

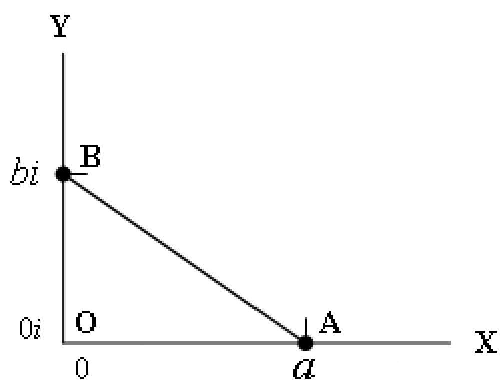

Figure 4. Positions of the material segment $\overline{AB}$ and the right-angled triangle ΔAOB in the material Cartesian coordinate system XOY. a is the abscissa of the point A; bi is the ordinate of the point B.

As is known, all points of the material rectilinear segment $\overline{AB}$ are identical material points in the material Cartesian coordinate system XOY. If the positions (coordinates) of material points A and B in the system XOY are measured by non-identical rulers OX and OY, then the essence of these measurements is as follows:

(a) such measurements are an inadmissible (impermissible) operation;

(b) the identical material points A and B turn into non-identical material points. Really, the coordinate of the point A is the real number a, and the coordinate of the point B is bi which is not a real number. In this case, the qualitative determinacy of the numbers a and bi is different in the system XOY. This leads to the following contradiction: identical points A and B become non-identical points in the system XOY. Therefore, the point B cannot belong to the material segment in the system XOY.

Consequently, the ordinate bi of the point B does not exist in the geometric coordinate system XOY. Thus, the standard statement that the ordinate bi of the point B exists in the material Cartesian coordinate system XOY is a gross formal-logical error.

(c) Existence of bi contradicts to the Pythagorean theorem in the case of the right-angled triangle ΔAOB (Figure 4):

$$a^2 + (bi)^2 \neq \left(d^{(\overline{AB})}\right)^2, \quad a^2 - (bi)^2 = \left(d^{(\overline{AB})}\right)^2$$

where $d^{(\overline{AB})}$ is the length of the hypotenuse.

Thus, the standard geometric representation (interpretation) of complex numbers is a gross methodological error.

Discussion

Thus, the theory of complex numbers is wrong. As the history of mathematics and theoretical physics shows, scientists made mistakes because scientists rely on intuition, and not on the correct methodological basis (truth criterion): the unity of formal logic and rational dialectics. Formal logic and rational dialectics are interrelated (interconnected, interdependent) general sciences about correct methods of thinking and cognition of the world. Mathematicians ignore the dialectical principle of knowledge: “practice $\to$ theory $\to$ practice”. Mathematicians ignore the philosophical category of measure as the unity of the qualitative and quantitative determinacy of a material object. This is the root of gross errors in pure mathematics and geometry [14-47]. The theory of complex numbers – an achievement of pure mathematics – is absurd, because this theory operates with a meaningless symbol. Mathematical (quantitative) operations on a meaningless symbol are meaningless, because the symbol $i \equiv \sqrt{-1}$ is not a real number. A complex number $a + bi$ is a meaningless concept (for example, like the expression $a + b\Delta$, where Δ is the triangle symbol). The operation of conversion of the symbol $i \equiv \sqrt{-1}$ into the number $i^2 = -1$ is a logical error. This operation is an inadmissible operation, because a mathematical (quantitative) operation i^2 is an inadmissible operation on the qualitative determinacy (i.e., on the meaningless essence) of the symbol $i \equiv \sqrt{-1}$. Therefore, the theory of complex numbers is not the correct way to calculate.

A complex number $\alpha = a + bi$ cannot be represented (interpreted) in the Cartesian geometric coordinate system, because the Cartesian coordinate system XOY is a system of two identical rulers (scales) OX and OY. The standard geometric interpretation (representation) of a complex number leads to the following contradiction: $0i = 0$, $1i = 1$, $i = 1$, if the scales OX and OY are not identical. This means that the imaginary number scale OY cannot exist in the Cartesian geometric coordinate system.

Consequently, the theory of complete numbers and the use of the theory of complex numbers in mathematics and physics (electromagnetism and electrical engineering, fluid dynamics, quantum mechanics, relativity) represent a gross methodological error and lead to gross errors in mathematics and physics.

Conclusion

Thus, the critical analysis of the starting point of the theory of complex numbers within the framework of the correct methodological basis leads to the following main results:

1) the definition $\alpha = \mathrm{Re}(\alpha) + \mathrm{Im}(\alpha)$ of a complex number $\alpha = a + bi$ contradicts to the laws of formal logic, because this definition is the union of two contradictory concepts: the concept of a real number $\mathrm{Re}(\alpha)$ and the concept of a non-real (imaginary) number – an image – $\mathrm{Im}(\alpha)$. The concepts of $\mathrm{Re}(\alpha)$ and $\mathrm{Im}(\alpha)$ are in the logical relation of contradiction: the essential feature of the concept of $\mathrm{Re}(\alpha)$ completely negate the essential feature of the concept of $\mathrm{Im}(\alpha)$. These concepts do not have common feature (i.e. these concepts have nothing in common with each other), therefore one cannot compare these concepts with each other.
Consequently, the two concepts $\mathrm{Re}(\alpha)$ and $\mathrm{Im}(\alpha)$ cannot be united and contained in the definition of a complex number α. The concept of a complex number is a gross formal-logical error;

2) the real part (i.e. number α) of a complex number is the result of the measurement. But the imaginary part (i.e. symbol bi) of a complex number is not the result of the measurement. The imaginary part bi is a meaningless symbol, because the mathematical (quantitative) operation of multiplication of a real number b by a meaningless symbol is a meaningless operation. This means that complex number theory is not a correct method of calculation. Consequently, mathematical (quantitative) operations on meaningless symbols i, bi and $\alpha = a + bi$ is a gross formal-logical error;

3) a complex number $\alpha = a + bi$ cannot be represented (interpreted) in the Cartesian geometric coordinate system XOY, because the Cartesian coordinate system XOY is a system of two identical rulers (scales) OX and OY. The standard geometric interpretation (representation) of a complex number leads to the following contradiction: $0i = 0$, $1i = 1$, $i = 1$ if the scales OX and OY are not identical. This means that the imaginary number scale cannot exist in the Cartesian geometric coordinate system XOY.

Consequently, the theory of complete numbers and the use of the theory of complex numbers in mathematics and physics (electromagnetism and electrical engineering, fluid dynamics, quantum mechanics, relativity) represent a gross methodological error and lead to gross errors in mathematics and physics.

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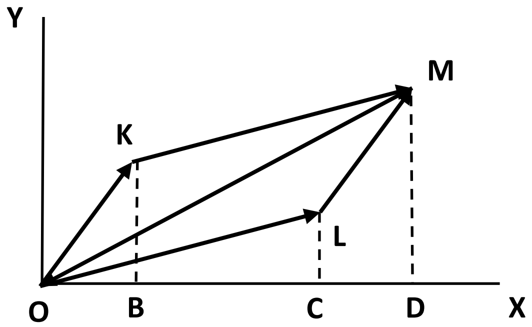
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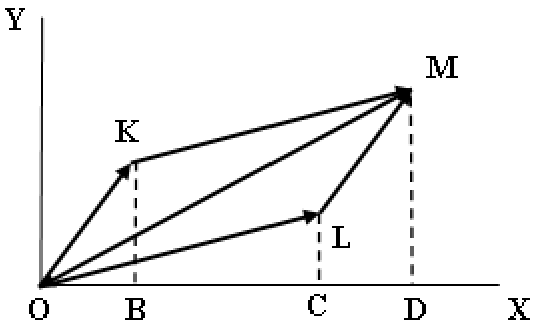
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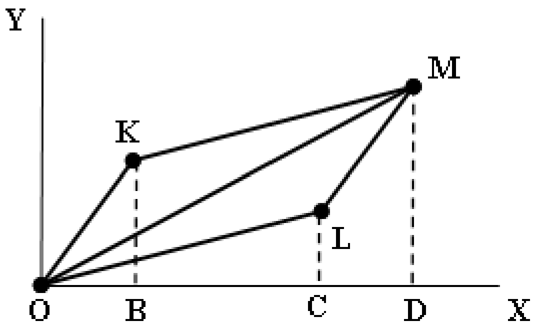
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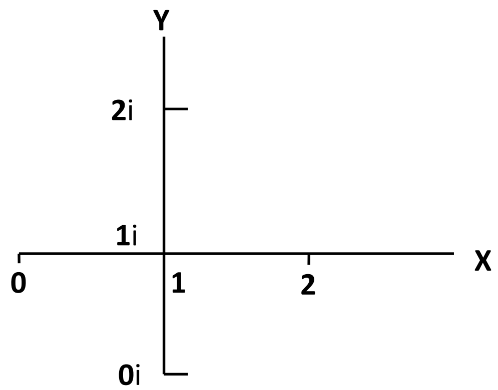
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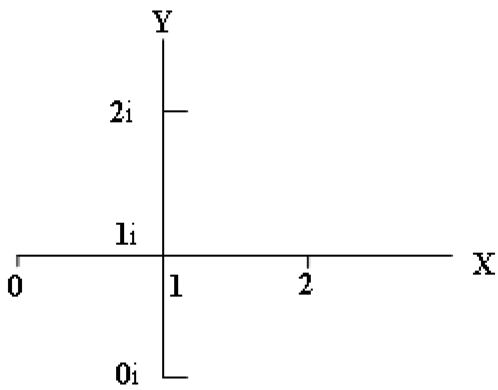
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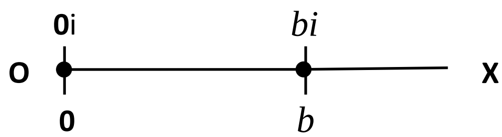
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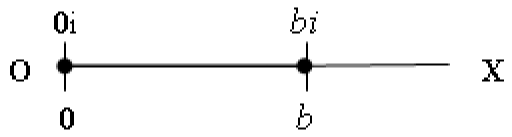
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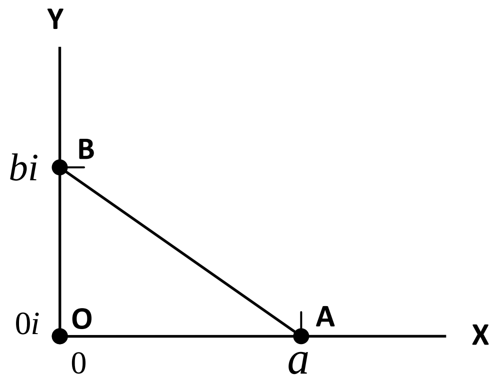
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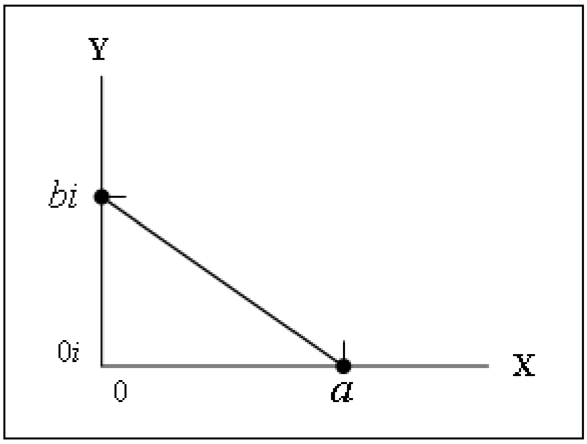
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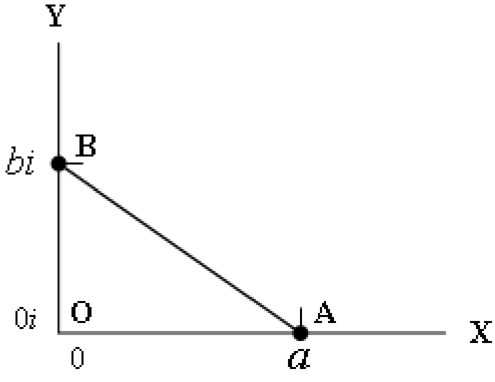
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Abstract. The critical analysis of the foundations of standard trigonometry is proposed. The unity of formal logic and rational dialectics is methodological basis of the analysis. The analysis leads to the following main results.

Keywords: general mathematics, philosophy of mathematics, methodology of mathematics, trigonometry, geometry, mathematical physics, physics, engineering, formal logic, dialectics, philosophy of science.

MSC: 00A05, 00A30, 00A30g, 00A35, 00A69, 00A79, 03A05, 03A10, 03B42, 03B44, 03B80, 33B10, 03F50, 97E20, 97E30, 97G60, 97G70, 97M50, 51M15, 51N35, 51P05.

Introduction