the general science journal
The General Science Journal. July 2025.

# A Physical Proof of the Inconsistency of Infinity

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**Abstract**.-The overwhelming empirical evidence on the decreasing intensity of radial force fields with distance from the center of the field contradicts the impossibility of deducing that decreasing when applying transfinite arithmetic to the corresponding inverse-square laws, such as Coulomb's Law or Newton's Law of Universal Gravitation. This amounts to an empirical demonstration of the invalidity of this infinitist perspective for interpreting the physical world. It would be the first empirical proof of the inconsistency of the actual infinity.

**Keywords**: Radial force fields, Newton's Law of Gravity, Coulomb's Law, transfinite arithmetic, actual infinity, Axiom of Infinity.

## 1. Introduction

The image of the lines of force of a radial force field with arrows opening with the distance to the origin of the field in the spacetime continuum is one of the great graphic topics of contemporary physics. It justifies and explains in this way, as intuitively as it is wrong, the loss of field strength with the distance to the origin of that field (Figure 1). I dealt with this matter in another article in which the impossibility of any two lines of force of a radial field opening (separating) with the distance to the center of the field without there ceasing to be between them the same infinite non-numerable number of other lines of force, exactly $2^{\aleph_0}$ lines of force [1], was demonstrated.

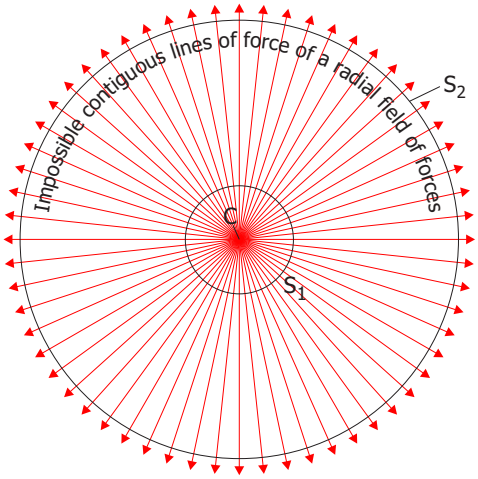


**Figure 1** – The usual and wrong way to represent, in the densely ordered spacetime continuum, the decrease with distance of the intensity of a radial force field: between any two of its lines of force there is always the same non-numerable infinite number ($2^{\aleph_0}$) of other lines of force (two-dimensional representation). $C$ center of the field; $S_1$ and $S_2$ plane sections of two concentric spheres with $C$.

The conclusion of this new paper is much more significant, shattering I would say, for the formal consistency of the mathematical infinitism assumed by both contemporary mathematics and physics. On the one hand, the overwhelming empirical evidence indisputably confirms the loss of intensity of radial force fields with distance from the center of such fields; but on the other hand, the dense order of the spacetime continuum and transfinite arithmetic make it impossible. So either all this overwhelming empirical evidence must be discarded, or the inconsistency of mathematical infinitism must be accepted.

## 2. A physical proof of the inconsistency of transfinite arithmetic

The argument developed here could be applied to other force fields, but it is not necessary to do so because it is only intended to prove physically a mathematical inconsistency, for which only one argument suffices, which is the following. Let $S_1$ and $S_2$ be two concentric spheres with the center of a radial force field $F_r$, and suppose that $S_1$ is close to that center while $S_2$ is arbitrarily far away from it (Figure 1). Obviously, if $r_1$ and $r_2$ are their respective radii and $A_1$, $A_2$ their respective areas, we can write:

$$A_1 = 4\pi r_1^2 \tag{1}$$

$$A_2 = 4\pi r_2^2 \tag{2}$$

$$0 < r_1 < r_2 \implies 0 < A_1 < A_2 \tag{3}$$

Where $r_1$, $A_1$, $r_2$ and $A_2$ are finite quantities greater than zero. The number of lines of force of the field $F_r$ traversing the surface of area $A_1$ is the same as the number of such lines traversing

the surface $S_2$ of area $A_2$: exactly $2^{\aleph_0}$ lines. And being $A_1 < A_2$ it is easy to be tempted to conclude that the density of $F_r$ lines of force in $A_1$ is greater than the density of such lines in $A_2$. And that is what physics usually does, ignoring the peculiarities of transfinite arithmetic.

Indeed, the densities $d_1$ and $d_2$ of the lines of force of the radial field $F_r$ in the respective concentric spheres $S_1$ and $S_2$ considered are given respectively by:

$$d_1 = 2^{\aleph_0} \times (4\pi r_1^2)^{-1};\ d_2 = 2^{\aleph_0} \times (4\pi r_2^2)^{-1} \tag{4}$$

And, as just noted, one would think that being $r_1 < r_2$ it should be $d_2 < d_1$. However, and according to transfinite arithmetic, what we can write is:

$$2^{\aleph_0} \times (4\pi r_1^2)^{-1} = 2^{\aleph_0} \times (4\pi r_2^2)^{-1} = 2^{\aleph_0};\ (\forall r_1, r_2 \in \mathbb{R}) \wedge (\ 0 < r_1 < r_2) \tag{5}$$

So that:

$$d_1 = d_2 \tag{6}$$

Which contradicts the enormous empirical experience of $d_2 < d_1$. We have, then, a contradiction:

$$d_1 = d_2 \text{ (according to transfinite arithmetic)} \tag{7}$$

$$d_1 > d_2 \text{ (according to empirical evidence)} \tag{8}$$

The empirical evidence for (8) is so enormous (all confirmed empirical data deduced from of the physical laws of the inverse of the square, such as Coulomb's Law, or Newton's Law of Universal Gravitation) that it allows us to affirm the falsity of (7), and consequently the inconsistency of the Axiom of Infinity which formally legitimizes the existence of infinite sets and then transfinite arithmetic. An inconsistency for which the author has found more than forty formal proofs in the last fifty years [2] (here [3] you can see one of the shortest and simplest ones).

We are, then, before the first empirical proof of the inconsistency of the actual infinity. But,experience tells me that this empirical proof will be of no consequence either in physics or in mathematical infinitism which, in the form of infinitesimal calculus (differential and integral), is so successful in contemporary physics. Although it can be shown very easily that this supposedly continuous calculus is in reality a discrete calculus, a jump calculus: there are no successive real numbers in the dense order of the real numbers, so that the approximation to the mathematical limit (decisive in infinitesimal calculus) is always discontinuous, discrete; and the jumps are always over the same number of different real numbers, since between each two of them, whatever they are, there are always $2^{\aleph_0}$ different real numbers over which one must jump [3] .

## Bibliographical References

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