

The Reality of Centrifugal Force

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Abstract. Centrifugal force is a well-known concept. It is the familiar outward acting force that is induced in rotating systems. It has many practical applications in engineering, and its mathematical formula plays a crucial role in planetary orbital analysis.

It is important therefore to investigate why the physics establishment shies away from embracing this concept to the full. We hear talk about centrifugal force not being a real force, or that it is merely an equal and opposite reaction to a centripetal force, or that it only exists in a rotating frame of reference, or that it doesn't exist at all. This unfortunate attitude will now be investigated in full.

The Physical Significance of Centrifugal Force

I. Some say that centrifugal force is merely a consequence of a body undergoing or striving to follow its uniform straight-line inertial path. And so it is. They are correct. But that doesn't mean that centrifugal force is not a real force. Others then say that it doesn't matter whether it's a real force or not, and that it is a waste of time arguing about it.

It matters however to those who believe that Einstein was wrong in discarding Maxwell's luminiferous aether. If an elastic medium acting as the carrier of electromagnetic waves does pervade all of space, as Maxwell and others believed, [1], [2], [3], [4], [5], then it becomes important that this physical medium exactly accounts for the inertial forces, otherwise its presence in the paths of the planetary orbits could cause the planets to fall into the Sun. See the added note below reference [1] in the reference section, which comments on the likeness between the inertial forces and the electromagnetic forces.

Why wouldn't Centrifugal Force be Real?

II. It is often argued that when a weight is being swung in circular motion on the end of a string and we cut the string, that since the weight flies off tangentially and not radially, this therefore proves that there is no such a thing as centrifugal force, and/or, that centrifugal force is merely the equal and

opposite reaction of the weight on the string to the action which the string exerts on the weight. But in fact, the string is only pulling inwards on the weight because the string is taut, and the string would not be taut unless the weight was first pulling on the string. And the weight is only pulling on the string because there is already an inertial centrifugal force acting on the weight due to the weight wanting to carry on at a tangent. It is the very fact of the weight wanting to carry on at a tangent that is causing the centrifugal force to act on the weight, which then pulls the string taut. And when the string is cut, the weight flies off both radially and tangentially. It's impossible that it could fly off tangentially only without also flying off radially.

While centrifugal force (*from Latin:- centrum-fugere = to flee away from a centre*) is commonly associated with circular motion, it is in fact an applied centripetal force (*from Latin:- centrum-petere = to seek the centre*), and not a centrifugal force, that is always responsible for curved path motion. Meanwhile centrifugal force is most certainly never a reaction to an applied centripetal force. The equal and opposite reactions to both of these two forces is on the pivot. The inward acting centripetal force in the example above exists due to the tension in the string and this tension is due to the string being taut, and it only becomes taut in the first place because of an already existing inertial centrifugal force. The centripetal force due to the tension in the string then cancels this inertial centrifugal force. There will still however remain a net centripetal force as will come clear through the polar analysis in Section III below.

It is the radial effect induced by the tendency to carry on tangentially that causes the water in Newton's rotating bucket to press against the inside of the walls of the bucket and to then rise upwards against gravity. Who could in all honesty deny that this is a demonstration of centrifugal force as a real force? In the case of a uniformly rotating body of water, it can even be shown that centrifugal force derives from rotational kinetic energy, otherwise known as centrifugal potential energy, $\frac{1}{2}mr^2\dot{\theta}^2$, where r is the radial distance from the centre of rotation, $\dot{\theta}$ is the angular speed, and where $v = r\dot{\theta}$ is the transverse speed at that radius. The centrifugal force acting on an element of water of mass m is obtained by differentiating the rotational kinetic energy with respect to r . The greater the centrifugal potential energy, the higher the surface of the water rises against gravity. Since centrifugal potential energy is maximum where the water touches the inside walls of the bucket, the surface of the water becomes concave. Centrifugal force therefore derives from an absolute quantity, kinetic energy, which is accepted to be real.

Yet some still say that centrifugal force doesn't exist at all because it disappears in an inertial frame of reference. Others say that it is a fictitious force because it is only observed in a rotating frame of reference. But anybody who has ever observed the action of a centrifuge machine will know that this action is absolute and does not disappear in an inertial frame of reference. There is no

frame of reference that can mask the effects of a centrifuge machine because there is nothing fictitious about centrifugal force.

Polar Analysis in an Inertial Frame of Reference

III. When we express Newton's second law of motion in an inertial frame of reference using polar coordinates, with respect to any arbitrarily chosen point origin, it takes the form,

$$m\ddot{\mathbf{r}} = m(\ddot{r} - r\dot{\theta}^2)\hat{\mathbf{r}} + m(2\dot{r}\dot{\theta} + r\ddot{\theta})\hat{\boldsymbol{\theta}} \quad (1)$$

and we can see that centrifugal force features. The second term on the right-hand-side of equation (1), as in $mr\dot{\theta}^2$, is the centrifugal force, while $m\ddot{\mathbf{r}}$ is the applied force. See **Appendix I** for the full derivation of equation (1) and for an explanation of the symbols. In planetary orbital analysis where the transverse components cancel because of Kepler's second law, see **Appendix II**, the total acceleration in the inertial frame is radial and it is equated with the acceleration due to gravity, \mathbf{g} , which in turn equates with the second time derivative of the radial length, $\ddot{r}\hat{\mathbf{r}}$, minus the centrifugal acceleration, $r\dot{\theta}^2\hat{\mathbf{r}}$. The radial force equation then takes the form,

$$m\mathbf{g} = m(\ddot{r} - r\dot{\theta}^2)\hat{\mathbf{r}} \quad (2)$$

which is normally rearranged as,

$$m\ddot{r}\hat{\mathbf{r}} = m(g + r\dot{\theta}^2)\hat{\mathbf{r}} \quad (3)$$

In other words, the second time derivative of the radial length is the resultant of gravity and the centrifugal force. Gravity and centrifugal force are the two active forces which determine the shape of a planetary orbit.

Since angular momentum is a conserved quantity under Kepler's second law (the law of equal areas in equal times), the centrifugal term, $r\dot{\theta}^2$, can be rewritten as an inverse cube law term, dependent on angular momentum. See **Appendix II**. Leibniz first presented this equation in the basic form,

$$\ddot{r} = a/r^3 - b/r^2 \quad (4)$$

where a is a constant related to angular momentum and b is a gravitational factor. See the derivation of Leibniz's equation from an elliptical orbit in *"Leibniz's Radial Planetary Orbital Equation"* [6]. In general, equations (3)

and (4) can be solved to an ellipse, circle, parabola, hyperbola, or a straight line, and in the case of the straight line, it can either be radial or transverse to the polar origin.

Some say that if we were to re-write these equations in Cartesian coordinates that centrifugal force would disappear. Not so. The radial distance r would simply be replaced by $r^2 = x^2 + y^2$. Centrifugal force is always measured to a point and so we need to use position vectors as opposed to displacement vectors in the analysis. Using Cartesian coordinates will not undermine the reality of the centrifugal force.

Equation (4) indicates that centrifugal force is dependent on angular momentum and the inverse cube of distance from the origin while being totally independent of any applied centripetal forces such as gravity. Centrifugal force cannot therefore be an equal and opposite reaction to an applied centripetal force, especially since an action-reaction pair acts across two bodies. This was something that even Sir Isaac Newton himself failed to realize. Newton objected to Leibniz's equation on these grounds, but the reality is that it is Leibniz's equation which is used today in planetary orbital analysis [7]. This was a matter that Newton got wrong, which is perhaps at least part of the reason why there is so much confusion today surrounding this topic.

Conclusion

IV. Centrifugal force is a real force that gives rise to real physical effects. It acts outwards from a centre in a rotating system. So, first and foremost, we should remember that while mathematics is used to formulate centrifugal force, it is arrant nonsense to suggest that centrifugal force is merely a creation of mathematics.

Yet, according to what level is being taught, the schools and universities will either deny the existence of centrifugal force altogether, or claim that it is merely the reaction to a centripetal force, or pass it off as an artefact of making observations from a rotating frame of reference. And all three of these perspectives are mutually exclusive! When the first two of these three perspectives are being taught, the argument is invariably restricted to the special limiting case of circular motion, where the equality in magnitude between all the radial forces involved has the ability to mask the centrifugal force altogether, or to make it deceptively appear as part of an equal and opposite action-reaction pair. In uniform circular motion, equation (1) in Section III above reduces to,

$$m\ddot{\mathbf{r}} = -mr\dot{\theta}^2\hat{\mathbf{r}} = -mv^2\hat{\mathbf{r}}/r \quad (5)$$

where v is the transverse speed. The total force involved, relative to the inertial frame, is a centripetal force which is equal to the negative of the centrifugal force. This limiting circular motion example is enough to convince most students that centrifugal force doesn't exist. When we remove the applied centripetal force and note that the object flies off at a tangent to the point of release on the circle, this further convinces them that no outward radial force is or was ever involved. They tend however to miss out on the fact that the object flies off both radially and tangentially.

However, when we generalize the motion to ellipses, parabolas, hyperbolas, curved motion generally, and straight-line motion, the centrifugal force emerges as a distinct force in its own right and it becomes clear that $-mv^2\hat{r}/r$ does not correspond to the applied centripetal force as might have been believed based on equation (5). Unfortunately, the vast majority of physicists today who enter the centrifugal force debate are still only at equation (5) level, and they tend to dig in there as if circular motion were the whole story.

Centrifugal force is an inertial force induced by a body as a result of it either following its uniform straight-line inertial path or striving to do so. Naturally of course, it then follows that centrifugal force can't be counted among the forces that would cause a body to deviate from its inertial path, and so some might say that centrifugal force is not therefore a Newtonian force. It should be noted however that when a body is travelling along in its uniform straight-line inertial path, that it will have a centrifugal force to every point in space and that these will all cancel. When this fact is taken into consideration, then centrifugal force does become a Newtonian force. Indeed, it starts to take on a likeness to magnetic repulsion when we see how Maxwell attempted to explain Ampère's circuital law using centrifugal force in a sea of molecular vortices that pervades all of space. These vortices press against each other with centrifugal force while striving to dilate, and their mutual rotation axes trace out the prevailing magnetic lines of force. See "***Straight Line Motion***" and "***The Significance of the Inertial Forces***" [8], [9].

What centrifugal force is definitely not, is a reaction to a centripetal force as part of an equal and opposite action-reaction pair. It can however form part of an action-reaction pair with another centrifugal force. In circular motion examples where some textbooks claim that centrifugal force is part of an action-reaction pair with a centripetal force, the commonly used example is that of a weight being swung around on the end of a string. They say that the centrifugal force pulling outwards on the string is an equal and opposite reaction to the string pulling inwards on the weight. However, the string is only taut in the first place because of the already existing centrifugal force acting on the weight due to its transverse motion, and an action-reaction pair must act across two bodies. The equal and opposite reaction to the induced centripetal force acting on the weight is actually the centripetal force that now acts on the pivot.

Finally, it is quite common for people who do actually accept the reality of centrifugal force, to avoid using the very name itself. For example, they might prefer to talk about it as being merely the effects of inertia. “*Inertia*” is a vague archaic term that can variously mean kinetic energy, momentum, or inertial mass. It has no unique mathematical description that we could use in an equation. Using vague language or using a formula of words designed to avoid using the specific term “*centrifugal force*”, does not however undo the reality of it.

References

[1] Clerk-Maxwell, J., “*On Physical Lines of Force*”, Philosophical Magazine, Volume XXI, Fourth Series, London, (1861)

http://vacuum-physics.com/Maxwell/maxwell_oplf.pdf

† Equation (77) in this paper is Maxwell’s electromotive force equation and it exhibits a strong correspondence to equation (1) above. The centrifugal and Coriolis terms in equation (1) correspond to the compound centrifugal term $\mu\mathbf{v} \times \mathbf{H}$, while the other transverse term corresponds to $-\partial\mathbf{A}/\partial t$. Meanwhile Coulomb’s law, which is also in Maxwell’s equation (77), corresponds in form to the gravity term in equation (4).

[2] Tombe, F.D., “*The Double Helix Theory of the Magnetic Field*” (2006)

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<http://gsjournal.net/Science-Journals/Research%20Papers-Mathematical%20Physics/Download/6371>

[3] O’Neill, John J., *PRODIGAL GENIUS, Biography of Nikola Tesla*, Long Island, New York, 15th July 1944

<http://www.rastko.rs/istorija/tesla/oniell-tesla.html>

“Long ago he (mankind) recognized that all perceptible matter comes from a primary substance, of a tenuity beyond conception and filling all space - the Akasha or luminiferous ether - which is acted upon by the life-giving Prana or creative force, calling into existence, in never ending cycles, all things and phenomena. **The primary substance, thrown into infinitesimal whirls of prodigious velocity, becomes gross matter; the force subsiding, the motion ceases and matter disappears, reverting to the primary substance**”.

[4] Lodge, Sir Oliver, “*Ether (in physics)*”, Encyclopaedia Britannica,

Fourteenth Edition, Volume 8, Pages 751-755, (1937)

<http://gsjournal.net/Science-Journals/Historical%20PapersMechanics%20/%20Electrodynamics/Download/4105>

In relation to the speed of light, “*The most probable surmise or guess at present is that **the ether is a perfectly incompressible continuous fluid, in a state of fine-grained vortex motion**, circulating with that same enormous speed. For it has been partly, though as yet incompletely, shown that such a vortex fluid would transmit waves of the same general nature as light waves— i.e., periodic disturbances across the line of propagation—and would transmit them at a rate of the same order of magnitude as the vortex or circulation speed*”

[5] Whittaker, E.T., “*A History of the Theories of Aether and Electricity*”, Chapter 4, pages 100-102, (1910)

“All space, according to the younger Bernoulli, is permeated by a **fluid aether, containing an immense number of excessively small whirlpools**. The elasticity which the aether appears to possess, and in virtue of which it is able to transmit vibrations, is really due to the presence of these whirlpools; for, owing to centrifugal force, each whirlpool is continually striving to dilate, and so presses against the neighbouring whirlpools.”

[6] Tombe, F.D., “*Leibniz’s Radial Planetary Orbital Equation*” (2017)

https://www.researchgate.net/publication/320694417_Leibniz's_Radial_Planetary_Orbital_Equation

[7] Swetz, Frank J., “*Learn From The Masters*” ‘An Episode in the History of Celestial Mechanics’, page 269

Mathematical Association of America (1995)

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[8] Tombe, F.D., “*Straight Line Motion*” (2018)

https://www.researchgate.net/publication/325472420_Straight_Line_Motion

[9] Tombe, F.D., “*The Significance of the Inertial Forces*” (2019)

https://www.researchgate.net/publication/332912193_The_Significance_of_the_Inertial_Forces

Appendix I

Consider a particle moving in an inertial frame of reference. We write the position vector of this particle relative to any arbitrarily chosen polar origin as,

$$\mathbf{r} = r\hat{\mathbf{r}} \quad (1A)$$

where the unit vector $\hat{\mathbf{r}}$ is in the radial direction, and where r is the radial distance. Taking the time derivative and using the product rule, we obtain the particle’s velocity,

$$\dot{\mathbf{r}} = \dot{r}\hat{\mathbf{r}} + r\dot{\theta}\hat{\boldsymbol{\theta}} \quad (2A)$$

where $\hat{\boldsymbol{\theta}}$ is the unit vector in the transverse direction, and where $\dot{\theta}$ is the angular speed about the polar origin. Taking the time derivative again we obtain the expression for the particle’s acceleration in the inertial frame,

$$\ddot{\mathbf{r}} = \ddot{r}\hat{\mathbf{r}} + \dot{r}\dot{\theta}\hat{\boldsymbol{\theta}} + \dot{r}\dot{\theta}\hat{\boldsymbol{\theta}} + r\ddot{\theta}\hat{\boldsymbol{\theta}} - r\dot{\theta}^2\hat{\mathbf{r}} \quad (3A)$$

which can be rearranged as,

$$\ddot{\mathbf{r}} = (\ddot{r} - r\dot{\theta}^2)\hat{\mathbf{r}} + (2\dot{r}\dot{\theta} + r\ddot{\theta})\hat{\boldsymbol{\theta}} \quad (4A)$$

Appendix II

Kepler's second law is the law of areal velocity, which is essentially a statement that angular momentum is conserved, and it means that only radial forces are involved in planetary orbits. This can be proven in reverse as follows. If the total transverse acceleration is zero, then,

$$2\dot{r}\dot{\theta} + r\ddot{\theta} = 0 \quad (5A)$$

hence, multiplying across by r ,

$$2r\dot{r}\dot{\theta} + r^2\ddot{\theta} = 0 \quad (6A)$$

Since,

$$2r\dot{r}\dot{\theta} + r^2\ddot{\theta} = d/dt(r^2\dot{\theta}) = 0 \quad (7A)$$

then,

$$r^2\dot{\theta} = L \quad (8A)$$

where L is a constant which is twice the areal velocity, and so the areal velocity must be constant. Hence,

$$\dot{\theta} = L/r^2 \quad (9A)$$

substituting $\dot{\theta}$ in (9A) into (4A) in conjunction with (5A) we obtain,

$$\ddot{\mathbf{r}} = (\ddot{r} - L^2/r^3)\hat{\mathbf{r}} \quad (10A)$$

which expresses the centrifugal force in terms an inverse cube law in radial distance from the polar origin and a constant, L^2 , that is related to angular momentum. In the case of the uniform straight-line inertial path, where we have $\ddot{\mathbf{r}} = 0$, equation (10A) becomes,

$$\ddot{\mathbf{r}} = +a\hat{\mathbf{r}}/r^3 \quad (11A)$$

where a is a constant related to angular momentum. This means that relative to any arbitrarily chosen point in space, a particle undergoing uniform straight-line motion experiences a centrifugal acceleration to this point, dependent on the angular momentum relative to it, and this acceleration diminishes as per the inverse cube of the distance from the point.

A centrifugal force, being angular momentum dependent, is of course always rotating, but this doesn't mean that we have to be rotating with it in order to observe it. It's a case of a rotating vector that is universally observable in an inertial frame of reference. Unfortunately, modern physicists feel the need to employ the concept of a rotating frame of reference and then to argue that centrifugal force can only be observed in such a rotating frame. This is arrant nonsense. The rotating frame of reference serves only to obfuscate the analysis.