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COMMENT TO MY WORK, "SOLUTION OF THE PROBLEM OF MULTISECTING AN ANGLE"

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Being engaged in critical analysis of the foundations of natural sciences for 30 years, I understood the main scientific truth: there are logical errors in the statement and solution of fundamental problems. In other words, the foundations of sciences contain logical errors. This truth has not yet completely comprehended by scientists. These statements also concern classical geometry. From this point of view, I would like to explain my work on multisecting (trisecting) an angle.

1. In my opinion, the problem of multisecting (in particular, trisecting) an angle, i.e. the problem of division of a given arbitrary angle into the given set of equal parts using only a compasses and an unmarked straightedge, differs essentially from the problem of multisecting a straight line. A straight line (as the simplest form of a line) can be divided into a set of identical and of equal parts with only a compasses and an unmarked straightedge. However, an angle (as a system of two intersecting straight lines) and a curved line (for example, an arc of circle) cannot be divided into a set of identical and equal parts with only a compass and an unmarked straightedge in all cases because a curved line is not a sum and consequence of set of straight line segments. (Russian mathematician N. Lobachevsky was probably the first who realized this fact). Difficult practical problems can not be solved using only simple tools and methods. The solution of complex practical problem is the unity of practical and theoretical results. Criterion of truth of the solution is a practice.

2. Geometry studies the form of a material object. Form is a system of parts (elements). System (i.e. whole, made of parts) is a set of the elements which are in relations and connections with each other and form certain integrity, unity. Part (i.e. element, line) is characterized by a form (i.e. qualitative determinacy) and size (i.e. quantitative determinacy). Therefore, the problem of dividing the line has two aspects: qualitative and quantitative aspects. The qualitative aspect is decomposition of a given form in the set of non-identical forms (for example, Fourier expansion) or transformation of a given form to another form (i.e. transformation of an arc to a straight line segment, i.e. straightening of an arc). The quantitative aspect is division of a given form into a set of identical forms. For example, decomposition of an arc into a set of identical arcs is a quantitative aspect, the quantitative operation, and decomposition of an arc in the set of chords or straightening of an arc is a qualitative aspect, a qualitative operation. In carrying out the operation of division, one should take into consideration the following requirement of formal logic: the arc should be measured by a unit arc, the angle should be measured by a unit angle, and the straight line should be measured by a unit straight line. But if one takes into account only the length of the line (for example, if one neglects the difference between the form of the arc and the form of the chord), the operation of division into parts is both qualitative and quantitative operation.

3. An angle is a geometric figure formed by two intersecting straight lines. This form can be measured and divided into parts, if there is a unit angle. A unit angle exists if the angle is a part of the more complex system. In this case, there is a relationship between the angle and other elements of the system. For example, a central angle is an element of the system “circle + central angle + chord”. The relation between the quantity of the central angle, length of corresponding chord, and radius of the circle exists and leads to the concepts “radian measure of the angle”, “grade measure of the angle”. In the case of a rectangular triangle, there is a relation (functional relationship) between the quantity of the angle and the ratio of the lengths of sides of the triangle. If an angle is not a part of a more complex system, it would be impossible to formulate the scientific concept “quantity of angle” and to determine the unit of measurement of the angle.

4. The theoretical basis of classical geometry is Euclid's axiomatic system. These axioms contain the following concepts: point, line, straight line segment, circle with radius and center, right angle, etc. But Euclid's text "Elements" does not contain logically correct definitions. For example, the concepts “point”, “line”, and “straight line” are defined in “Elements” as follows: the point is that which has no parts; line is length without width; straight line is a such line which is equally situated with respect to all its points. Really, these definitions do not satisfy the requirements of formal logic. Furthermore, Euclid's axiomatic system does not contain a definition of the concept “line form”. Therefore, from a formal-logical point of view, the concepts “arc”, “chord”, and “diameter” are indistinguishable (i.e. are identical) concepts in Euclid's “Elements”.

5. From a formal-logical point of view, the concepts “arc length” and “chord length” are identical ones if these concepts do not take into consideration the difference between the forms of lines (i.e. if a form is not a essential sign of line). In this case, these concepts characterize only the common (quantitative) aspect of geometric objects. Then the geometric objects (for example, arc and chord) can be compared with each other. But the difference between the forms of lines can be completely taken into consideration in neither practical classical geometry nor theoretical classical geometry. That is why there is a discrepancy between the practical solution (with a compasses and an unmarked straightedge) and the theoretical solution (based on mathematics). The most perfect approach is computational geometry as a unity of theory and of practice.

6. One can partially eliminate the discrepancy between a practical solution and a mathematical solution if one will combine practical and theoretical approaches to solution. In this case, one should take into consideration the following requirement of formal logic: the arc should be measured by a unit arc, the angle should be measured by a unit angle, and the straight line should be measured by a unit straight line. However, if the arc and the chord (diameter) are described in terms of the “arc length” and “chord length” (“diameter length”) in theoretical geometry, then such a description leads to the appearance of a coefficient of proportionality (i.e. form factor) k in the formulas. Form factor is defined as follows:

$$k = \frac{l}{h}$$

(where l and h are the length of arc of a circle and length of corresponding chord of a circle, respectively). Form factor is an increasing function of the angle α : if $l = L/6$, then $h = R$ (where L and R are the length of the circle and the radius of the circle, respectively) and

$$k = \frac{L}{6R} = \frac{2\pi R}{6R} = \frac{\pi}{3};$$

if $l = L/2$, then $h = 2R$ and

$$k = \frac{L}{4R} = \frac{2\pi R}{4R} = \frac{\pi}{2}.$$

7. Taking into consideration the existence of the logical invisibility of lines in Euclid's text “Elements” and the existence of the form factor k, one can prove Theorem 10: diameter passed through an arbitrary point of the arc divides this arc and the chord which ties this arc into proportional parts. In order to do it, one should reformulate this theorem as follows: if an arbitrary point of an arc divides the arc into two parts in the ratio of q (where $0 < q < 1$), then there exists a point of the chord that divides the chord into two parts in the same ratio; the line passing through these points is called diameter. The proof is as follows: multiplying the definition of form factor k by an arbitrary fractional number q, one can obtain the result: $ql = qhk$. But this result cannot be obtained by constructing. Thus, one can formulate the proportions (proportions are a mathematical relationships between the lengths of geometric objects). But the method of proportions is not a substitute for a method of practical construction.

8. The essence of multisecting (in particular, trisecting) an angle (arc) is manifested with the following operations: (a) transformation of the arc to a straight line segment (i.e. straightening the arc); (b) division of this straight line segment into a set of equal parts and marking of the points obtained; (c) transformation of this straight line segment to the arc (i.e. the bending the straight line to the arc); (d) passing straight lines from the center through the obtained points. The result of these operations is the division of an angle into a set of identical and equal parts.

Thus, scientific and cognitive significance of my work is that my work promotes a critical analysis of geometry within the framework of the correct methodological basis – unity of formal logic and of rational dialectics. In order to understand why theoretical geometry is differs from practical geometry, or, in general, why theory differs from practice, one should analyze the geometry on the base of the philosophical categories “theory and practice”, “content and form”, “quality and quantity”, “part and whole”. Then this will lead to the realization that theoretical geometry (for example, the axiomatic approach) should contain definitions of concepts and should represent a field of natural sciences.