Evaluation of Four Energy-Momentum Relations

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Abstract

The aim of this research is to identify and classify the different formulas that describe the relationship between energy and momentum. We examine all the three currently known relations in addition to a newly proposed relation in terms of their range of validity and their physical significance. It is found that the proposed relation is the most broadly valid and strongest in terms of physical significance.

Introduction

Formulating physical theories in the form of conservation laws is of great importance, because such a formulation clearly reveals the symmetries underlying the theory. This makes the theory simpler and more amenable to development, as well as easier to connect with and compare to other theories.

The Definitions of Energy and Momentum and the Significance of their Relation

First, we must clarify what is meant by energy and momentum. Energy and momentum are two quantities constructed from the variables describing the motion of bodies in fields: energy is associated with time, while momentum is associated with the spatial dimensions. However, this alone is not sufficient to define them, because an infinite number of quantities can be constructed from these variables. The fundamental feature that distinguishes energy and momentum is the conservation law: any quantity that is not conserved cannot be called energy or momentum. A quantity may be conserved only within a certain domain, and in that case it may be called energy or momentum only within that domain. For example, the quantity $\mathrm{E} = \frac{1}{2}mv^2$ is conserved for low velocities and in the absence of a gravitational field, so it may be called energy as long as we remain within this framework. But if we move outside this framework, it becomes merely a constructed quantity—similar to $\frac{1}{2}mv^{\frac{1}{2}}$ or $3mv^4$, and so on—which has no physical meaning.

There is another requirement in defining energy and momentum, although it is less important than the requirement of conservation. This requirement concerns the unity of the mathematical structure in their definitions, such that the only difference between the definition of energy and that of momentum is that the first is related to time while the second is related to spatial dimensions. Relations whose definitions of energy and momentum satisfy this requirement are preferable to others because they reveal the symmetry between time and spatial dimensions.

As for the relationship itself, its importance lies in its origin. If the relation between energy and momentum is derived directly from the definitions themselves, then the relation is trivial. This does not mean that it is completely useless or meaningless, since it still represents a relation between physical quantities that do have meaning, but its importance is reduced. However, when the relation results from a physical law independent of the definitions—together with the definitions themselves—then the relation becomes strong and highly significant.

Now, based on these objective concepts, we seek to classify the known relations between energy and momentum and to add to them a newly proposed relation. This classification will show that the proposed relation represents the complete and ideal form.

(1)Newtonian Mechanics in the Absence of Gravity

Definitions of Energy and Momentum: $E = \frac{1}{2}mv^2$, $P = mv$.

Energy-Momentum Relation[1]:

$$\boxed{\mathbf{E = \frac{1}{2m}P^2}}$$

Domain of Validity: Low Velocity, No Gravity.

Structural Similarity to Definitions: No Similarity.

Source of the Relation (Triviality Criterion): Definitions, the Relation is Trivial.

Additional Notes: In Newtonian Mechanics in the Presence of Gravity there is no Energy-Momentum Relation, because there is no Definition for the Conserved Momentum.

(2)Special Relativity

Definitions of Energy and Momentum :$E = \gamma mc^2$, γmv.

Energy-Momentum Relation:[2]

$$\boxed{\mathbf{E^2 = (Pc)^2 + (mc^2)^2}}$$

Domain of Validity: Any velocity, No Gravity.

Structural Similarity to Definitions: Similar Structure **(** if we write γmvc instead of γmv.).

Source of the Relation (Triviality Criterion): Definitions, the Relation is Trivial.

Additional Notes: These definitions are most closely associated with the names energy and momentum, therefore, in the coming steps confusion may arise when we call other conserved quantities energy and momentum.

(3)Relativistic Static Gravitational Field

Definitions of Energy and Momentum: $E = \sqrt{(\gamma mc^2)^2 - \frac{2\gamma^2 m^2 MGc^2}{r}}$, $P = \sqrt{(\gamma mvc)^2 - \frac{2\gamma^2 m^2 MGc^2}{r}}$.

Energy-Momentum Relation[3]:

$$\boxed{\left((\gamma mc^2)^2 - \frac{2\gamma^2 m^2 MGc^2}{r}\right) = \left((\gamma mvc)^2 - \frac{2\gamma^2 m^2 GMc^2}{r}\right) + (mc^2)^2}$$

Domain of Validity: Any Velocity, Any Static Gravitational Field.

Structural Similarity to Definitions: Definitions are Similar in Structure.

Source of the Relation (Triviality Criterion): Definitions, the Relation is Trivial.

Additional Notes: Someone who looks only at the mathematical aspect of this relation might think it is pointless to have two identical terms on both sides of the equation. Why not simply cancel them and obtain a simpler equation? However, from a physical point of view, if we did that, the equation would become meaningless, because it would merely relate quantities that are not conserved; therefore it would have no physical significance.

(4)New General Relation (Proposed)

Definition of Energy and Momentum:

$$E^2 = m^2\gamma^2 c^2 (g_{00} v^0 v^0 - g_{00} U^0 U^0), \qquad P^2 c^2 = \sum_{i,j=1,2,3} m^2 \gamma^2 c^2 (g_{ij} v^i v^j - g_{ij} U^i U^j)$$

Where U^p is a purely radial vector quantity whose magnitude is given by:

$$U = \sqrt{\frac{2GM}{r}}$$

$$U^t = 0 \quad , \quad U^r = \frac{U}{\sqrt{g_{rr}}} \quad , \quad U^\theta = 0 \quad , \quad U^\emptyset = 0$$

The physical meaning of this quantity can be understood as follows: Suppose a body is located very far from the source of gravity and begins to fall from rest ($v^t = c$, $v^r = 0$, $v^\theta = 0$, $v^\emptyset = 0$) toward that source. When it reaches a distance r from the source, the components of its velocity will be ($\boldsymbol{v^t} = \mathbf{c}$, $\boldsymbol{v^r} = \boldsymbol{U^r}$, $\boldsymbol{v^\theta} = \mathbf{0}$, $\boldsymbol{v^\emptyset} = \mathbf{0}$)., indicating that it has acquired velocity components U^p of magnitude ($U^t = 0$, $U^r = \frac{U}{\sqrt{g_{rr}}}$, $U^\theta = 0$, $U^\emptyset = 0$.)

We can generalize this idea if the field is changing and calculate components U^p in that case.

Energy-Momentum Relation[4]:

$$\boxed{\boldsymbol{m^2\gamma^2c^2}(\ g_{00}\boldsymbol{v^0v^0} -\ g_{00}\boldsymbol{U^0U^0}), + \sum_{i,j=1,2,3}\boldsymbol{m^2\gamma^2c^2}(\ g_{ij}\boldsymbol{v^iv^j} -\ g_{ij}\boldsymbol{U^iU^j}) = \boldsymbol{m^2c^4}}$$

Domain of Validity: Any Velocity, Any Gravitational Field.

Structural Similarity to Definitions: Structure of Definitions is Similar.

Source of the Relation (Triviality Criterion): The relation is Not Trivial because the source of the relation is not the definitions of Energy and Momentum but General Relativity itself, this can be shown as follows:
We have : $U^2 = \sum_{i,j=1,2,3} g_{ij}U^iU^j$, $v^2 = \sum_{i,j=1,2,3} g_{ij}V^iV^j$

This gives: $E^2 = \boldsymbol{m^2\gamma^2c^4}\, g_{00}$ and $P^2\boldsymbol{c^2} = \boldsymbol{m^2\gamma^2c^2}\left(v^2 - U^2\right)$,
In Schwarzschild solution $g_{00} = -\left(1 - U^2lc^2\right)$,

Thus; $E^2 + P^2\boldsymbol{c^2} = \boldsymbol{m^2c^4}$

Additional Notes: This relation indicates that the gravitational field, represented by the quantity U, the geometry of spacetime represented by the metric, and the motion of the body represented by the velocity and the Lorentz factor all combine to form an invariant quantity that depends only on the mass.

The square of the body's mass is a constant quantity distributed between two containers: a temporal container that holds the square of the energy, and a spatial container that holds the square of the momentum. The spatial container itself is further divided into three internal containers.

The interactions of the body with its environment, whatever they may be, do not involve any exchange of the quantity represented by the square of the momentum or the square of the energy. Rather, the exchange occurs only internally, where part of the energy may transform into momentum or vice versa, or momentum may be redistributed among its internal components.

References

[1]See:Isaac Newton, Philosophiæ Naturalis Principia Mathematica (London: Royal Society, 1687).

[2]See:Albert Einstein, “Zur Elektrodynamik bewegter Körper,” Annalen der Physik 17 (1905): 891–921.

[3]See:Albert Einstein, “Die Feldgleichungen der Gravitation,” Sitzungsberichte der Preußischen Akademie der Wissenschaften (1915): 844–847. This relation is not known in the complete form presented in this paper. Some references contain the conserved energy resulting from time symmetry, but the remaining information about the conserved form of the total momentum and the relation between the conserved energy and the conserved total momentum can be easily inferred to complete the picture and facilitate comparison with other relations.

[4]See:Mueiz Gafer KamalEldeen , The Expansion of the Universe is a Transformation of Energy into Momentum, viXra:2512.0077.