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A brief argument with profound implications for physics

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Abstract.-This article develops a new and brief argument on a classic problem that science has yet to resolve. It discusses the uniform motion of a macroscopic object from a new perspective that inevitably leads to a contradiction involving the assumed continuous nature of space and time. The only solution to this contradiction is to consider a discrete nature for both space and time, and consequently the discrete nature of motion.

Keywords: change, uniform motion, dense order, discrete space, discrete time, discrete motion.

1. Introduction

One of the most notable characteristics of the set $\mathbb{R}$ of real numbers is its dense order: between any two of these numbers there are always another $2^{\aleph_o}$ different real numbers. And the same is true for points on any line in the spacetime continuum: between any two of them there are always another $2^{\aleph_o}$ different points. An immediate consequence is that any line of, for example, one angstrom in length, has the same number of points as another of 95 billion light years. Or that one trillionth of a second contains as many instants as the 13.8 billion years of the supposed complete history of the universe. Another immediate consequence of the dense order is that no point in space, nor any instant in time, has an immediate successor, as is the case, for example, between successive natural numbers, where each number n has an immediate successor $n + 1$. This difference has very negative consequences for contemporary infinitist physics, as will be seen in this article.

A classic and well-known paradox of the dense order of real numbers is Zeno's paradox about the impossibility of change (although Zeno obviously does not refer to the dense order of real numbers), paradoxe that remain unresolved [6, p. 655-656]:

> Change is the most pervasive characteristic of our incessantly evolving universe. But change is also the most elusive and difficult question we have ever been faced with (for a general background see [12, 14] and the particular view of H. Bergson in [1, 2]). So elusive that no one has been able to explain how a simple change in position of a physical object occurs. So elusive that it could be inconsistent, as claimed at least since the time of Parmenides. But not only pre-Socratic authors as Parmenides, Zeno of Elea or Mellisus claimed the impossibility of change [12], modern authors as J. E. McTaggart also defended the impossibility of change [11]; and Hegel its inconsistent existence [3, p. 382]. If that were the case, the task of explaining the physical world in consistent terms would be impossible because the physical world is, essentially, change. And being physics the science of change, the science of the regular succession of events in Maxwell's words [10, p. 1], it should be concerned with the solution of this fundamental step in the understanding of physical reality.
>
> It seems reasonable to assume that we model reality as a continuous system because we perceive it as a continuous system. The problem is that this perceived continuity is illusory. In fact, our brain takes a time greater than zero (≈13 ms [13]) to process each visual image (the base of the well known α, β, γ and δ movements, and of ϕ-phenomenon), so that *a continuum* of visual images is physiologically impossible. The same illusory perception happens with motion when observed in a film. And in the same way a film is a discontinuous sequence of images, natural motion could also be a discontinuous sequence of changes in position, which is perceived as continuous by

our brains and our physical instruments.

Obviously, motion is a type of change: a change of position over successive instants in time. This article discusses the uniform motion of a macroscopic object along a straight trajectory, but from a new perspective, different from Zeno's discussions, which are invariably based on the non-existence of the next position for any given position (due to dense order of points and real numbers). The consequence of our new discussion is inevitable: the continuous motion modeled in the spacetime continuum is inconsistent.

2. Neither points have extension, nor instants have duration

Although the zero spatial extension of points in space and the zero temporal duration of instants of time are commonly accepted, it is worth providing a formal proof that this is so. As will be seen below, the proof could not be simpler. To do this, we will first state and accept the following axiom, the statement of which is usually taken for granted without the need to declare it as such an axiom:

Axiom 1 (of the null Extension) *All points (instants) on the spacetime continuum have the same extension (duration).*

Comment: If this were not the case, it would be necessary to specify the spatial extension of each point in space and the temporal duration of each point (instant) in time, which is obviously not the case. Axiom 1 of the Null Extension allows us prove the following:

Theorem 1 *The spatial points of the spacetime continuum have zero extension, as does the duration of the instants of that same continuum.*

Figure 1 – The dense order of real numbers, and therefore of points on the real line, implies that between points P and Q, between points P and R, and between points R and Q there exists the same uncountable infinite number ($2^{\aleph_o}$) of different points.

Proof.-Let P and Q be any two points on a line separated by a distance δ/n, where $\delta > 0$ is any arbitrarily small real number, and $n \geq 1$ is any arbitrarily large natural number. Let R be a point on the segment PQ located at the same distance from P as from Q (Figure 1). Since between P and Q, between P and R, and between R and Q there is the same uncountable infinite number of points ($2^{\aleph_o}$ points), the extension of R must be less than δ because otherwise R would occupy the entire segment PQ, or the entire segment PR, or the entire segment RQ, which is not the case because between P and Q, between P and R, and between R and Q, there is the same uncountable infinite number of different points, all of the same extension (Axiom 1 of the Null Extension). And since δ is any positive real number, the extension of R must be less than any positive real number. Therefore, the extension of R can only be zero. And considering again Axiom 1 of the Null Extension, it must be concluded that all points (instants) of the spacetime continuum must have the same null extension (duration). □

The statement of the above theorem is usually accepted without proof. Here, proof has been provided to avoid unnecessary discussions related to the content of the following section, which addresses the central problem of this article on the supposed continuous nature of uniform motion.

3. On the assumed continuous nature of uniform motion

It can be proved that Zeno's Dichotomy Paradox is actually a contradiction that can be formally deduced from the dense order of real numbers and therefore from the points of the real line and of the spacetime continuum. [4, 5, 7]. In fact, it is the dense order of real numbers (and of points/instants in the spacetime continuum) that is formally responsible for the fact that no point of the continuum that serves as a model for physical space has an immediate successor point in any direction. In other words, points in space do not touch each other in any spatial direction. The same applies to moments in time. Between any two of these points (instants) there always exists the same uncountable infinite number of other different points (instants). The following argument also makes use of the dense order of real numbers, but from a new perspective that is very different from Zeno's classical arguments. And much more decisive, as will be seen immediately.

Although it is not necessary for the argument that follows, we will assume the physical reality of space deduced from its ability to vibrate, transmit its vibrations (gravitational waves), and interact with real interferometers that detect its real vibrations: that which does not exist cannot vibrate, transmit its vibrations, or interact with real physical instruments.

To avoid unnecessary discussions related to the dual wave-particle nature of photons (supposedly point-like physical objects), in this argument we will use any macroscopic physical object Ob animated with a uniform motion of, for example, 100 km/h. That said, let P be any point of the physical object Ob[1] and S its straight trajectory. Let a and b be any two points on that trajectory separated, for example, by 1 km; and let t_{ab} be the time it takes P to go from a to b at its uniform velocity of 100 km/h (Figure 2).

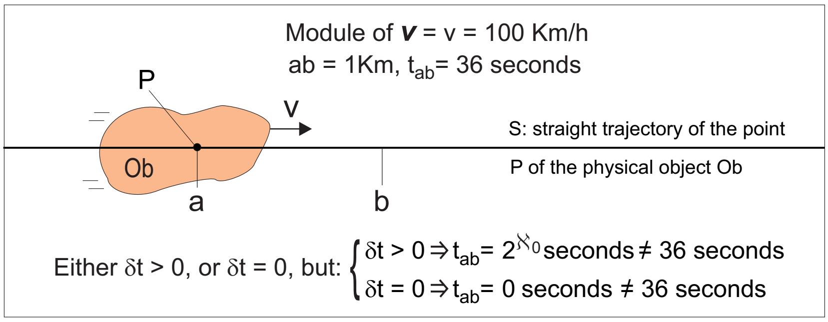

Figure 2 – The supposed continuous and uniform motion of a macroscopic object Ob, where t_{ab} is the time it takes the point P of the physical object Ob to go from point a to point b of its straight trajectory S, and δt is the time that this point P remains at each point of its straight trajectory, which can only be either zero or greater than zero. As suggested in this figure (and proved below), both possibilities are impossible. In consequence, it should be considered the possibility that motion can only be discrete, i. e. discontinuous.

The main idea of this argument is to consider the time δt that P remains at each point of its straight trajectory S, assuming that it passes through all the points of its trajectory. Otherwise, if that were not the case, it would be necessary to specify which points P passes through and which it does not, and why this is so. Naturally, this is not the case in the assumed continuous motion of P along its continuous trajectory S. It is therefore assumed that P passes through all points of its straight continuous trajectory S. Let us therefore consider the only two possible cases:

1.- P remains for a time $\delta t > 0$ at each point of its trajectory S.

2.- P remains for a time $\delta t = 0$ at each point of its trajectory S.

Note that the third possible alternative for the indeterminacy of δt simply implies ignorance of not being able to know which of the two previous cases applies, but it can be stated that it will be one of the two previous cases. This is because there are no other possibilities in the set of non-negative real numbers that represent the passage of time into the future.

Let us now consider the closed interval $[a, b]$ of the trajectory of P, and suppose that the distance ab is exactly 1 km. Since the uniform velocity of P is 100 km/h, we will obviously

[1] Although it would be very interesting to do so, I will not discuss here whether points without extension can represent parts of physical objects with extension.

have:

$$t_{ab} = \frac{1\text{Km}}{100 \text{ Km/h}} = 0.01 \text{ hours} = 36 \text{ seconds} \tag{1}$$

Like any other interval on the real line, $[a, b]$ contains $2^{\aleph_0}$ different and densely ordered points. Therefore, if P remains for a time $\delta t > 0$ seconds at each of these points, we can write:

$$t_{ab} = \delta t \, \frac{\text{seconds}}{\text{point}} \times 2^{\aleph_0}\text{points} = \delta t \times 2^{\aleph_0}\text{seconds} = 2^{\aleph_0}\text{seconds} \neq 36 \text{ seconds} \tag{2}$$

And if P remains for a time $\delta t = 0$ seconds at each of these points, we can write:

$$t_{ab} = 0 \, \frac{\text{seconds}}{\text{point}} \times 2^{\aleph_0}\text{points} = 0 \times 2^{\aleph_0}\text{seconds} = 0 \text{ seconds} \neq 36 \text{ seconds} \tag{3}$$

Therefore, in the infinite spacetime continuum, neither alternative can explain why P takes 36 seconds to travel one kilometer at a uniform velocity of 100 km/h. The same argument would apply to any other finite uniform velocity and any other finite distance traveled. The spacetime continuum is therefore not a valid model for representing the actual motion of real physical objects through real physical space and real physical time. The spacetime continuum is a mathematical artifact that has been accepted without question by physicists for more than 125 years and is based on the inconsistency of the Axiom of Infinity [8, 9]. From the indeterminacy of $\delta t \times 2^{\aleph_0}$ or $0 \times 2^{\aleph_0}$ (or both) defended by a small number of authors, the same conclusion would be drawn about the inadequacy of the spacetime continuum to model the actual movement of real physical objects through real physical space.

As expected, the impossibility represented by equations (2) and (3) disappears in a discrete and finite space and time, with a minimum and indivisible unit of space (qusit, quantum space unit), a minimum and indivisible unit of time (qutit, quantum time unit), and a maximum and finite velocity of 1 qusit / 1 qutit. In this discrete space, qusits are contiguous: each one has an immediate successor in every direction of space. And each qutit has an immediate successor in the only direction that marks the passage of time.

Bibliographical References

[1] Henri Bergson. *Creative Evolution*. Dover Publications Inc., New York, 1998.

[2] Henri Bergson. The Cinematographic View of Becoming. In Wesley C. Salmon, editor, *Zeno's Paradoxes*, pages 59 – 66. Hackett Publishing Company, Inc, Indianapolis/Cambridge, 2001.

[3] Georg W.F. Hegel. *The Science of Logic*. Cambridge University Press, Cambridge, UK, 2010.

[4] A. León Sánchez. The aleph-zero or zero dichotomy. *Cogprints*, pages 1–7, September 2006. https://arxiv.org/abs/0804.2934.

[5] A. León Sánchez. *Infinity put to the test*, chapter Zeno Dichotomies, pages 196–202. 2021.

[6] A. León Sánchez. *Apparent relativity*. Amazon's Kindle Direct Publishing, 2022. PDF.

[7] A. León Sánchez. Achilles, the tortoise and the speed of light. *The General Science Journal*, 2023.

[8] A. León Sánchez. *Infinity put to the test*. Amazon's Kindle Direct Publishing, 2023 (2021). PDF.

[9] A. León Sánchez. The Axiom of Infinity Is Inconsistent. *The General Science Journal*, 2024. PDF.

[10] James Clerk Maxwell. *Matter and Motion*. The Macmillan Co., New York, 1920.

[11] J. E. McTaggart. The unreality of time. *Mind*, 17:457 – 474, 1908.

[12] Chris Mortensen. Change. In E. N. Zalta, editor, *Stanford Encyclopaedia of Philosophy*. Stanford University, URL = http://plato.stanford.edu, 2020.

[13] Mary Potter, Carl Hagmann, and Emily McCourt. Banana or fruit? Detection and recognition across categorical levels in RSVP. *Psychonomic Bulletin and Review*, 22(2):578–585, 2015.

[14] Steven Savitt. Being and Becoming in Modern Physics. In Edward N. Zalta, editor, *The Stanford Encyclopedia of Philosophy*. The Stanford Encyclopedia of Philosophy, 2008.