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The Derivation of the Time Dilation Equation

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Abstract. This article has been written in order to terminate the endless arguments that arise over the issue of how to derive the time dilation equation from the Lorentz transformation time equation.

Time Dilation

I. There seems to be a lot of confusion as to how to get rid of the unwanted vx/c^2 term in the Lorentz transformation time equation when deriving the time dilation equation. Here is a standard textbook derivation starting with the time equation from the Lorentz transformations,

$$t_1' = \gamma[t_1 - vx_1/c^2] \quad \textbf{(1)}$$

where $\gamma = 1/\sqrt{(1 - v^2/c^2)}$, t_1 is the time as measured in a stationary frame of reference by a stationary observer, and t_1' is the time that this same stationary person observes in another frame of reference that is moving at speed v along the x-axis of the stationary frame.

At time t_2 the equation becomes,

$$t_2' = \gamma[t_2 - vx_2/c^2] \quad \textbf{(2)}$$

hence,

$$\Delta t' = t_2' - t_1' = \gamma[\Delta t - v(x_2 - x_1)/c^2] \quad \textbf{(3)}$$

and since,

$$\Delta x = v\Delta t \quad \textbf{(4)}$$

then,

$$\Delta t' = \gamma[\Delta t - v^2\Delta t/c^2] \quad \textbf{(5)}$$

hence,

$$\Delta t' = \gamma \Delta t[1 - v^2/c^2] \quad \textbf{(6)}$$

Hence,

$$\Delta t' = \Delta t/\gamma \quad \textbf{(7)}$$

Time Intervals and Coordinate Time

II. Equation **(7)** refers to time intervals, whereas t_1' and t_1 are understood to be points in time. However, if the equation $x = vt$ had been substituted directly into equation **(1)** the result would have been,

$$t_1' = t_1/\gamma \quad \textbf{(8)}$$

and one might well have asked, *"what is the difference?"*. It's very difficult to understand the concept of coordinate time unless it too refers to a time interval relative to some arbitrarily chosen time origin. It's a bit like the difference between a position vector and a displacement vector. They both measure a length and a direction in space, and likewise coordinate time and time intervals both measure time intervals.

Conclusion

III. Equation **(7)** means that $\Delta t'$ is smaller than Δt and meanwhile equation **(8)** means that t_1' is less than t_1. The Einsteinian interpretation of these facts is that the stationary observer observes time to slow down in the moving reference frame. Meanwhile, in the Global Positioning System (GPS) it means that the moving orbital clocks tick at a slower rate due to their motion, *although they actually tick faster because of the dominating effect of the weaker gravity at the higher altitudes*, [1].

The controversy, however, is whether we should treat the time terms in the Lorentz transformations as referring to astronomical time, in the sense that one complete orbit of the Earth around the Sun, relative to the background stars, corresponds to one year, or whether the time terms should be referred locally to the frequency of the physical mechanism in the GPS caesium clocks, [2]. In the latter interpretation, all the paradoxes that are associated with symmetry will be removed once we identify a physical basis for the *Earth Centred Inertial Frame* (ECI). Assuming this frame to be the electromagnetic wave-carrying medium entrained within the Earth's magnetic and gravitational fields, then motion through this medium should cause a physical interaction with the caesium atoms, as well as regulating inertial motion rather than causing dissipative friction. Motion through this medium should actually induce the inertial forces that keep the satellites in orbit, [3]. *See the note in the appendix after the reference section.* Meanwhile, the v and c terms in the above equations will then specifically be measured relative to the ECI frame and there will be no clock paradox. It is proposed that the latter interpretation is the correct interpretation.

References

[1] Tombe, F.D, ***"Atomic Clocks and Relativity"***, (2023)
https://www.researchgate.net/publication/372165749_Atomic_Clocks_and_Relativity

[2] Tombe, F.D., ***"The Lorentz Aether Theory"***, (2020)
https://www.researchgate.net/publication/339696770_The_Lorentz_Aether_Theory

[3] Tombe, F.D., ***"Aether Friction in the Planetary Orbits"***, (2021)
https://www.researchgate.net/publication/350873624_Aether_Friction_in_the_Planetary_Orbits

Appendix

When a caesium clock accelerates linearly through the electromagnetic wave-carrying medium, the intrinsic energy of the caesium atoms will increase as a result of the physical interaction. This is the physical basis of kinetic energy. It's therefore tempting to believe that this increase in intrinsic energy will be accompanied by an increase in the angular frequency of the caesium atoms. However, since we can't get a clear picture of what actually does go on inside these atoms, we can't be absolutely sure. But if the angular frequency does increase with motion, then, since we also know that the clock frequency decreases, it follows that the transition energy that matches the radiation in the clock mechanism must also decrease, and it is this energy, $\boldsymbol{E}$, and its associated frequency, f, through the Planck equation, $\boldsymbol{E} = hf$, that we would use in the time dilation equation, for $v << c$, in connection with GPS clocks, [1].