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# Physicists Calculate But Do Not Explain 4/7

## Physics and infinity

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**Abstract**.-Through a simple supertask performed with a collection of disks and a hollow cylinder, the inconsistency of the actual infinity and, consequently, of the infinitist mathematical language of contemporary physics is demonstrated here. This mathematical inconsistency is the basic reason why physicists can calculate but ultimately cannot explain the physical world in logically consistent terms.

**Keywords**: Actual infinity, supertask theory, inconsistency of the actual infinity.

## 1. The tragedy of the actual infinity

In a more or less exclusive way, I have devoted more than 40 years to the critical study of mathematical infinity, i.e. to the actual infinity. During that time I have been able to verify that most contemporary physicists find the critical debate on infinity alien and unnecessary. They take it for granted that its formal language cannot be other than that of contemporary infinitist mathematics. For more than a century, physics has been built on a blind faith in infinitism. Imagine what a tragedy it would be for physics to demonstrate the inconsistency of (the actual) infinity. A demonstration that faces, among others, two major problems. The first is the deception of our own brain, which makes us see a continuous physical world when in reality it only offers us an image every 13 milliseconds (the time it takes to process each image); we thus perceive a discontinuous succession of images as if it were a continuous succession. The second problem is the success of differential and integral calculus in physics. A success that is explained by the fact that this calculus is in fact a discrete calculus (see the first article of this series), the same as the physical world would have to be if the actual infinity is inconsistent.

## 2. New theoretical instruments

The purely logical and philosophical discussions about the actual infinity and the potential infinity lasted more than 23 centuries. Until in the early years of the 20th century, set theory (which includes the definition of infinite set and the Axiom of Infinity) was imposed in a fulminating way, and the discussions about infinity ended in a radical way. Ironically, it has been the new infinitist mathematics that provided the author with the mathematical tools for the critical analysis of the actual infinity, especially the $\omega$-order of the natural numbers, and the dense order of the rational numbers and the real numbers.

In the 1950s, and after a famous debate between J. F. Thomson and P. Benacerraf [5, 1] about a certain lamp that turns on and off an $\omega$-ordered sequence[1] of times, a new infinitist theory emerged: the theory of supertasks. A theory that has also been very useful to the author in developing critical arguments about the actual infinity. One such argument, although very summarized, is the one developed in the following section. The reader interested in the inconsistency of the Axiom of infinity from a set theory perspective can have a look at [3, Link].

## 3. A conflicting supertask

As is usual in contemporary science, from which the potential infinity has disappeared, the word infinity will always denote the actual infinity. For this type of infinity, infinite sets exist as complete totalities, which are defined as follows:

**Definition 1 (of Complete Totality)** *A complete totality is a set defined by comprehension in which every element that satisfies the corresponding membership definition of the set is in the*

[1] Ordered as the set of all natural numbers in their natural order of precedence

*set.*

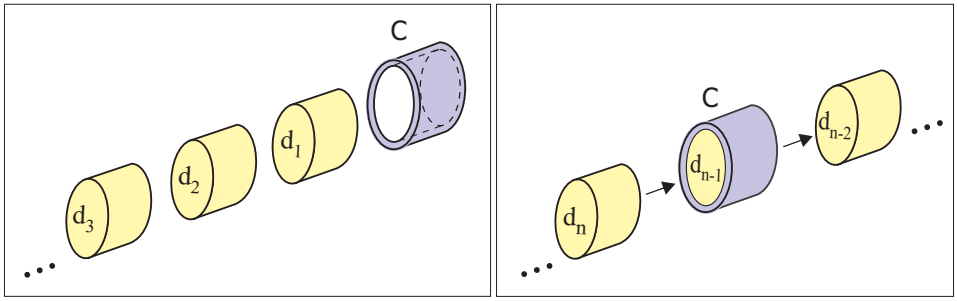


**Figure 1** – Left: The cylinder $C$ and the $\omega$-ordered sequence of disks $\langle d_i \rangle$ before starting the supertask. Right: The $n$-th task of the supertask will introduce the disk $d_n$ into the cylinder $C$ by ejecting from the cylinder $C$ the disk $d_{n-1}$ previously introduced in the $(n-1)$-th task.

In consequence, to a complete totality of a certain type of elements, it is not possible to add new elements of that type because it already contains *all of them*.

Let then $C$ be a straight, circular, hollow cylinder. Let also $D_omega = \{d_1, d_2, d_3, \dots\}$ be a set of disks indexed with the natural number in their natural order of precedence ($\omega$-order); all disks being identical, compact and with the same shape and the same volume as the internal hollow of the cylinder $C$, hollow in which each of them fits perfectly. And let $\langle t_i \rangle$ be an $\omega$-ordered, strictly increasing and convergent sequence of instants within the finite real interval $(t_a, t_b)$, whose mathematical limit is just the precise instant $t_b$. And then, consider the following supertask $S_\omega$:

> At each successive instant $t_n$ of the sequence $\langle t_i \rangle$ perform the following task, and only the following task: completely insert the disk $d_n$ of the set $D_\omega$ into the hole of the cylinder $C$, ejecting the disk $d_{n-1}$ introduced previously (except in the case of disk $d_1$, which is inserted with the cylinder $C$ being empty), so that $d_n$ is completely inside the hollow of the cylinder $C$ and $d_{n-1}$ outside the cylinder $C$.

Being instant $t_b$ the limit of the convergent sequence of instants $\langle t_i \rangle$, the instant $t_b$ is the first instant after all the instants of $\langle t_i \rangle$; and then the first instant after executing all the tasks of the supertask $S_omega$; i.e. the first instant after having introduced successively and one by one all the disks of the set $D_\omega$ into the hole of the cylinder $C$. Let us now consider the two possible states of cylinder $C$ at instant $t_b$:

State 1: At instant $t_b$ the cylinder $C$ is empty. This state is impossible because only the successive tasks of the supertask $S_\omega$ have been performed, and each and every one of these tasks leaves the hole of the cylinder $C$ completely occupied by a disk from the collection $D_\omega$.

State 2: At instant $t_b$ the cylinder $C$ is occupied by a disk $d_n$. This state is also impossible because the disk $d_n$ would be the last disk inserted in $C$, and therefore the impossible last disk of the $D_omega$-ordered set of disks.

From the supertask $S_\omega$ follows, then, a logical inconsistency: at the instant $t_b$ the hollow of the cylinder $C$ can neither be empty nor can it be non-empty, it being necessarily logical that it is either empty or non-empty. There is no third alternative.

If instead of the $S_\omega$-ordered sequence of tasks $S_\omega$, one considers the finite $n$-ordered sequence of tasks $S_n$ which consists of successively inserting into $C$ the first successive $n$ disks of a finite collection of disks $D_n = \{d_1, d_2, d_dotsd_n\}$, the finite sequence of tasks $S_n$ always ends consistently with the last disk $d_n$ inside the cylinder $C$. Conclusion that holds for every natural, and therefore finite, number $n$. We can then state that for any natural number $n$, the finite sequence of tasks $S_n$ is consistent. Only the infinite sequence $S_\omega$ is inconsistent. Therefore, it must be the $\omega$-order that is the reason for that inconsistency. And recall that $S_\omega$ is the least of the transfinite ordinals.

As it is to be expected, many other results are formally deduced from the previous contradiction, results that suppose insurmountable difficulties for contemporary mathematical infinitism. And with very serious theoretical consequences for contemporary physics (for example the formal inconsistency of the spacetime continuum). The only problem here is how to

combat the blind faith in infinitism practiced by both mathematicians and physicists, the latter being moreover satisfied with the agreement between their calculations and their experiments because, as pointed out above, in the end these calculations are not infinitist but finitist and discrete. I recommend to the reader a glance at [4, 2, 3]. And excuse the self-citations, I do not know of other authors critical of the actual infinity in the same sense that I am.

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