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Physicists Calculate But Do Not Explain 1/7

Physics and infinitesimal calculus.

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Abstract.–The same dense order of real numbers that underpins mathematical continuity is used here to demonstrate that, contrary to common belief, infinitesimal calculus can only be a discrete, discontinuous calculation. Its discreteness explains its practical success in physics since the 17th century; and its supposed theoretical continuity explains its inability to explain the physical world, which, if consistent, can only be discrete, as will also be demonstrated in this series of mini-articles.

1. Presentation of this series of mini-articles

I write in this format, a series of mini-articles, because as a dissenter from some major currents of thought dominant in contemporary science, it seems to me that this is the most appropriate way to have the reasons for my dissent read and considered. In this case, the series will consist of seven mini-articles:

Article 1: *Physics and infinitesimal calculus*, which demonstrates the unexpectedly discrete nature of infinitesimal calculus, and explains its operational success in physics.

Article 2: *Inconsistent fields in the spacetime continuum*, denouncing a blunder in the definition and use of physical fields in the spacetime continuum.

Article 3: *Interconverting between space and time*, the absurdity of this interconversion in view of the enormous qualitative physical differences between space and time.

Article 4: *Physics and infinity*, the shortest demonstration of the inconsistency of the actual infinity I have been able to construct.

Article 5: *The atrocities of physics' ordinary language*, denouncing the unsustainable corruption of ordinary language used in physics.

Article 6: *Black holes or black stars*, an unacceptable example of physics' misuse of ordinary language.

Article 7: *The shames of physics*, including, among others, its oblivion of the problem of change, its inability to discover preinertia, or its absolute submission to infinitist mathematics.

2. Physicists and their blind faith in mathematical infinity

The very word "infinitesimal," which qualifies a type of mathematical calculus, and the discussion that follows, justifies talking about infinitesimal calculus instead of mathematical analysis, the more modern, general, and abstract version that includes set theory and the Axiom of Infinity, whose infinitude is the actual infinity (it can be demonstrated that it cannot be the potential infinity [8]). An actual infinitude also present, albeit not explicitly stated, in the classic infinitesimal calculus originating from the works of Newton and Leibniz in the 17th century.

I confess that I find no explanation for the fact that the controversy between the actual infinity and the potential infinity, intensely active for more than 25 centuries and involving some of the most brilliant minds in the history of thought, suddenly ceased to be a controversy in the early 20th century (with some exceptions, like H. Poincaré [10, p. 121], [4, p. 1]). Since then, potential infinity has disappeared from mathematics and physics, despite the enormous calculation and formal consistency problems that the actual infinity inevitably brings to physical theories, including quantum mechanics (quantum corrections and renormalizations: tedious, incredibly lengthy, and unattractive calculation processes).

Despite this, it must be recognized that infinitesimal calculus works very well even in experimental physics. In the next section, the unexpectedly discrete reasons for this good functionality will be explained. The purpose of this brief section is simply to highlight two very important aspects of the relationship between physics and infinitesimal calculus:

1.- The importance of infinitesimal calculus in the development of modern and contemporary physics: derivatives, differential equations, integrals, etc. An importance that cannot be overstated and is undoubtedly founded on the undeniable functional and practical success of the mathematical-infinity-based language of physics.

2.- The acceptance by contemporary physicists of the actual infinity as subsumed in infinitesimal calculus. An infinity that they have never questioned and do not question. They do not find it questionable that there exist bijective and non-bijective mappings between the elements of the same pair of sets, where one is a proper subset of the other, and both are infinite.

As is their duty, physicists discuss everything debatable in their theories, with the rigorous exception of the actual infinity, about which, as far as I know, they have never debated and do not debate. On this blind faith of physicists in infinity, which so hinders the development of their theories, readers may peruse [9].

3. The deceptive continuity of infinitesimal calculus

As is well known, infinitesimal calculus (differential and integral) is considered a very important instrumental part of contemporary infinitist mathematics, whose foundation lies in continuous functions, in turn based on the continuity (dense order) of real numbers: between any two of them, in their natural order of precedence, there always exists the same number of different real numbers greater than the first and smaller than the second: exactly $2^{\aleph_0}$ different real numbers.

The dense order of real numbers gives rise to what is known as the Dimension Problem demonstrated by Cantor [1, 5, 11, 12, 7, 4, 2, 3], which I repeat whenever I can as a provocation seeking a reaction from physicists to this entirely unnecessary mathematical eccentricity foreign to the physical world: the dense order of real numbers allows us to affirm, for example, that light traverses in one millionth of a second the same number of points in space as in 14.8 billion years (exactly $2^{\aleph_0}$ points). Or that light takes the same number of instants to traverse one-millionth of a millimeter as it does to traverse 90 billion light-years (exactly $2^{\aleph_0}$ instants).

Perhaps the most universal and widely used procedure in infinitesimal calculus is the calculation of continuous functions when some of their continuous variables approach a limit value (approaching the limit). This calculation, as is well known, consists of successively approximating the successive values of a continuous variable of a continuous function to a specific final value (the limit) without ever reaching that specific final value. Reaching it invariably implies a physically or logically unsustainable situation, rendering the approximation process itself unnecessary: one would merely assign that final value directly to the variable. However, despite being a calculation based on the concept of continuity in infinitist mathematics, we shall see that it cannot be a continuous procedure. The reason lies in the dense order of real numbers in their natural order of precedence.

Indeed, the value of the continuous variable of any continuous function can only jump from its current value to a nearby value but not to the next or contiguous value, because in the natural order of precedence of real numbers (from which continuous variables draw their values), no such next or contiguous value exists for the current value of the variable, whatever its current continuous value may be. Thus, approaching the limit is a jump-based, discontinuous, discrete process. Moreover, according to infinitist orthodoxy, between the value (real number) from which the jump is made and the value (real number) to which the jump is made, there always exists the same uncountably infinite number of different values (real numbers): exactly $2^{\aleph_0}$ different values (real numbers), all greater than the first and smaller than the second of these two values (real numbers). Continuous approximation to the limit is impossible; one can only approach the limit through jumps, all of which involve skipping the same number of numbers with each jump. Continuous infinitesimal calculus is impossible; it is a farce. And I ask myself: How is it possible to be writing this at the end of 2024?

Infinitesimal calculus is explained, studied, and used as if it were a continuous calculus based on the infinitist continuity of contemporary mathematics. But, as we have just seen, it is actually a discrete, discontinuous, jump-based procedure. We believe we are applying continuous mathematics, but we are actually applying discrete mathematics, though we have not yet realized this mistake even in 2024. Approaching the limit works very well because it neither reaches the limit nor is it a continuous process; it is discontinuous and reaches its value before arriving at the limit. These theoretical facts are entirely compatible with a physical world in which all magnitudes are discrete, with indivisible minima (quanta) whose values must be of the order of those reached at the end of processes involving approaching the limit. "Infinitesimal" values are possibly related to some of the fundamental constants of physics.

The problem is that the error just highlighted here contributes to maintaining the continuous conception of the physical world, which could be discontinuous, discrete, and must be so if it is a physical world consistent with the fundamental laws of logic (Principle of Identity and Principle of Non-Contradiction). Unfortunately, this continuous conception is confirmed by another deception, this time from our own brain, which provides us with a continuous sensory perception of the physical world that masks its possible discrete reality: our brain takes 13 milliseconds to process each image it receives from the external world [6], yet makes us see this discrete succession of images as if it were continuous. The same happens with the discrete sequence of images in a film projected onto a screen, as everyone knows.

4. Conclusion

The success of infinitesimal calculus in modern physics lies more in the discrete nature of its most basic procedure (approximation to the limit) than in the supposed mathematical continuity of the functions involved, which in turn is based on the dense order of the real numbers. The dense order, as will be shown in mini-article 4 of this series, is inconsistent in the case of both rational and real numbers. Finite and discrete mathematics would be as useful to physics as infinitesimal calculus, and would not be inconsistent. Indeed, physicists calculate accurately, but they do not explain the real physical world.

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