

Value of the Exponent in the Gravitational Formula

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Abstract: A more accurate value of the exponent in the gravitation formula was calculated based on Ruđer Bošković's theory.

Key words: Newton, Bošković, Planck, gravity, force, acceleration

1. Introduction

The aim of this paper is: determination of a more accurate value of the exponent over distance in Newton's gravity formula.

The results are expected to have their utility value from cosmological to quantum scales.

Some believe that Newton's law of gravity is not ideally correct, [3, *Extensions*]:

Extensions

Newton was the first to consider in his *Principia* an extended expression of his law of gravity including an inverse-cube term of the form

$$F = G \frac{m_1 m_2}{r^2} + B \frac{m_1 m_2}{r^3}, \quad B \text{ a constant}$$

attempting to explain the Moon's apsidal motion. Other extensions were proposed by Laplace (around 1790) and Decombes (1913):^[49]

$$F(r) = k \frac{m_1 m_2}{r^2} \exp(-\alpha r) \quad (\text{Laplace})$$

$$F(r) = k \frac{m_1 m_2}{r^2} \left(1 + \frac{\alpha}{r^3} \right) \quad (\text{Decombes})$$

Bošković expresses this in several paragraphs; let's quote from [1]:

170. *If universal gravity obeys the law of a force inversely proportional to the square of the distance (which, as I remarked in the first part, it only obeys as nearly as possible, but not exactly), sensibly unchanged only throughout the planetary & cometary system, it will certainly be the case that the curve of forces will not have the last arm PV asymptotic with the straight line AC as the asymptote, but will again cut the axis & wind about it.*

In order to achieve the above goal, I believe that it is necessary to view the Universe as a whole that generates itself through continuous movement. Understanding Bošković's philosophy in the

sense that you at least partially manage to think in his way is desirable. A very small number of people have achieved this, which has led to the fact that in the twenty-first century the problems that he solved three centuries ago are being discussed. At least I think so.

Here we will not delve into the consideration of his Theory, but we will try to solve the problem from the title with the conviction that a very small correction in the exponent in the gravitational formula can have far-reaching consequences for the understanding of the unity of cosmological and quantum mechanical phenomena. I hope: that I partially managed to think in his way and that the obtained value of the exponent will renew interest in this topic.

1. Bošković's understanding of forces

Bošković's capital book is about forces: "*Theoria philosophia naturalis redacta ad unicam legem virium in natura existentium*", [1]. The key figure by Bošković: (with color corrections) with an explanation is transferred from [2], where you can also refer to the importance, achievements, recognitions and references to Bošković's philosophy.

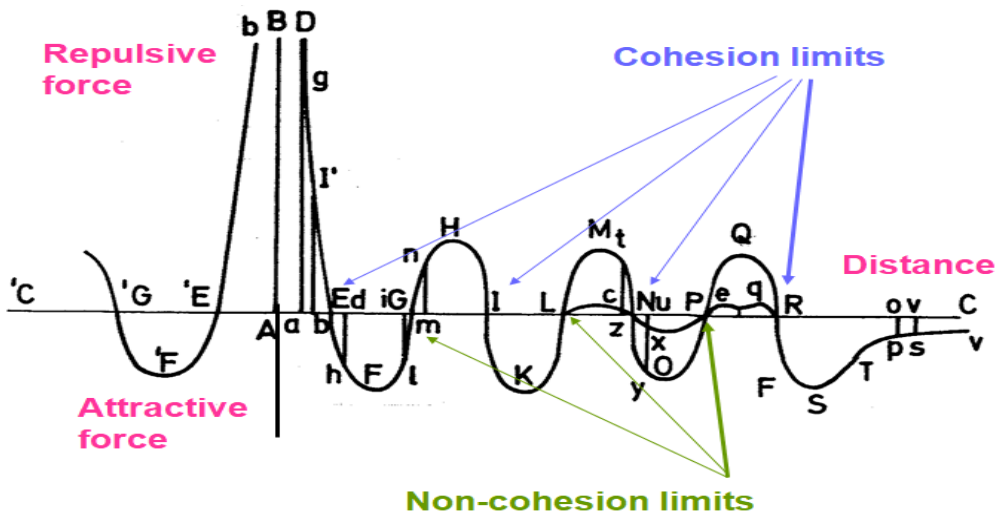


Figure 1 - Shows the general shape of Bošković's curve, which illustrates the change of attractive and repulsive forces with the change of distance between entities (bodies or particles). The boundaries of cohesion and non-cohesion correspond to intersections with the abscissa where there is no repulsion or attraction on the curve [2].

Here we will go to the subject of this paper:

2. Non-cohesion limits

Let's take the unit of acceleration (without attraction and repulsion) from [4, tables 1 and 1b] in the [kg-m-s] system: $a_0 = 6.95818 * 10^{-10} m/s^2$.

The value of a_0 is obtained by assuming a finite universe; not by our idea of spatial boundaries, but by the limited nature of the phenomena that characterize it. We will observe it: in the vicinity of points G, L, P on the abscissa of the Bošković curve, Figure 1. This is where recession and attraction: they approach this value, and on the abscissa their difference is zero. Note that in natural units, $a_0 = 1$ and that the acceleration can be n times greater than the unit (Attraction) or n times less (Repulsion).

Let's also apply a hypothetical fundamental particle (mass - $m_f = 1.08862 \cdot 10^{-28}$ kg and radius $R_f = 3.23131 \cdot 10^{-15}$ m), which has no attraction or repulsion, [4, f 3b and 4], and is related to Planck's values [4, f 11] because it holds $\hbar / c = m_{pl} * l_{pl} = m_f * R_f$: where : m_{pl}, l_{pl}, \hbar are Planck mass, length, Reduced constant and c – speed of light.

Let's also note that in the vicinity of Bošković's abscissa, acceleration is not the only thing that produces force, that's why Bošković uses the term "**propensity**". Let's quote from [1]:

The law of forces does not necessitate action at a distance, nor is it some mysterious quality.

101. *The objection is frequently brought forward against mutual forces that they are some sort of mysterious qualities or that they necessitate action at a distance... Moreover these forces are of the following nature. The idea of a **propensity** to approach or of a **propensity** to recede is easily formed. For everybody knows what approach means, and what recession is; everybody knows what it means to be indifferent, & what having a **propensity** means; & thus the idea of a **propensity** to approach, or to recede...*

The previous quote: as well as the results that will be obtained in the future solve some controversies that still exist in science, but we will not deal with it further because it is not the subject of this paper.

Let's write Newton's gravitational acceleration (m and R are mass and distance) in several ways:

$$g_n = -G * m * R^{-2} \quad (1)$$

$$g_n = -c^2 * m * l_{pl} / (m_{pl} * R^2) \quad (2)$$

$$g_n = -c^2 * m * R_u / (M_u * R^2) \quad (3.1)$$

$$g_n = -\frac{m}{m_f} * \left(\frac{R}{R_f} \right)^{-2} * a_o \quad (4.1)$$

None of these formulas are in Newton's "**Principia**", [5] because he expressed gravitational acceleration in relative terms.

Formula (1) was invented later: and the constant G was named in honor of Newton: "Newton's gravitational constant", which prevailed in the literature. Not only: that it is not suitable for the needs of this paper: but the objections to it, presented in [6], are also evident.

Formula (2) is known after Planck: and from it one can conclude much more than from (1).

In formula (3.1) is: M_u – the upper limit of the mass and R_u – the upper limit of the distance of the universe, which in my previous papers obtained in various ways, for example:

$$M_u = 2\pi * \left[2^{\exp(2\pi)/3} * (m_{pl}^4 / m_p) \right]^{1/3} = 1.7394492721 * 10^{53} kg \quad (3.2)$$

$$R_u = 2^{\exp(2\pi)/3} * (m_{pl} / m_p)^{-2/3} * \lambda_p = 1.2916529930 * 10^{26} m \quad (3.3)$$

Where m_p and λ_p are: the mass and the Compton wavelength of the proton.

From (3.1), the value a_0 of the acceleration with limits, M_u and R_u , is obtained:

$$a_0 = g_n = -c^2 * M_u * R_u / (M_u * R_u^2) = -c^2 / R_u = 6.95818 * 10^{-10} m * s^{-2} \quad (3.4)$$

Formula (4.1) is: the most suitable because masses and distances are multiplied by a constant value of acceleration a_0 , and over the dimensionless: m/m_f and R/R_f , any exponent is possible.

For an example of using formula (4.1): let's calculate the gravitation in the [kg-m-s] system, on the Earth's surface we get (5):

$$g_n = \frac{5.97356 * 10^{24}}{1.08862 * 10^{-28}} * \left(\frac{6.37597 * 10^6}{3.23131 * 10^{-15}} \right)^{-2} * 6.95818 * 10^{-10} = 9.80661 m * s^{-2} \quad (5)$$

We got the same value as the most commonly applied form of the formula: (1) and what we would get with: (2) and (3).

If we put $g_n = a_0$ in (4) on the left side, we get (6):

$$r = R_f * \left(\frac{m}{m_f} \right)^{1/2} \quad (6)$$

When applied to planet Earth we get (7).

$$r = 3.231311 * 10^{-15} * \left(\frac{5.97356 * 10^{24}}{1.08862 * 10^{-28}} \right)^{1/2} = 7.569341 * 10^{-11} m \quad (7)$$

Lowercase letter r is: **Bošković's non-cohesion limit**: (points G, L, P on the force curve of figure 1, which is a discrete quantity that is valid only for the acceleration a_0 . We obtained that a particle that is at a distance from the Earth of $7.569341 \cdot 10^{11} m$ will not have **propensity** approaching the Earth.

Applying formula (6): to the Sun, we can easily calculate that the small bodies of the Ort cloud [7] are at a distance from the Sun of about $10^{14} m$ where the unit acceleration is a_0 .

Formulas (4) and (6) are valid for all celestial bodies and also for sets of celestial bodies:

It is known that Gravitation: applies to all masses of the universe, so the question arises what happens when the attraction by the Earth stops. I claim:

It is rational that it is also gravitation, but the gravity of the rest of the universe (g_{os}) because nature does not care what mass is involved, so let's assume (8):

$$g_{os} = \frac{(M_u - m)}{m_f} * \left(\frac{R_u}{R_f} \right)^{-2} * a_o \quad (8)$$

Let us add to this topic a quote from Bošković, [1]:

108. Next, as regards attraction & repulsion being of different species, even if it were a fact that they were so, it would not matter in the slightest degree, since by rigorous argument the existence of both attraction & repulsion is proved, as we have seen; but really the supposition is untrue. Both kinds of force belong to the same species; for one is negative with regard to the other, & a negative does not differ in species from positives.

Considering: the gravity of the rest, g_{os} is opposite to the positive gravity of the body; it is of the same type as the gravity of the body, only of the opposite sign.

In the case of galaxies, separation was observed: which is an expected phenomenon at distances greater than the non-cohesion limit. For $R < r$, the following is possible: the attraction of neighboring galaxies, and there is also the possibility that the attraction of one galaxy turns into the attraction of a neighboring galaxy. This was mostly explained by Bošković in articles **169** and **170** from [1]: although galaxies were not known in his time. Let's quote:

169. The intermediate arcs, which wind about the axis, can also, at any point where they reach it, return backwards & touch it; and they can do this on either side of it; they may also be reflected and recede before actual contact, the approach being altered into a recession, as is to be seen in Fig. 1 with regard to the arc PefqR.

3. Value of the exponent in the gravitational formula

I think that Bošković is right, as in everything else, and that the formulas for: Gravity (4.1) and the gravity of the rest of the universe (8) can demonstrate the correctness of Bošković's position. Considering that: a_0 is a constant in both formulas we can temporarily omit it and finally use it as a transformation coefficient for any system of measurement units.

Since the terms for masses and distances are dimensionless, we use mathematics without paying attention to dimensional correctness. The resultant force can be defined as the sum of these two:

$$g = g_n + g_{os} \quad (9)$$

That is, if we simplify the writing with:

$$\mu = m/m_f, \quad \mu_{os} = (M_u - m)/m_f, \quad \rho = R/R_f \quad \text{и} \quad \rho_u = R_u/R_f$$

Including it in (4) and (8) and adding we get (10):

$$g = (\mu + \mu_{os}) * \rho^{-2} * a_0 \quad (10)$$

Which we can replace with the unique exponent x :

$$g = \mu * \rho^x * a_0 \quad (11)$$

Equating (10) and (11), we get (12):

$$\mu * \rho^x = (\mu + \mu_{os}) * \rho^{-2} \quad (12)$$

By logarithming and ordering gives the formula for the exponent over the distance (13):

$$x = -\log[\mu / (\mu / \rho^2 - \mu_{os} / \rho_u^2)] / \log(\rho) \quad (13)$$

And the gravitational acceleration in the developed form instead of: (1) to (4.1) we get more precisely (4.2).

$$g = -\mu * \rho^{-\log[\mu / (\mu / \rho^2 - \mu_{os} / \rho_u^2)] / \log(\rho)} * a_0 \quad (4.2)$$

It is possible: that my assumption in front of formula (8) is inappropriate, but not [1] because there: are many confirmations and I hope that this paper is one of the confirmations.

The entire procedure applied: to the Sun, Jupiter and Earth are shown in Table 1.

Table 1

Determining the exponent in the gravitational formula

$$c = 2,99792458E+08 \quad a_o = c^2/R_u \quad 6,95818E-10$$

Fund.part.	1,08862E-28	1,00000E+00	3,23131E-15	1,00000E+00
name	m	$\mu = m / m_f$	R	$\rho = R/R_f$
Univerzum	1,73945E+53	1,59785E+81	1,29165E+26	3,99731E+40
Sun	1,98900E+30	1,82708E+58	6,95500E+08	2,15238E+23
Jupiter	1,89860E+27	1,74404E+55	7,14920E+07	2,21248E+22
Earth	5,97360E+24	5,48731E+52	6,37597E+06	1,97319E+21

Continuation of Table 1

						[m*s ⁻²]
F.part.	-1,00000E+00	1,00000E+00	0,00000E+00	#DIV/0!	#DIV/0!	#DIV/0!
name	$g_n = -\mu/\rho^2$	$g_{os} = \mu_{os}/\rho_u^2$	$g = g_{os} + g_n$	x	$g = -\mu * \rho^x$	$g = a_o * g$
Univ.	-1,00000E+00	0,00000E+00	-1,00000E+00	-2,0000000000000000	-1,00000E+00	-6,95818E-10
Sun	-3,94385E+11	1,00000E+00	-3,94385E+11	-2,0000000000000005	-3,94385E+11	-274,42016
Jupiter	-3,56286E+10	1,00000E+00	-3,56286E+10	-2,0000000000000055	-3,56286E+10	-24,79100
Earth	-1,40936E+10	1,00000E+00	-1,40936E+10	-2,000000000000145	-1,40936E+10	-9,80661

- In the table, the calculation with *dimensionless* quantities is shown in red and bold black: it is in the [kg - m - s] system.
- From the Table, we see agreement with Bošković's position that Gravitation: *is noticeably unchanged only in the entire planetary and cometary system.*
- For the virtual fundamental particle, we see an undefined state for the acceleration and for the exponent, which is the same as in point, A, in Bošković's figure 1.
- In Table 1 for the planets the exponent x differs from "-2": at the twelfth decimal place.
- The exponent deviates less: from "-2", for more massive planets and the Sun.
- The exponent deviates more with: increasing distance from the planet.
- It can be calculated from formulas (6) and (7): that in the vicinity of the non-cohesion limit it reaches a value of about -2.5 for smaller planets.
- It makes no sense to calculate the value of the exponent for: $R > r$ because then the point is in the rest of the universe and the rest becomes the observed body for which we calculate gravity.
- It can be checked that formula (4.2) supports the following position of Bošković, [1]:

10. Now the law of forces is of this kind; the forces are repulsive at very small distances, & become indefinitely greater & greater, as the distances are diminished indefinitely, in such a manner that they are capable of destroying any velocity, no matter how large it may be, with which one point may approach another, before ever the distance between them vanishes. When

the distance between them is increased, they are diminished in such a way that at a certain distance, which is extremely small, the force becomes nothing. Then as the distance is still further increased, the forces are changed to attractive forces; these at first increase, then diminish, vanish, & become repulsive forces, which in the same way first increase, then diminish, vanish, & become once more attractive; & so on, in turn, for a very great number of distances, which are all still very minute; until, finally, when we get to comparatively great distances, they begin to be continually attractive & approximately inversely proportional to the squares of the distances.

4. Conclusion

- The formula for the exponent x in the gravity formula, (13) was determined.
- Checking the formula for planets is almost impossible because the deviation from "-2" is small.
- Those with cosmological data sets could more easily check (4.2) using also (6).
- Checking small structures such as molecules, atoms and at the subatomic level is easier, but other forces also appear there. Let's quote Bošković, [1]:

121. ... *Especially, in the case of extremely small distances, the whole force, which the particles exert upon one another, will differ very much in every case from the force of gravity, if that is supposed to be inversely proportional to the squares of these distances. For, in the case of gases, which exercise such a mighty force of self-expansion, there is certainly repulsion at those very small distances from one another, & not attraction; again, the attraction that arises in cohesion is immensely greater than it ought to be according to the law of universal gravitation.*

5. References

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