the general science journal

The General Science Journal. July 2023.

GRAVITATIONAL WAVES AS AN EMPIRICAL PROOF OF SPACE REALITY

Antonio León Sánchez

Retired Professor. Independent researcher in the foundations of science.

Abstract.-This article presents an unexpected formal consequence of the experimental detection of gravitational waves. It proves that space is not a theoretical fiction useful for discussing the relative positions of physical bodies, but a real physical object. It also proves that the physical space is not formed by a non-denumerable infinitude of inextensive points, but by a finite number of discrete, indivisible and contiguous units of non-zero extension. The demonstration is based both on the detectable reality of gravitational waves, and on the inconsistency of the Axiom of Infinity, inconsistency that is very briefly demonstrated in Appendix B of this paper. I invite you to examine that short demonstration, and if you do not find it correct, you can stop reading this article right there.

Keywords: gravitational waves, spacetime continuum, dense order, discreteness, physical space, physical time, real objects versus useful fictions, fundamental physics, operational physics.

1. Introduction: Gravitational waves

Some precedents for the idea of gravitational waves may be found somewhat forcibly in the suggestion made by M. Faraday in 1847 that gravity might involve some kind of radiating phenomenon [16], and somewhat more explicitly in those made in 1870 by W.K. Clifford on the vibrations of space caused by mass [9, p. 158]:

> (1) That small portions of space are in fact of a nature analogous to little hills on a surface which is on the average flat ; namely, that the ordinary laws of geometry are not valid in them.
>
> (2) That this property of being curved or distorted is continually being passed on from one portion of space to another after the manner of a wave.
>
> (3) That this variation of the curvature of space is what really happens in that phenomenon which we call the motion of matter, whether ponderable or etherial.
>
> (4) That in the physical world nothing else takes place but this variation, subject (possibly) to the law of continuity.

In June 1905, H. Poincaré also predicted the existence of gravitational waves [42, p. 1507]:

> ... I was first led to assume that gravitational propagation is not instantaneous, but takes place at the speed of light.
>
> ... in this case, we're talking about the position or speed at the instant when the gravitational wave left the body.

Gravitational waves also appear in 1916 as a deducible possibility of general relativity [12], and become an experimentally confirmed reality in 2015. The history of gravitational waves during those 100 years was full of lights and shadows, with more shadows than lights (Einstein himself was not sure of their existence, although he was sure of their interest [15]). But confidence in its existence was never completely lost, from the failed detection by J. Weber with his detector (electromechanical antenna) in 1969 [49, 50], to its definitive confirmation in 2015. In recent months, primordial gravitational waves possibly originated in the first events in the history of the universe have been detected. The detection has made headlines in print and broadcast media around the world.

An acceptable way to explain the nature of gravitational waves is to compare them with the well-known electromagnetic waves generated by interactions between electric charges: As the saying goes, gravitational waves are to gravitational interactions what electromagnetic waves, e.g. light, are to electromagnetic interactions. If interactions between electric charges produce electromagnetic waves, interactions between gravitational masses produce gravitational waves. The enormous difference between the intensities of the two interactions explains the enormous difficulties that have had to be overcome in order to detect gravitational waves experimentally: the gravitational interaction is 10^{30} times weaker than the electromagnetic interaction. For this reason, we can only hope, at least for the moment, to detect the gravitational waves generated by the gravitational interactions between the most massive objects in the universe, such as neutron stars or black holes.

Unlike the electromagnetic interaction, which can be both attractive and repulsive, the gravitational interaction is always attractive, so although much more weaker, its additivity extends it over enormous areas of the universe, which does not occur with the electromagnetic interaction due to the approximate cancellation between electromagnetic attractions and repulsions. What they do resemble is that both types of waves are transverse (they vibrate in directions perpendicular to the direction of their propagation), although gravitational waves are exclusively quadrupole waves.[1]. Both propagate at the same speed, the speed of light: 299792.458 Km/s.

Obviously, the gravitational interactions capable of generating detectable gravitational waves must be those of the highest intensity, which in turn originate from interactions between the most massive objects in the universe, for example:

1. Supernova explosions.
2. Binary stars (revolving around each other).
3. Binary pulsars.
4. Irregular neutron stars rotation.
5. Neutron star collisions.
6. Rotating black holes.
7. Binary black holes in spiral approach.
8. Collision of black holes.

As usual in these cases, there is an abundant primary and secondary literature on gravitational waves, including that related to their experimental detection and to the latest discoveries on primordial gravitational waves, e.g. [1, 3, 5, 6, 7, 8, 15, 17, 18, 21, 38, 39, 44, 46, 53]. The interested reader can also visit on the Internet the sites of some important gravitational wave research projects:

2. Space deformations

In addition to the corresponding mathematical equations, the gravitational deformations of space are usually explained in ordinary language with the help of some metaphors (almost always the same) and more or less simplifying drawings (also almost always the same). What is never done is to describe these deformations in ordinary language, in terms consistent with the dense order of the points that supposedly constitute space. This dense order is that of the set $\mathbb{R}^3$ of all 3-tuples of real numbers that model space. One of the most important consequences of this dense order is that it is impossible for two points in space to be contiguous, i.e. adjacent: between any two points in space there must always be the same non-numerable infinite number ($2^{\aleph_o}$) of different points. So points do not touch each other. A really uncomfortable infinitist requirement for the explanation of the physics of space and its deformations (local curvatures, extensions, vibrations).

Moreover, points have neither extension nor shape, which can be proved almost immediately [32, Link p. 61]. And the extension of a non-denumerable number of densely ordered points of null extension is a mathematical indeterminable ($0 \times 2^{\aleph_o}$). So, it is an indeterminable extension. Whatever region of space we consider has the same number of points as the whole universe, and its extension will always be indeterminable if it is to be expressed as a consequence of the extension of its individual constituents (points). For this reason, arbitrary metrics must be defined without any relation to the inextensive points that presumably compose the space.

When all the peculiarities of points and dense order are taken into account, practically insurmountable difficulties appear in explaining in physical and consistent terms how a space formed by points can be deformed. For example, the followings:

1. Points cannot be deformed because they have no shape to deform. Therefore a space of points cannot be deformed by deforming its points.
2. Points have no extension:
   a) Therefore a space of points cannot contract by the contraction of its points: points have no extension to contract.
   b) Neither can the space be stretched by stretching its points: those points would cease to be inextensive, and therefore would cease to be points.

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[1] Like the sides of a square that contract and stretch alternately in the two orthogonal directions of its orthogonal sides

3. A space of points cannot be deformed by sliding its points: they would leave gaps of indeterminable extension and space would no longer be a continuum of points.
4. Furthermore, contacts between points would occur, which is impossible because the contacting points would have to be contiguous, which is not possible because contiguous points are impossible in a densely ordered set of points.
5. The deformation of a space of points cannot occur either by destruction of points or by creation of new points:
    a) In the first case, gaps of indeterminable extension would be created and the continuum would cease to be a continuum.
    b) The second case is also impossible, since there would have to be previous holes for the new points, which is impossible in a continuum of points.
6. A deformed space would be indistinguishable from an non-deformed space: in any direction of space, whether or not deformed, there would be the same number of inextensive, formless, non-deformable and non-adjacent points: always $2^{\aleph_o}$ of such points.

Consequently, it seems impossible to deform a space constituted by a continuum of points without size, without form, without contiguity, and of which the same number would exist in any region of the universe, whatever its size, from a Planck volume to the whole three-dimensional universe.

It is formally objectionable, therefore, the ease and lack of infinitist consistency with which contemporary physicists speak in ordinary language of the deformations of the spacetime continuum of points and instants. It could be said that they make a schizophrenic use of language: consistent in their mathematical language, inconsistent in their ordinary language. I have the impression that by forcing physicists to express themselves in ordinary language consistent with the infinitism of their mathematical language, they would end up rejecting that mathematical infinitism.

3. Physical space is a real physical object

I agree with T. Maudlin that it is impossible to exaggerate the importance of explaining what space is for physics [35, p. 25]. Although I disagree with the majority opinion of contemporary physicists, according to which space is only an unreal instrument useful for describing the relative positions of physical objects (see Appendix A). An opinion that will be seriously compromised by recent experimental confirmations of some of its physical properties. Indeed, the experimental detection of gravitational waves has an immediate formal consequence on the nature of physical space:

> Physical space can vibrate and be the transmitting medium of its own vibrations, which are of transverse, quadripolar type, of a great variety of frequencies and with a velocity of propagation through space itself of $299792458\,ms^{-1}$. This implies that the space must have the necessary physical properties to enable these vibratory and transmitting capabilities.

But a necessary condition for an object to have empirically detectable physical properties is that the object actually exists, for what does not exist cannot have empirically detectable physical properties. Vibrations of something that does not exist also do not exist, and therefore cannot be empirically detected. Consequently, if they are empirically detectable, they do exist. This simple argument (Modus Tollens) leads us to the conclusion that physical space is real.

Since gravitational waves are transmitted through the enormous empty spaces between material objects, we must conclude that space is a physical object, not a property of material objects. In addition, gravitational waves are vibrations of space that interact with the arms of the interferometers that detect them, modifying, even minimally, the distances between their mirrors. So they can only be real vibrations of real stuff. We have, therefore, proved the following:

eTheorem 1 (of the Physical Space) *Physical space is a real physical object with certain physical properties that can be tested and measured in experimental terms.*

Where an e-theorem is a statement whose veracity is supported by both empirical data (in this case the empirical detection of gravitational waves) and logical inferences.

To some readers, the above e-Theorem may seem superfluous. They may even think that its deduction and emphasis is unnecessary. But it turns out that for most contemporary physicists, neither space nor time are real. As noted above, for them they are mere fictions useful for explaining the RELATIVE spatial and chronological positions of the physical objects included in the observable universe. Therefore, after the above brief analysis of the immediate consequences that the detection of gravitational waves will have on the (real or fictitious) nature of physical space, it is worth taking a brief look at Appendix A, which summarizes the position of some relevant physicists on the ontological nature of space.

4. Physical space is discrete

It is formally proved in this section that if physical space is real and formally consistent, then it must be discrete, i.e. it must consist of contiguous units of a non-zero extension, rather than inextensive and densely ordered points. The proof makes use of the inconsistency of the actual infinity, a formal result demonstrated by the author (more than 25 years ago) with more than 40 different proofs that the interested reader can find in [31, Link]. With a few exceptions, whom I thank for their support[2], the hegemonic infinitist stream in modern mathematics ignores all these proofs. One of them is reproduced, very simplified, in Appendix B of this paper. You can examine it in less than two minutes, and if it does not seem to be a correct argument, you can stop reading this article right there.

Let us prove that the real physical space cannot be continuous, but discrete, i.e. consisting of basic units of non-zero extension, hereafter called discrete units, or qusits (quantum space units). Indeed, if space were a continuum of inextensive points, then it would be formally equal to the continuum R^3, which is a non-denumerable infinite set. And being non-denumerable it contains a non-denumerable infinitude of denumerable infinite proper subsets. We may therefore consider the following argument:

(a) If R^3 is consistent, then there exist consistent denumerable sets.

(b) If there exist consistent denumerable sets, then the Axiom of Infinity is consistent.

(c) The Axiom of Infinity is not consistent (Theorem 2, Appendix B)

(d) Therefore, the continuum R^3 is not consistent (Modus Tollens a-b-c)

(e) So, the continuum of points of physical space, which formally equals R^3, is inconsistent.

(f) Consequently the real physical space, if consistent, cannot be a continuum of points.

On the other hand, physical space cannot be constituted by an infinite number (denumerable or non-denumerable) of discrete units either, since it would be formally equivalent to an infinite set, and since infinite sets are inconsistent (Corollary 1, Appendix B), so would be physical space. The finiteness of the number of discrete units of physical space has at least three consequences:

(g) As noted above, the extension of any infinite and densely ordered set of points with a null extension (as would be the case of the entire three-dimensional space of points) is a mathematical indetermination. The situation is now much clearer: A totality formed by a finite number of discrete units of null extension, has exactly a null extension, which is not the case of physical space. Therefore, the discrete units of physical space cannot have a null extension. So, they can only have an extension greater than zero.

(h) The discrete units of the physical space cannot be densely ordered, because in that case the physical space would be constituted by an infinite number of such units. Therefore, and being inconsistent all infinite sets (Corollary 1, Appendix B), in any considered spacial direction the discrete units forming the physical space must be contiguous, adjacent.

(i) The discrete units of the real physical space, as such discrete units, cannot be divided, since the new discrete units arising from the division would be equally divisible and we would have an infinite, and then inconsistent (Corollary 1, Appendix B), number of such discrete units, and therefore the real physical space would be inconsistent.

The above argument (a)-(i) proves the following:

eTheorem 2 (of the Discrete Space) *Physical space can only consist of discrete units which, as such units, are of a non-null extension, indivisible and contiguous in all directions.*

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[2] For mental health reasons (social anxiety disorder) I do not correspond with anyone, but I thank the messages.

As extravagant as the e-Theorem of the Discrete Space may seem to some readers, let us remember that this scenario of the physical world was already considered by the early pre-Socratics [34]. And that the Arabic philosophical-theological school of thought known as Kalām (IX-X centuries) developed a discrete cosmology, with discrete units for mass, space, and time, so that motion had to occur in leaps and bounds separated by a certain number of discrete units of time, less time units the faster the motion is [22, p. 62-68].

Although overly speculative, one might consider the possibility that the universe functions in a manner similar to the functioning of cellular automata. In such a case, its discrete units would have different states that could give rise to the fundamental interactions and elementary particles from which all material objects would arise. In certain extreme states of gravitational interactions the discrete units of space could deform (which, for the reasons given above, is not possible with points) and propagate the deformations as gravitational waves; or they could originate some change in the content of the discrete units which would also propagate as do the detected gravitational waves, with the same consequences on the transmitting space.

On the other hand, it does not seem to be a coincidence that gravitational waves propagate with exactly the same velocity as electromagnetic waves: $299792458\, ms^{-1}$. The coincidence could be due to the fact that both velocities represent the maximum possible velocity in a discrete space and time: one discrete unit of space per one discrete unit of time (the discrete nature of time is demonstrated in the next section).

For obvious reasons, I do not intend to explain here gravitational waves from the perspective of a discrete space (simply because I would not know how to do it), but to point out that the discrete scenario, so far only considered by a few authors (curiously making use of infinitist mathematics) is the only consistent scenario if the actual infinity is inconsistent (I remind you the proof of that inconsistency included at the end of this article and the more than 40 included in [31]). At most, I dare to indicate that in a discrete and finitist scenario things could be very different in the fundamentals, while being compatible with all known experimental data.

5. Time is a discrete magnitude

Physical objects can be defined in terms of their composition, structure and properties. This is the case, as seen in the two previous sections, of space, once it has been proved that it is a real physical object (e-Theorem 1 and Definition). Time, on the contrary, does not seem to be a physical object but a magnitude that measures a basic and universal property of all physical objects: their ability to persist in a given state. But it is very likely that time, being a magnitude that measures such a basic and universal property, is a primitive concept that can only be defined in operational terms, which in this case would be related to the permanence and evolution of the successive states of physical objects, including their changes in position.

Since the states and changes of state of objects are real, they can be empirically detected, and their permanence can be measured in comparative terms, for example by comparison with any arbitrarily chosen permanence, it makes sense to assign a magnitude that measures the permanence of each state of each material object. That magnitude would be time, and it would be real in the same sense that any other empirically detectable property of material objects is real.

The difficulty, perhaps insurmountable, of defining time in non-operational terms suggests that time is not, in fact, a physical object like space (see Definition 1 below), for if it were, we would expect a non-circular and descriptive definition of that object, which has never happened, nor does it seem likely to happen. What can be given, as will be shown below, is an operational definition of time which clearly indicates that it is a measurable magnitude of a fundamental and universal property of all material objects. From its operational definition one could define operational units of time and other related concepts, such as time interval, simultaneity, flow of time, etc.

On the other hand, modern physics considers that time has the mathematical structure of a one-dimensional continuum of instants (the “points” of time) equivalent in formal terms to the continuum $\mathbb{R}$ of the real numbers. Exactly the same argument (a)-(i) developed in the previous sections for the case of space can be made for the case of time, which leads us to state the following:

eTheorem 3 (of the Discrete Time) *Time is a discrete magnitude whose discrete units, as such units, have to be indivisible, contiguous and of a non-null extension.*

The discrete units of space could all have the same extension, or not. If they did not all have the same extension it could happen that the general physical laws would vary with spatial directions, which does not seem to be the case. It is therefore reasonable to assume that in terms of the extension of its discrete units (which we could call *qusits*) physical space is isotropic. The same is true for the discrete units of time taken as a reference to measure time, although here there are only two directions (past and future), all the discrete units of time (which we could call *qutits*) should have the same extension, otherwise we would have to consider the possibility that the physical laws vary with the evolution of the universe.

It is interesting to note at this point that there is an exclusively formal way, independent of the empirical detection of gravitational waves, which allows us to demonstrate the existence of the above discrete and real units of space. An argumentation whose starting point is the Principle of the Directional Evolution: *The universe always evolves independently of its observers and in the same direction of increasing its entropy.* A principle that, as we know, has enormous inductive support. The following results, among others, can be immediately deduced from it [28, Link] [29, Link]:

1. The universe evolves under the control of a unique set of invariant and consistent physical laws.
2. There is an indivisible minimum of space (time) of which all space (time) intervals are an integer multiple.
3. The indivisible units of space and time are physical, and then real and absolute.
4. Every space interval (or time interval) is finite and can only be divided into an integer number of adjacent qusits (qutits).
5. The continuum densely ordered spacetime cannot be used to model uniform motion.
6. The laws of physics do not apply in spaces smaller than the indivisible unit of space nor in times smaller than the indivisible unit of time, both being of non-zero extension (duration).

6. Absolute motion

The fact that we cannot observe qusits does not mean that they do not exist (some authors, as G. Cantor, said the same thing about atoms in the first half of the 20th century). According to the above arguments they must exist if the universe is consistent. And it can be proved that the universe is formally consistent on the basis of the above Principle of Directional Evolution. If we could observe qusits, it would be possible to choose any of them and refer to it the motion of all material objects in the universe. Naturally, all these motions would be absolute: motions THROUGH a unique and absolute space. The differences of these absolute motions would be the cause of the relative motions of all material objects to each other. These relative motions are the only motions we can observe due to PREINERTIA.[3]) and the fact that, as mentioned above, it is not possible, at least for the time being, to observe qusits and establish an absolute reference frame with them.

But the absolute or relative nature of natural motions should not depend on the biological fact that we humans can or cannot perceive qusits: motion existed long before living things appeared in the universe. So, although it is anathema in contemporary physics, we would have to consider the possibility that, indeed, all motions of all material objects are absolute in nature, motions THROUGH a finite, discrete, real and absolute space. Contemporary physics would then be operational physics, not fundamental physics. To develop a fundamental physics one would have to change the current paradigm of the infinitist spacetime continuum to a new paradigm based on the finite discreteness of space and time.

It is really significant that material objects are not altered by the physical space they continuously traverse, and that space is modified by the presence of material bodies, which in relativity is called spacetime curvature, and that in reality could only be an alteration in the content of the qusits closest to the material objects, an alteration produced by the nearby presence of these material objects. The preinertia of electromagnetic waves and this alteration of qusits would be sufficient to explain gravity without having to modify the shape of space, perhaps too much of an apparatus. Although the gravitational deformation of the discrete qusits would also be possible, what is not possible, for the reasons given above, is the deformation of the spacetime continuum of densely ordered points. A model to start thinking about the new discrete paradigm could be the cellular automata like models (CALMs) [27, Link].

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[3] All physical objects, including photons, inherit the relative velocity vector (with respect to any other inertial reference frame, including absolute space) of the inertial reference frame in which they are set in motion [30, Link].

An additional attraction of finitist and discrete models, such as CALMs, is that they could solve the old problem of change, which has been posed for 25 centuries without having found a solution. A problem that physics has been ignoring for the last centuries. But the physical world will not be properly explained (fundamental physics) until the problem of change has been resolved. With all its mathematical apparatus (always infinitist), the physics of our days has not yet been able to explain how a simple change in position of any material object occurs. Moreover, it can be formally proved that in the spacetime continuum, change is an inconsistent process [27], which had already been anticipated by some philosophers such as H.W.F. Hegel [19, 20, 37, 40, 43, 48], while others, as J.M.E. McTaggart, came to the same conclusion as Parmenides [41] on the impossibility of change [36]. All of which points to the fact that it would be convenient to start considering the possibility of changing the infinite and continuous scenario of contemporary physics for a finite and discrete one.

At this point, it is interesting to recall the following (very simplified) Aristotelian argument about the non-existence of the vacuum [2, Book IV, 209a, p. 216], in force for more than 2000 years:

> If the vacuum were something and we placed anything else in it, we would have two somethings in the same place, which is impossible.

The reality of gravitational waves and the interpretation of the universe as a CALM invalidates this Aristotelian conclusion: Space is something that contains and generates all material objects. Therefore, an object at a place in space is a material object generated by space at that place. There is then no inconsistent superposition of *somethings*.

I end this paper by proposing the following definition of physical space as a recapitulation of all that has been said and all that has been proven up to this point in this paper:

Definition 1 (of Physical Space) *Space is a real physical object formed by a finite number of indivisible and contiguous units of a non-null extension that contains, and possibly generates, all the material objects of the universe, to which it offers no resistance to their motions and makes possible their mutual interactions.*

AMP A.L.P.

Appendixes

A. Physicists and physical space

(Text taken from [32, p. 119-121])

> The dominant idea in contemporary physics is that neither space nor time are real physical objects. Answers like the next one can be found on some physics well-known FAQ websites (obviously answered by 'expert' physicists):
>
> > Spacetime is not a fabric, it is not material. Space is just an illusion, time is just an illusion therefore spacetime is just an illusion and a good way of simplifying the concept of general relativity to the public.
>
> This has also been the opinion of many relevant authors in the history of science and thought (particularly empiricists): G. Leibniz, D. Hume, C. Huygens, E. Mach, H. Poincaré, E. Borel, L. Wittgenstein etc. And of the vast majority of contemporary physicists. For example [45, p. 266]:
>
> > ... space and time, like society, are in the end also empty conceptions. They have meaning only to the extent that they stand for the complexity of the relationships between the things that happen in the world.
>
> Although, on the other hand, we can also read the contrary opinion. For instance, according to:
>
> 1. A. Einstein
>   - I agree with you that the general theory of relativity is closer to the ether hypothesis than the special theory [26, p. 68].
>   - According to the general theory of relativity, space is endowed with physical qualities... [26, p. 98].
> 2. F. Wilczek:
>   - Spacetime is also a form of matter [52, p. 180].
>   - Spacetime has a life of its own [52, p. 180].
>   - According to general relativity, spacetime is extremely rigid [52, p. 181].

  - Dark energy could be a universal density of space itself [52, p. 194].
  - What appears to our eyes as empty space is revealed to our minds as a complex medium full of spontaneous activity [51, p. 1].

3. N.A. Tambakis:
   - It seems to me that in this way we can confirm the well-known epistemological assumption that space and time are not fictions but rather modes of the dynamic existence of matter [47, p. 146].
4. M. Kaku:
   - In a sense, gravity does not exist; it is the distortion of space and time that moves the planets and stars (cited in [5, p. 63]).

The case of A. Einstein is a bit more complex. Let's remember some of his words through the years about space and time:

> 1905: The introduction of a "luminiferous ether" will prove to be superfluous inasmuch as the view here to be developed does no longer need an absolute space, at absolute rest, with physical properties [10, p. 891].
>
> 1913: For me it is absurd to attribute physical properties to "space" (Letter to E. Mach cited in [25, p. 135]).
>
> 1914: As much I am not disposed to believe in ghosts so I do not believe un the enormous thing about which you are talking and which you call space [11, p. 345].
>
> 1915: Thereby (through the general covariance of the field equations) space and time lose the last remnant of physical reality (Letter to M. Schlick cited in [25, p. 134]).

However, in 1916 Einstein changed his mind about the physical nature of space and the existence of the ether:

> 1916: I agree with you that the general theory of relativity is closer to the ether hypothesis than the H.A. Lorentz cited in [25, p. 135]).
>
> 1919: Thus, once again "empty" space appears as endowed with physical properties, i.e. no longer as physical empty, as seemed to be the case according to special relativity. One can thus say that the ether is resurrected in the general theory of relativity, though in a more sublimated form (Morgan Manuscript, cited in [25, p. 137]).
>
> 1938: Our only way out seems to be to take for granted that space has the physical property of transmitting electromagnetic waves... We may still use the world ether, but only to express some physical property of space... At the moment it no longer stands for a medium built up of particles [13, pp. 159-160] [14, p. 115]

In later writings he defended that the physical notion of space is linked to the existence of rigid bodies, but he rejects the idea that space is an *a priori form of intuition* [4], as Kant defended [24]. Einstein *"always supported an objective description of physical reality, without interference of the observer"* [23, p. 128].

B. The actual infinity is inconsistent

(This demo is an abbreviated version of [31, p. 59-63 Link], where the reader can find another 40 different proofs. Another slightly more detailed version can be found at [33, liink]. If you need to remember the differences between the actual infinity and the potential infinity see [31, p. 35-38 Link].)

Definition 2 (of the Types of Sets) *A set is finite if it has a definite and finite number of elements. A set is potentially infinite if it always contains a finite number of elements of a certain type and any finite numbers of new elements of that type can always be added to it, without the set ceasing to be potentially infinite and without it being necessary to change its name. Two sets are equipotent (have the same number of elements) if, and only if, there is a bijection between their respective elements.*

Definition 3 (of Infinite Set) *A set is infinite if it can be put into one-to-one correspondence with one of its proper subsets.*

Definition 4 (of Inconsistent Set) *A set is inconsistent if a contradiction can be deduced from the number of its elements, or from the number of elements of at least one of its proper subsets.*

Definition 5 (of Denumerable Set) *A set is denumerable if its cardinal is the smallest infinite cardinal* $\aleph_0$ *of the infinite set of all natural numbers. An infinite set is non-denumerable if its cardinal is greater than the smallest infinite cardinal* $\aleph_0$*.*

Theorem 1 (of Denumerable Sets) *There is always at least a one-to-one correspondence between any two denumerable sets.*

Proof.- Let A and B be any two denumerable sets. Assume there is no one-to-one correspondence between their respective elements. In consequence, A and B would not have the same number of elements (Definition 2), which is not the case because, being both denumerable sets, they have exactly the same number of elements: just $\aleph_0$ elements (Definition 5). Therefore, there must be at least a one-to-one correspondence between the sets A and B, and then between any two denumerable sets. □

Theorem 2 *Every non-denumerable set has denumerable proper subsets.*

Proof.- Let X be any non-denumerable set. Since its cardinal is greater than $\aleph_0$ (Definition 5), X contains proper subsets with only $\aleph_0$ elements, all of which are denumerable proper subsets of X (Definition 5). □

Theorem 3 *The infinity subsumed in the Axiom of Infinity can only be the actual infinity.*
Proof.- Since potentially infinite sets do not exist as complete totalities, only two proper subsets with the same number of elements of the same potentially infinite set could be put into one-to-one correspondence, and then we would have a one-to-one correspondence between two proper subsets of a potentially infinite set, instead of a one-to-one correspondence between a set and one of its proper subsets, as required by the definition of an infinite set (Definition 3). Therefore, the potential infinity cannot be the infinity of an infinite set. Only the actual infinity can be the infinity of the infinite sets whose existence is established by the Axiom of Infinity. □

Theorem 4 *The elements of a denumerable set can be reordered with the same order as the elements of any other denumerable set.*
Proof.-Let $A = \{a, b, c, \dots\}$ and $B = \{\alpha, \beta, \dots\}$ be any two denumerable sets. There exists at least one bijection f between the elements of A and B (Theorem 1). Consequently, f pairs each element k of A with a unique and exclusive element, say δ, of B, which can be used to exclusively index that element k of A, so that element k can be rewritten as a_δ. Consequently, the elements of the set A can be reordered and rewritten to define the set $A' = \{a_\alpha, a_\beta, a_\gamma, \dots\}$ which has exactly the same elements as A, and ordered in the same way as the elements of B. □.

Theorem 5 *The denumerable sets are inconsistent.*
Proof.- Let A be any denumerable set. The set A allows us to define the set A' with the same elements as A but reordered as the set $\mathbb{N}$ of natural numbers in their natural order of precedence: $A' = \{a_1, a_2, a_3, \}$ (Theorem 4). The open interval of rational numbers $(0, 1)$ is densely ordered in the natural order of precedence (represented by the symbol <) defined by the natural values of the rational numbers. It is also a denumerable set, so there exists a bijection f between A' and $(0, 1)$ (Theorem 1). Consequently, $(0, 1)$ can be reordered and rewritten as the set $\mathbb{Q}_{01} = \{q_{a_1}, q_{a_2}, q_{a_3}, \dots\}$, where $q_{a_i} = f(a_i), \forall a_i \in A'$, and the successive elements $q_{a_1}, q_{a_2}, q_{a_3}, \dots$ of $\mathbb{Q}_{01}$ are ordered by the successive natural numbers in their natural order of precedence, and not by their respective values as rational numbers. Let x now be a rational variable defined initially as q_{a_1}. And let the value of x be <-compared (i.e., compared according to the values of the rational numbers) with the successive elements of the set $\mathbb{Q}_{01}$, with x being redefined as the compared element q_{a_i} if, and only if, $q_{a_i} < x$.

For short, let us call comparison* this <-comparison and redefinition of x if, and only if, the value of the compared element is smaller than the current value of x. It is immediate to prove that for each natural number v it is possible to perform the first v comparisons* of x with the first v successive elements of $\mathbb{Q}_{01}$. Indeed, if it were not possible, there would be at least one natural number $n \leq v$ such that x could not be compared* with q_{a_n}, which is impossible because q_{a_n} is a rational number of $\mathbb{Q}_{01}$ that can be compared* with the current value of x, which is also a rational number. Once all possible comparisons* of x with the successive elements $q_{a_1}, q_{a_2}, q_{a_3}, \dots$ of $\mathbb{Q}_{01}$ have been made, the current value of x, whatever it may be, could only be the smallest rational number of that set. Indeed, if once performed all possible comparisons* of x with the successive elements of $\mathbb{Q}_{01}$ the current value of x were not the smallest rational number of $\mathbb{Q}_{01}$, there would be at least one element q_{a_n} in $\mathbb{Q}_{01}$ such that $q_{a_n} < x$. But that is impossible because n is a natural number; the first n comparisons* have been carried out; and therefore x was compared* with q_{a_n} and redefined as q_{a_n}; and in all subsequent comparisons*, x could only be redefined with values smaller than q_{a_n}. Therefore, it is impossible for $q_{a_n} < x$. But, on the other hand, it is also immediate to prove that once all possible comparisons* of x with the successive elements of $\mathbb{Q}_{01}$ have been made, the current value of x is not the smallest rational number of that set: every element of the infinite set $\{x/2, x/3, x/4 \ldots\}$ is an element of $\mathbb{Q}_{01}$ smaller than x. This contradiction proves that the set A', defined exclusively with the elements of A, is inconsistent. Therefore A' and A are inconsistent (Definition 4). And A being any denumerable set, it must be concluded that all denumerable sets are inconsistent. □

Corollary 1 *All infinite sets are inconsistent.*
Proof.-Let X be any infinite set. If X is denumerable, then it is inconsistent (Theorem 5). If X is non-denumerable, then it has denumerable proper subsets (Theorem 2), all of which are inconsistent (Theorem 5). Consequently X is inconsistent (Definition 4). Therefore, all infinite sets are inconsistent. □

Corollary 2 *The axiom of infinity is inconsistent.*
Proof.-This is an immediate consequence of Corollary 1. □

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