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The Origin of the Universe: A Formal Proof

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Abstract.-In this article it is formally demonstrated that the observable universe had to have an origin whose cause can only be external to the observable universe, and therefore unknowable to our science, because our science can only be constructed with knowledge extracted from the observable universe itself.

Keywords: Big Bang, eternal universe, finite universe, origin of the universe, entropy, Principle of Directional Evolution, Second Law of Thermodynamics.

1. Introduction

After a brief reflection on the negative reception of the Big Bang theory for its implications on the uncomfortable existence (for the dominant scientific materialism) of a first cause for the origin of the observable universe (universe from now on), a formal scenario is introduced in which precisely the necessity of a first external cause to explain the origin of the universe is proved in formal terms, where external cause means a cause that cannot be established by the scientific knowledge constructed from within the universe itself, which is the only knowledge with which our science can be built.

2. A short note on the Big Bang

A century after the first publications by A. Friedmann and G. Lemaître [6, 7, 14, 15, 16, 17, 18] on the foundations of the theory that would end up being known as the Big Bang (the initially derisive name by which F. Hoyle referred to it in 1949), this is the cosmological theory almost unanimously accepted by today's scientific community on the origin of the universe. But for almost half a century the theory was not well received by the dominant scientific materialism of that time (it still is in ours): the Big Bang suggested an act of initial creation, as opposed to the dominant idea since Greek times of a stationary and eternal universe. And yet the Friedmann-Lemaître theory had been deduced from Einstein's general relativity.

Things began to change with the experimental and accidental discovery in 1964 by A. Penzias and R. Wilson of the cosmic microwave background radiation, radiation that Lemaître called "l'éclat disparu de la formation du mondes" (quoted in [23, p. 67] and whose existence had also been announced by G. Gamow [8]. A new experimental discovery, the expansion of the universe detected through the redshift of the frequencies of the light coming from the different galaxies (Hubble-Lemaître Law) finally consolidated the necessity of an initial *Big Bang* for the origin of the universe. There is, of course, an abundant literature on the Big Bang, e.g. [10, 26, 13, 9, 1, 11, 5, 12, 25, 24] etc.

The Big Bang put an end to the idea of a stationary and eternal universe. But at the same time that scientific materialism accepted its almost inevitable reality, the Big Bang was completed with other theoretical speculations with the sole objective of making this original act of creation unnecessary. Thus, infinite cycles of Big Bangs and Big Crunches, infinite universes, universes arising from a fluctuation of nothing, etc., appeared. However, and as will be seen in this article, it is possible to formally demonstrate that the universe must necessarily have a first cause external to the universe, a first cause that cannot be established from the knowledge constructed from within the universe itself. In what follows, the formal necessity of that first cause external to the universe is demonstrated simply by making use of the two fundamental laws of logic and of an inductive principle of the maximum empirical evidence.

3. The formal scenario

The following discussion is based on three primitive and unproven statements: the first two laws of logic and the following inductive principle which, taking into account that for most of the history of the universe there were no rational observers, generalizes and extends the Second Law of thermodynamics:

Principle 1 (of Directional Evolution) *The observable universe always evolves independently of its rational observers and in the same direction of increasing its global entropy.*

where the term entropy could be replaced by the term isotropy [19, pdf]. It is immediate to recognize in the statement of the above principle a universal generalization of the Second Law of Thermodynamics [2, p. 547]:

> The entropy of a closed system increases with time.

A law that was not initially well received, although it was eventually declared by A. Einstein to be *the first law of all the sciences* [4]. From the above Principle of Directional Evolution the following two results can be immediately deduced:

Theorem 1 (of the Consistent Universe) *The universe evolves under the control of a unique set of invariant and consistent physical laws.*

Proof.-If the physical laws governing the evolution of the universe were not a unique set of invariant and consistent physical laws, changes would occur with the same frequency in all directions, and no progress would be possible in any of them. So, directional evolution would not be possible, which goes against the Principle 1 of Directional Evolution. So then, the universe evolves under the control of a unique set of invariant and consistent physical laws. □

Theorem 2 (of Formal Dependence) *No proposition proves itself; no concept defines itself; no object or physical process is the cause of itself; and no cause is the cause of itself.*

Proof.-If propositions proved themselves, then any proposition could prove itself, and consistent sets of laws would be impossible, which goes against Theorem 1. If concepts defined themselves, their meanings would be inaccessible to human knowledge, and they could not be used to establish the natural laws that we can only establish with those concepts, which also goes against Theorem 1. If physical objects, processes and causes were the cause of themselves, then anything could exist and anything could happen, so that the directional evolution of the universe would be impossible, which goes against the Principle 1 of Directional Evolution. □

Although the following theorem is exclusively mathematical, it is of great importance in the consistency of mathematics, and therefore in the consistency of the physical theories that are constructed with such infinitist mathematics (the interested reader can find another forty proofs in [20]). From here on, the word infinity will always refer to the actual infinity, not to the potential infinity (a definition of both infinities can be found in [20, Chapter 4. Pdf]). The proof makes use of the following formal elements, most of which are taken from [22].

Definition 1 (of Successors and Predecessors) *In strictly ordered sets, all elements that, in the ordering of the set, follow (precede) a given element of the set, are its successors (predecessors). If between the given element and one of its successors (predecessors) there is no other element, then this successor (predecessor) is the immediate successor (predecessor) of the given element.*

Definition 2 (of Complete Totality) *A complete totality is a set defined by comprehension in which every element that satisfies the corresponding membership definition of the set is in the set.*

In consequence, to a complete totality of a certain type of elements, it is not possible to add new elements of that type because it already contains *all of them*.

Definition 3 (of the Types of Sets) *A set is finite if it has a definite and finite number of elements. A set is potentially infinite if it always contains a finite number of elements of a certain type and any finite numbers of new elements of that type can always be added to it, without the set ceasing to be potentially infinite and without it being necessary to change its name. Two sets are equipotent (have the same number of elements) if, and only if, there is a bijection between their respective elements.*

Definition 4 (of Infinite Set) *A set is infinite if it can be put into one-to-one correspondence with one of its proper subsets.*

This is the well-known Dedekind's definition of infinite set [3, p. 115]. But giving a definition of infinite set does not justify its existence, so we need an axiom that formally legitimizes that existence: the Axiom of Infinity, which can be expressed in different more or less abstract ways, but all of them compatible with the following ordinary language expression :

Axiom 1 (of Infinity) *There exists at least one infinite set.*

Where an infinite set is one that satisfies Dedekind's definition of an infinite set (Definition 4).

Definition 5 (of the Types of Infinities) *The actual infinity is the infinity of the infinite sets. The potential infinity is the infinity of the potentially infinite sets.*

Definition 6 (of Inconsistent Set) *A set is inconsistent if a contradiction can be deduced from the number of its elements, or from the number of elements of at least one of its proper subsets.*

Corollary 1 (of Inconsistent Set) *A set with the same number of elements as an inconsistent set, is also inconsistent.*

Proof.-It is an immediate consequence of Definition 6

Definition 7 (of Denumerable Set) *A set is denumerable if its cardinal is the smallest infinite cardinal $\aleph_0$ of the infinite set of all natural numbers. An infinite set is non-denumerable if its cardinal is greater than the smallest infinite cardinal $\aleph_0$.*

Cardinals greater than $\aleph_0$ are, for example, $2^{\aleph_0}$ or $\aleph_1$.

Definition 8 (of ω-Ordered Sets) *A set is ω-ordered if being denumerable, it has a first element, each element has an immediate successor and an immediate predecessor, except the first one which has no predecessor.*

Now it is Immediate to Prove the Following Results:

Theorem 3 (of the Axiom of Infinity) *The infinity subsumed in the Axiom of Infinity can only be the actual infinity.*

Proof.- Since potentially infinite sets do not exist as complete totalities (Definitions 2 and 3), only two proper subsets with the same number of elements of the same potentially infinite set could be put into one-to-one correspondence, and then we would have a one-to-one correspondence between two proper subsets of a potentially infinite set, instead of a one-to-one correspondence between a set and one of its proper subsets, as required by the definition of an infinite set (Definition 4). Therefore, the potential infinity cannot be the infinity of an infinite set. Only the actual infinity can be the infinity of the infinite sets whose existence is established by the Axiom of Infinity. □

Theorem 4 (of Denumerable Sets) *There is always at least a one-to-one correspondence between any two denumerable sets.*

Proof.- Let A and B be any two denumerable sets. Assume there is no one-to-one correspondence between their respective elements. In consequence, A and B would not have the same number of elements (Definition 3), which is not the case because, being both denumerable sets, they have exactly the same number of elements: just $\aleph_0$ elements (Definition 7). Therefore, there must be at least a one-to-one correspondence between the sets A and B, and then between any two denumerable sets. □

Theorem 5 (of non Denumerable Sets) *Every non denumerable set has denumerable proper subsets.*

Proof.- Let X be any non-denumerable set. Since its cardinal is greater than $\aleph_0$ (Definition 7), X contains proper subsets with only $\aleph_0$ elements, all of which are denumerable proper subsets of X (Definition 7). □

Theorem 6 (of Indexation) *The elements of a denumerable set can be reordered with the same order as the elements of any other denumerable set.*

Proof.-Let $A = \{a, b, c, \dots\}$ and $B = \{\alpha, \beta, \dots\}$ be any two denumerable sets. There exists at least one bijection f between the elements of A and B (Theorem 4). Consequently, f pairs each element k of A with a unique and exclusive element, say δ, of B, which can be used to exclusively index that element k of A, so that element k can be rewritten as a_δ. Consequently, the elements of the set A can be reordered and rewritten to define the set $A' = \{a_\alpha, a_\beta, a_\gamma, \dots\}$ which has exactly the same elements as A, and ordered in the same way as the elements of B. □.

The infinity of infinite sets is the actual infinity, not the potential infinity (Theorem 3 of the Axiom of Infinity). This implies the existence of certain infinite sets that are also complete totalities (Definition 2). For example the set $\mathbb{N}$ of ALL natural numbers in their natural order of precedence. It is not possible, then, to add new natural numbers to the set $\mathbb{N}$ of natural numbers because it already contains them all. And the same is true of many other numerical or non-numerical sets. For many authors, the existence of these ordered and complete totalities without a last element that completes them (or without a first element that initiates them) is a proven conclusion independent of the Axiom of Infinity. It is not. It is an existence assumed and legitimized by the Axiom of Infinity. Their existence is, therefore, as debatable as the Axiom of Infinity itself. So it is as legitimate to argue about that axiom as it is to argue about the existence of those complete totalities. This fully justifies the following:

Theorem 7 (of the Denumerable Infinity) *The denumerable sets are inconsistent.*

Proof.- Let A be any denumerable set. The set A allows us to define the set A' with the same elements as A but reordered as the set $\mathbb{N}$ of natural numbers in their natural order of precedence: $A' = \{a_1, a_2, a_3, \}$ (Theorem 6). The open interval of rational numbers $(0, 1)$ is densely ordered in the natural order of precedence (represented by the symbol <) defined by the natural values of the rational numbers. It is also a denumerable set, so there exists a bijection f between A' and $(0, 1)$ (Theorem 4). Consequently, $(0, 1)$ can be reordered and rewritten as the set $\mathbb{Q}_{01} = \{q_{a_1}, q_{a_2}, q_{a_3}, \dots\}$, where $q_{a_i} = f(a_i), \forall a_i \in A'$, and the successive elements $q_{a_1}, q_{a_2}, q_{a_3}, \dots$ of $\mathbb{Q}_{01}$ are ordered by the successive natural numbers in their natural order of precedence, and not by their respective values as rational numbers. Let x now be a rational variable defined initially as q_{a_1}. And let the value of x be <-compared (i.e., compared according to the values of the rational numbers) with the successive elements of the set $\mathbb{Q}_{01}$, with x being redefined as the compared element q_{a_i} if, and only if, $q_{a_i} < x$.

For short, let us call comparison* this <-comparison and redefinition of x if, and only if, the value of the compared element is smaller than the current value of x. It is immediate to prove that for each natural number v it is possible to perform the first v comparisons* of x with the first v successive elements of $\mathbb{Q}_{01}$. Indeed, if it were not possible, there would be at least one natural number $n \leq v$ such that x could not be compared* with q_{a_n}, which is impossible because q_{a_n} is a rational number of $\mathbb{Q}_{01}$ that can be compared* with the current value of x, which is also a rational number. Once all possible comparisons* of x with the successive elements $q_{a_1}, q_{a_2}, q_{a_3}, \dots$ of $\mathbb{Q}_{01}$ have been made, the current value of x, whatever it may be, could only be the smallest rational number of that set. Indeed, if once performed all possible comparisons* of x with the successive elements of $\mathbb{Q}_{01}$ the current value of x were not the smallest rational number of $\mathbb{Q}_{01}$, there would be at least one element q_{a_n} in $\mathbb{Q}_{01}$ such that $q_{a_n} < x$. But that is impossible because n is a natural number; the first n comparisons* have been carried out; and therefore x was compared* with q_{a_n} and redefined as q_{a_n}; and in all subsequent comparisons*, x could only be redefined with values smaller than q_{a_n}. Therefore, it is impossible for $q_{a_n} < x$. But, on the other hand, it is also immediate to prove that once all possible comparisons* of x with the successive elements of $\mathbb{Q}_{01}$ have been made, the current value of x is not the smallest rational number of that set: every element of the infinite set $\{x/2, x/3, x/4 \dots\}$ is an element of $\mathbb{Q}_{01}$ smaller than x. This contradiction proves that the set A', defined exclusively with the elements of A, is inconsistent. Therefore A' and A are inconsistent (Definition 6). And A being any denumerable set, it must be concluded that all denumerable sets are inconsistent. □

Although the consistency of a mathematical proof of infinite steps is universally accepted without the need to perform all of its infinite steps, the theory of supertasks considers the possibility of performing them in finite time. In the case of the above successive comparisons* of x with each successive q_{ai} would be performed at each successive instant t_i of a strictly increasing and convergent sequence $\langle t_i \rangle$ of instants within the finite time interval (t_a, t_b), whose limit is t_b. The instant t_b is the first instant after all instants of $\langle t_i \rangle$, and therefore the first instant after having performed all possible comparisons* of x with the successive elements of Q_{01}. At the instant t_b the rational variable x will still be a rational variable with a certain value, whatever it is; and not, for example, an elephant (in which case anything could be proved). The problem is that the value of x at the instant t_b is and is not the least rational of Q_{01}. From the previous theorems, we can immediately deduce, among many others, the following results:

Corollary 2 (of ω-Ordered Sets) *All ω-ordered sets are inconsistent.*

Proof.-Since all ω-ordered sets are denumerable (Definition 8), all of them are inconsistent (Theorem 7). □

Corollary 3 (of Inconsistent Infinite Sets) *All infinite sets are inconsistent.*

*Proof.*Let X be any infinite set. If X is denumerable, then it is inconsistent (Theorem 7). If X is non-denumerable, then it has denumerable proper subsets (Theorem 5), all of which are inconsistent (Theorem 7). Consequently X is inconsistent Definition 6). Therefore, all infinite sets are inconsistent. □

Corollary 4 (of the Inconsistent Axiom of Infinity) *The axiom of infinity is inconsistent.*

Proof.-This is an immediate consequence of Corollary 3. □

Theorem 8 (of the Actual Infinity) *The actual infinity is inconsistent.*

Proof.-The actual infinity is the infinity subsumed in the Axiom of Infinity (Theorem 3). That axiom only establishes the existence of at least one infinite set, and therefore of a set whose only declared property is that of being actual infinite (Axiom 1). But the Axiom of infinity is inconsistent (Corollary 4). Therefore, the existence of a set whose only declared property is that of being actual infinite is inconsistent; which is only possible if the actual infinity (Definition 5) is inconsistent. □

Corollary 5 (of Infinite Divisibility) *The actual infinite divisibility of any formal or physical object is inconsistent.*

Proof.- From the actual infinite divisibility of any formal or physical object can only result an inconsistent infinite set of parts (Corollary 3). So that actual infinite divisibility is inconsistent. □

Theorem 9 (of the Inconsistent Continuum) *The spacetime continuum is inconsistent.*

Proof.- Being $\mathbb{R}$ the set of all real numbers, the spacetime continuum is, by definition, the Cartesian product $\mathbb{R}^4 = \mathbb{R} \times \mathbb{R} \times \mathbb{R} \times \mathbb{R}$ of all quadruples of real numbers (x, y, z, t). And since $\mathbb{R}$ is an infinite set (Definition 4), it is inconsistent (Corollary 3). Therefore, the spacetime continuum $\mathbb{R}^4$, of which $\mathbb{R}$ is a part, is also inconsistent (Definition 6). □

4. The origin of the universe

From the formal scenario introduced in the previous section, the following results can be immediately deduced:

Theorem 10 (of the Finite Entropy) *The entropy of the observable universe is finite.*

Proof.-Suppose that the entropy S of the observable universe is infinite, and let u be any finite unit of entropy. Since the quantity $n \times u$ is always finite for any finite number n, the number of u units of entropy contained in the entropy S of the observable universe could only be an infinite number. Therefore, the set of all those u units of entropy contained in S would be an infinite set and therefore inconsistent (Corollary 3 of the Inconsistent infinity), which is impossible in a consistent universe (Theorem 1 of the Consistent Universe). Therefore, the entropy of the observable universe can only be finite. □

Theorem 11 (of non-Eternal Universe) *The observable universe cannot be eternal.*

Proof 1.-If the universe were eternal then it would be possible to define an infinite set of instants each separated from the next one into the past by, say, one second. Such a set of instants would thus be an infinite set, and therefore inconsistent (Corollary 3 of infinite sets). Consequently, the eternal universe would contain inconsistent sets, which is impossible if the universe is consistent (Theorem 1 of the consistent universe). Therefore, the universe cannot be eternal. □

Proof 2.-Since entropy grows towards the future (Principle 1 of Directional Evolution), it decreases towards the past. Considering that the total entropy of an isolated system such as the universe cannot be negative, and its present value being finite (Theorem 10), the entropy of the universe would reach its minimum possible value in a finite time from the present and into the past. Therefore the universe cannot be eternal. □

Theorem 12 (of the First Cause) *The observable universe had an origin whose cause is external to the universe itself and scientifically unknowable.*

Proof.- Since it is not eternal (Theorem 11 of the Non-Eternal Universe), the universe must have had an origin. And the cause of that origin must have been external to the universe itself; otherwise, the universe would be an object that causes itself, which is impossible (Theorem 2, of Formal Dependence). If instead of a single cause for the origin of the universe there was a succession of causes, that succession could not be infinite and consistent (Corollary 3 of infinite sets), nor could that succession originate itself (Theorem 2, of Formal Dependence). Finally, the universe could not have originated from a fluctuation of nothingness, because then, contrary to the First Law of logic, nothingness would not be nothingness but something with the capacity to fluctuate and create universes, and that something could not originate itself either (Theorem 2, of Formal Dependence). In all cases, we would have a cause external to the object whose origin we are trying to explain, and being an external cause to that object, that external cause cannot be explained in terms of scientific knowledge extracted from within that same object. Therefore, the cause of the origin of the universe is scientifically unknowable. □

5. Alternatives

From Aristotelian times until well into the twentieth century, the universe was believed to be eternal. But the Big Bang theory suggests an origin of the universe, and the origin of the universe suggests something that neither scientific materialism nor political materialism was willing to accept. Hence the rejection of the Friedmann-Lemaître theory. In the second half of the 20th century the experimental observation of the universe [1] only confirmed that the universe began with something that onomatopoeically could be described as a Big Bang, as a kind of outburst of something that Lemaître called the "*primitive atom*" [18], something of enormous density and temperature. Although, as is well known, physics cannot describe the actual origin of the universe, but what must have happened from 5.39124×10^{-44} seconds later.

But the rejection of an origin for the universe continues in our days with the proposal of theories that try to dodge the thorny issue of its origin. These theories can be divided into two groups: those that make use of infinity (infinite universes or infinite cycles of creation and destruction of universes) and those that defend its origin from nothing. The first group of theories faces the inconsistency of the actual infinity. The second group faces the First Law of logic (Principle of Identity), because nothingness would have to be different from nothingness, it would have to be SOMETHING with the capacity to create universes (see [21, pdf]

Bibliography and References

[1] M. Bojowald. *Antes del Big Bang. Una historia completa del universo.* Debate, 2011.

[2] John Daintith, editor. *Dictionary of Physics.* Oxford University Press, New York, 2009.

[3] Richard Dedekind. *Qué son y para qué sirven los números (Was sind Und was sollen die Zahlen (1888)).* Alianza, Madrid, 1998. Definición de conjunto infinito p. 115.

[4] A. Einstein. Eine Theorie der Grundlagen der Thermodynamik. *Annalen der Physik*, 316(5):170–187, 1903.

[5] Enrique Fernández Borja. *El vacío y la nada.* RBA Editores, Barcelona, 2015.

[6] A. Friedmann. Über die Krümmung des Raumes. *Zeitschrift für Physik*, 10 (1):377–386, June 1922.

[7] A. Friedmann. Über die Möglichkeit einer Welt mit konstanter negativer Krümmung des Raumes. *Zeitschrift für Physik*, 21 (1):326–332, 1924.

[8] G. Gamow. The Expanding Universe and the Origin of Elements. *Physical Review*, 70:572–573, Oct 1946.

[9] Stephen Hawking and Leónard Mlodinov. *El gran diseño.* Editorial Crítica S.L., Barelona, 2010.

[10] Stephen W. Hawking. *Historia del tiempo. Del big bang a os agujeros negros.* Crítica, Baercelona, 1988.

[11] Manuel Lallena Rojo. *El Big Bang y el origen del universo.* RBA Editores, Barcelona, 2015.

[12] P. Landa. *Georges Lemaître, La teoría del Big-Bang y el origen del universo.* Historia en 50 minutos, 2017.

[13] León Lederman and Dick Teresi. *LA PARTÍCULA DIVINA Si el universo es la pregunta ¿cuál es la respuesta?* Editorial Crítica, Barcelona, 2008.

[14] G. Lemaître. Un univers homogène de masse constante et de rayon croissant, randant compte de la vitesse radiale des nèbuleuses extra-galactiques. *Annales de la Société Scientifique de Bruxelles*, 47:49–59, 1927.

[15] G. Lemaître. L'expansion de l'espace. *Publications du Laboratoire d'Astronomie et de Géodésie de l'Université de Louvain*, 8:101–120, 1931.

[16] G. Lemaître. L'univers en expansion. *Annales de la société scientifique de Bruxelles*, 53A:51–83, 1933.

[1] The expansion of the universe, the red shift of light from galaxies, the cosmic microwave background, the initial chemical composition of the universe

[17] G. Lemaître. Expansion de l'Univers. *L'Astronomie*, pages 153–168, 1935.

[18] G. Lemaître. *L'Hypothèse de l'atome primitif, essai de cosmogonie*. Dunod, 1946.

[19] A. León Sánchez. *The Physical Meaning of Entropy*. Amazon's KDP, 2021. PDF.

[20] A. León Sánchez. *Infinity put to the test*. Amazon's KDP, 2023 (2021). PDF.

[21] A. León Sánchez. Nothingness and the physical space. *The General Science Journal*, 2023. PDF.

[22] A. León Sánchez. The Axiom of Infinity Is Inconsistent. *The General Science Journal*, 2024. PDF.

[23] Jean-Pierre Luminet. Les commencements de la cosmologie moderne. *Études*, janvier(1):67–74, 2014.

[24] J. Rodríguez-Quintero and M.E. Gómez Santamaría. *¿Por qué hay algo en lugar de nada?* RBA Editores, Barcelona, 2017.

[25] José Alberto Rubiño. *El fondo cósmico de microondas*. RBA Editores, Barcelona, 2017.

[26] Lee Smolin. *The Life of the Cosmos*. Phoenix, London, 1998.