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Anomalous occultations:

A Review of the Apparent Projection of Stars on the Moon's Disk

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***Abstract*:**

In the present investigation, the reported apparent projection of occulted stars, on the Moon's disk, is, briefly, reviewed. In addition, G. B. Airy's hypothesis of refrangibility, as well as the travel time of moonlight, from the Moon to Earth, along with the aberration of moonlight and starlight, are, thoroughly, analyzed, on the basis of the assumption of constant speed of light, as defined within the framework of the classical wave theory, and on the basis of the ballistic assumption, as defined within the framework of the elastic-impact ballistic theory, respectively.

Keywords:

Lunar occultation; secular light aberration; annual light aberration; ballistic speed of light; Airy's hypothesis; uniform linear motion; optical image; motion relative to the CMBR; light travel time.

Introduction:

Undoubtedly, among the most bothersome questions, on the back of almost every astronomer's mind, during most of the 18th century, and throughout the 19th century, as well, must have been this one:

If the co-moving Moon, by definition, does not show any clear sign of planetary aberration, due to Earth's orbital velocity, around the Sun **[*Ref. #12*]**, then why can't stars, occulted, regularly, by the Moon, at the night side, be projected, on the Moon's disk, immediately, after the reappearance, on the trailing limb of the Moon, by, at least, a few seconds of arc, and a maximum of no more than 20.5 seconds of arc, at the end of each and every lunar occultation?

Such an obvious and quite disturbing — though largely implicit and unspoken — discrepancy was, in all likelihood, the primary motivation, behind a tremendous number of observational reports, in the published literature, during those two centuries, regarding the apparent projection of stars on the Moon's disk, at the start, as well as at the end of so many lunar occultations **[*Ref. #1]; [Ref. #2]; [Ref. #3]; [Ref. #4]; & [Ref. #5*]**..

By far, the clearest, the most comprehensive, and the best among those published reports, is the ***1859*** report, by G. B. Airy, entitled: "**On the Apparent Projection of Stars Upon the Moon's Disk in Occultations**" **[*Ref. #1.a*]**.

Airy's report divides its collection of observational data, on lunar occultations, into four classes:

- **Class *A*:** Which includes the distinct records of apparent projection of stars, upon the Moon's disk **[*Ref. #1.a*]**.

- **Class *B*:** Which includes the reported records of the occasional hanging of stars, to the Moon's limb, at the start of various lunar occultations **[*Ref. #1.a*]**.

- **Class *C*:** Which includes the distinct records of neither apparent projection, upon the Moon's disk, nor occasional hanging, to the Moon's limb, during lunar occultations **[*Ref. #1.a*]**.

- **Class *D*:** Which includes all records of conflicting reports, by different observers, at the same observatory, or in nearby places, during the same lunar occultations **[*Ref. #1.a*]**.

In order to explain away the apparent projection of occulted stars, on the Moon's disk, in accordance with the assumption of constant speed of light, as defined within the framework of the classical wave theory, G. B. Airy, initially, came up with the hypothesis of refrangibility, on the basis of which if the starlight is less refrangible than the moonlight, as in the case of red stars, then, at the instant of occultation, the apparent projection of the star can occur, only, at the upper limb of the Moon. And, conversely, if the starlight is more refrangible than the moonlight, as in the case of blue stars, then, at the instant of occultation, the apparent projection of the star can occur, only, at the lower limb of the Moon **[*Ref. #1.b*]**.

Based on the above report, however, G. B. Airy concluded that his working hypothesis, according to which stars appear on the Moon's disk, during occultations, provided that starlight is either less or more refrangible, in Earth's atmosphere, than moonlight, is no longer viable, because of its clear failure to pass the observational test **[*Ref. #1.a*]**.

In addition, it should be noted, in this regard, that the hypothesis of non-existing annual aberration of light, in the special case of the co-moving Moon, has failed to pass the above observational test, as well, due to the total absence of the clock-like regularity of apparent projection of stars, on the Moon's disk, at the end of each and every lunar occultation, in Airy's report..

Nonetheless, there is a good chance that the combined predictions of Airy's hypothesis of refrangible starlight and the hypothesis of non-existing annual aberration of moonlight may, well, be able, somehow, to pass, with flying colors, the above observational test.

And, therefore, it's necessary, within the current context, to examine the hypothesis of non-existing annual aberration of moonlight, more closely, in order to make sure that the effect of annual light aberration, on moonlight, is, indeed, completely, absent, before any attempt at combining its theoretical predictions with the theoretical predictions of Airy's hypothesis of less or more refrangible starlight.

In the following investigation of the effect of annual light aberration, on moonlight, these principal considerations will have to be taken into account:

I. The assumption of the classical wave theory of light, according to which the velocity of light is independent of the velocity of the light source.

II. The assumption of the elastic-impact ballistic theory of light, according to which the velocity of light is dependent upon the velocity of the light source.

III. The travel time of moonlight, from the co-moving Moon to topocentric observatories moving with the same orbital velocity of Earth, around the barycenter of the solar system.

IV. The travel time of moonlight, from the co-moving Moon to topocentric observatories moving with the velocity of Earth, relative to the Cosmic Microwave Background Radiation (CMBR).

V. The direction of the vector sum of the velocity of incident moonlight and the orbital velocity of Earth around the barycenter of the solar system.

VI. The direction of the vector sum of the velocity of incident moonlight and the velocity of Earth, relative to the Cosmic Microwave Background Radiation (CMBR).

And furthermore, the effect of annual aberration of moonlight, due to Earth's orbital velocity, around the barycenter of the solar system, and the effect of secular aberration of moonlight, due to Earth's velocity, relative to the Cosmic Microwave Background Radiation (CMBR), as well as the effect of the travel time of moonlight, from the Moon to Earth, on the libration of the Moon, and on the angular diameter of the Moon, as observed from Earth, will be calculated and discussed in detail.

1. The Annual Aberration of Moonlight According to the Wave Theory:

According to the classical wave theory of light, the velocity of sunlight, reflected by the Moon, is independent of the velocity of the emitting Sun, at the time of emission, as well as independent of the velocity of the reflecting Moon, at the time of reflection. And, therefore, the ballistic velocity resultants of sunlight have been excluded, entirely, right from the very beginning, in both cases, within the framework of this theory.

Let $\boldsymbol{v}$ denote the orbital velocity of the barycenter of the Earth-Moon system, around the gravitational center of the solar system.

And let $\boldsymbol{\theta}$ stand for the angle of the position of the Moon, with respect to the orbital velocity vector of Earth, $\boldsymbol{v}$, at the time of reflection.

Subsequently, during the travel time of reflected sunlight, from the Moon to Earth, $\boldsymbol{t'}$, the Earth makes a linear displacement equal to $\boldsymbol{vt'}$, upon the arrival of moonlight, at the time of reception.

And as a result, the observed position of the optical image of the Moon, relative to the instantaneous position of Earth, at the time of reception, is necessarily shifted, to the backward direction of Earth's orbital velocity vector, $\boldsymbol{v}$, by a small angle equal to $\boldsymbol{\Delta\theta}$.

And it follows, therefore, that, by applying the law of sines, to the displacement triangle, as illustrated in **Figure #*1***, below, the angle of the backward shift of the Moon's optical image, caused by the combined effects of Earth's orbital velocity and the finite velocity of reflected sunlight, from the Moon, can be computed, in accordance with the following trigonometric relation:

$$\sin(\Delta\theta) = \frac{v}{c}\sin(\theta)$$

where $\boldsymbol{v}$ is the orbital velocity of Earth, around the barycenter of the solar system; $\boldsymbol{c}$ is the velocity of reflected sunlight, from the Moon; and θ is the position of the Moon, at the time of reflection.

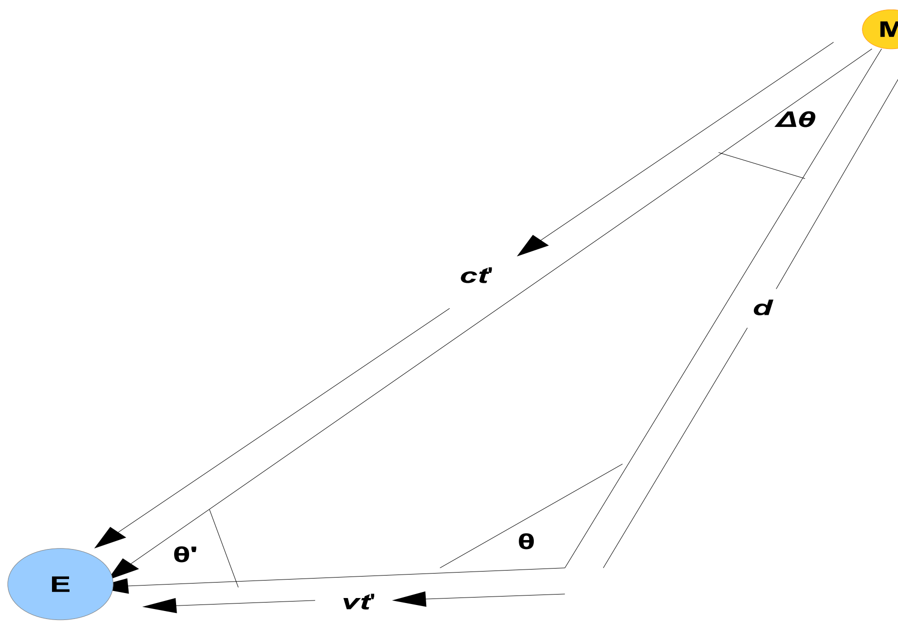

Figure #1: The backward shift of the optical image of the Moon, due to the travel time of moonlight

It should be noted, within this context, that the true position of the Moon, θ_{ref}, prior to the process of moonlight's travel time, at the time of reflection, is the same as the observed apparent position of the Moon, θ_{rec}, under the process of moonlight aberration, at the time of reception; i.e.

$$\theta_{ref} = \theta_{rec} = \theta$$

Immediately, upon the arrival of shifted moonlight, at the moving Earth, annual light aberration — the direction of the vector sum of the velocity of incident moonlight and the orbital velocity of Earth — takes the shifted image of the Moon, above, as an input, and shifts it, in the forward direction of the orbital velocity vector of Earth, $\boldsymbol{v}$, as an output, by the same small angle of $\Delta\theta$, and in accordance with the same trigonometric relation:

$$\sin(\Delta\theta) = \frac{v}{c}\sin(\theta)$$

where $\boldsymbol{v}$ is the orbital velocity of Earth, around the barycenter of the solar system; $\boldsymbol{c}$ is the velocity of reflected sunlight, from the Moon; and θ stands, in this case, for the position of the Moon, as seen from Earth, at the time of reception, as illustrated in **Figure #2**, below.

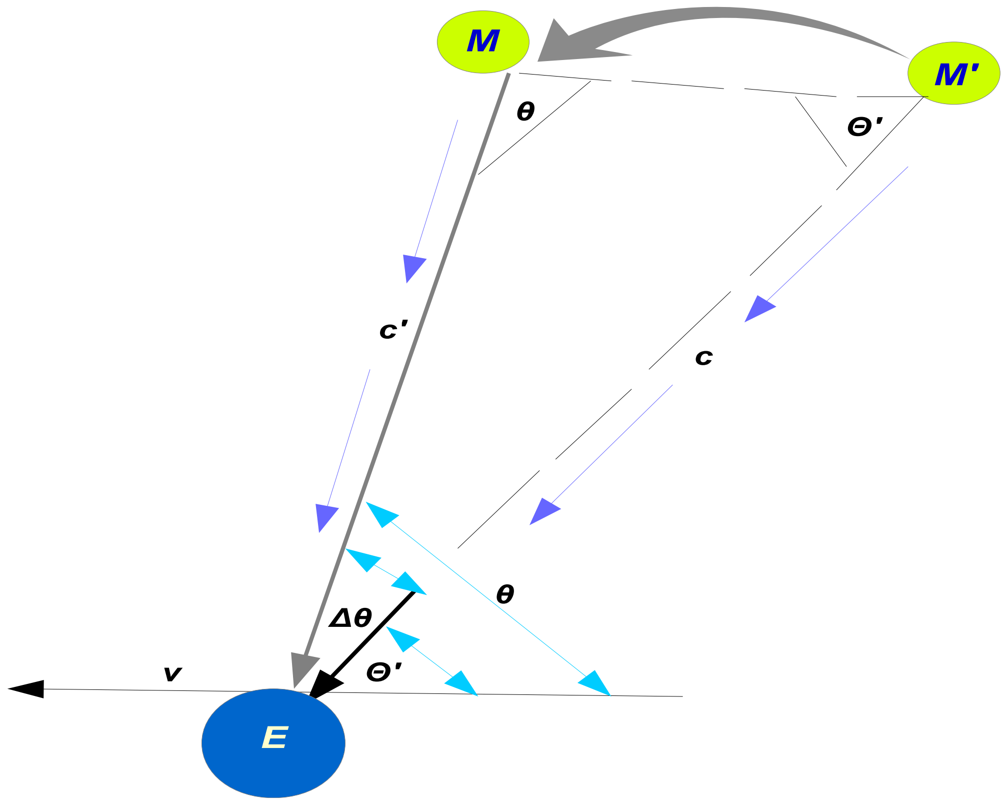

Figure #2: The forward shift of the optical image of the Moon, due to annual aberration of moonlight

On the basis of the assumption of constant speed of light, as defined within the framework of the classical wave theory, therefore, the following points have to be made explicit and clarified, further, with regard to the combined effects of the annual aberration of moonlight and the travel time of moonlight, from the Moon to Earth:

1. The co-moving Moon does not exhibit any sign of planetary aberration, due to Earth's orbital velocity, around the Sun, not because the annual aberration of moonlight is absent, but because the travel time of moonlight and the annual aberration of moonlight balance each other out and make the optical image of the Moon coincide, always, with the instantaneous position of the Moon, as observed from the moving Earth.

2. Since the optical images of stars, at their true positions, are occulted, only, by the backward-shifted optical image of the Moon, it is not possible for the optical images of stars to appear to be projected, on the Moon's disk, during lunar occultations, due to the presence of annual

aberration of moonlight.

3. The shifting of the optical image of the Moon, to the backward direction, with respect to Earth's orbital velocity vector, ***v***, by an angle equal to $\Delta\theta$, makes, at the same time, a minute fraction of the angular size of the far side of the Moon equal to $\Delta\theta$ visible from Earth, as well.

4. The annual aberration of moonlight takes the backward-shifted optical image of the Moon, caused by the travel time of moonlight, from the Moon to Earth, and rotates it, in the forward direction of Earth's orbital velocity vector, ***v***, by an angle equal to $\Delta\theta$; but it changes neither the direction of moonlight velocity, nor the direction of the orbital velocity of the Moon, around Earth, nor the visible fraction of an angle of $\Delta\theta$ from the far side of the Moon's angular size.

5. Since the maximum value of $\Delta\theta$ is equal to about 20.5 arc-seconds, throughout the course of a lunar month, the travel time of moonlight, from the Moon to Earth, contributes an amount of about 41 arc-seconds, to the total libration of the Moon, as observed from Earth.

6. More generally, as in the case of the co-moving Moon, the travel time of light and light aberration are, always, present and bound to balance each other out, in each and every case, in which the light source and the light receiver are moving with the same speed in the same direction.

2. The Annual Aberration of Moonlight According to the Ballistic Theory:

According to the ballistic theory of light, the velocity of sunlight, reflected by the Moon, is dependent upon the velocity of the emitting Sun, at the time of emission, as well as dependent upon the velocity of the reflecting Moon, at the time of reflection. And, therefore, the ballistic velocity resultants of moonlight have to be included, at all times, right from the start, in both cases, within the framework of this theory.

Since the Sun's rotational velocity, at every point of its visible surface, is within the velocity range ***{+1997, ..., -1997} m/s***, incident sunlight, on any point of the Moon's visible surface, must have velocity resultants within the velocity range ***{c + 1997, ..., c - 1997} m/s***.

And since sunlight is incident at right angles, with respect to both the orbital velocity of the Moon, around the barycenter of the solar system, ***v***, and the orbital velocity of the Moon, around the earth, which is equal to about ***1022*** m/s, one component of the velocity resultants of sunlight, reflected by the Moon, has to be within the velocity range ***{v + 1022, ..., v – 1022} m/s***.

And it follows, therefore, that the velocity resultants of moonlight, c', with respect to the barycenter of the solar system, are within the velocity range, which starts with the maximum velocity resultant c'_{max}:

$$c'_{\max} = \sqrt{(c+1997)^2 + (v+1022)^2} \quad \text{ms}^{-1}$$

and ends with the minimum velocity resultant $\boldsymbol{c'_{min}}$:

$$c'_{\min} = \sqrt{(c-1997)^2 + (v-1022)^2} \quad \text{ms}^{-1}$$

where $\boldsymbol{c}$ is the muzzle velocity of light; and $\boldsymbol{v}$ is the orbital velocity of the barycenter of the Earth-Moon system, around the barycenter of the solar system.

With respect to the moving Earth, however, the velocity component of moonlight, $\boldsymbol{v}$, at the time of reflection, is, necessarily, balanced out, at the time of reception, by Earth's orbital velocity, $\boldsymbol{v}$. And furthermore, the orbital velocity of the Moon, around Earth, is reduced, by varying amounts, in accordance with the following formula:

$$v_L = 465.278 \times \cos(\lambda) \quad \text{ms}^{-1}$$

where $\boldsymbol{v_L}$ is Earth's rotational velocity, at the geographical latitude of the observatory; ***(465.278 ms^{-1})*** is the rotational velocity of Earth, at the equator; and $\boldsymbol{\lambda}$ is the angle of the geographical latitude.

And, correspondingly, the velocity resultants of moonlight, $\boldsymbol{c'}$, with respect to observers, on Earth, are within the velocity range, which starts with the minimum velocity resultant $\boldsymbol{c'_{min}}$:

$$c'_{\min} = \sqrt{[c-1997]^2 + [1022 - (465.278)\cos(\lambda)]^2} \quad \text{ms}^{-1}$$

and ends with the maximum velocity resultant $\boldsymbol{c'_{max}}$:

$$c'_{\max} = \sqrt{[c+1997]^2 + [1022 - (465.278)\cos(\lambda)]^2} \quad \text{ms}^{-1}$$

where $\boldsymbol{c}$ is the muzzle velocity of light.

On the basis of the ballistic assumption, therefore, the optical image of the Moon, as seen from the moving Earth, is necessarily shifted, to the backward direction of Earth's orbital velocity vector, $\boldsymbol{v}$, by a small angle equal to $\boldsymbol{\Delta\theta}$, due to the combined effects of Earth's orbital velocity and the finite velocity of reflected sunlight, from the Moon, in accordance with the following trigonometric relation:

$$\sin(\Delta\theta) = \frac{v}{c'}\sin(\theta)$$

where $\boldsymbol{v}$ is the orbital speed of Earth, around the barycenter of the solar system; $\boldsymbol{c'}$ is the ballistic velocity resultant of reflected sunlight, from the Moon; and $\boldsymbol{\theta}$ is the position of the Moon, with respect to the co-moving Earth, at the time of reflection, as illustrated earlier in **Figure #*1***.

And subsequently, as moonlight, from the backwardly shifted optical image of the Moon arrives at the moving Earth, annual light aberration — the direction of the vector sum of the velocity of incident moonlight, c', and the orbital velocity of Earth, $\boldsymbol{v}$ — shifts it, in the forward direction of the orbital velocity vector of Earth, *v*. by the same small angle of $\boldsymbol{\Delta\theta}$, and in accordance with the same trigonometric relation:

$$\sin(\Delta\theta) = \frac{v}{c'}\sin(\theta)$$

where $\boldsymbol{v}$ is the orbital speed of Earth, around the barycenter of the solar system; $\boldsymbol{c'}$ is the ballistic velocity resultant of reflected sunlight, from the Moon; and $\boldsymbol{\theta}$ is the position of the Moon, as seen from the co-moving Earth, at the time of reception, as illustrated earlier, in **Figure #*2***.

Within the framework of the elastic-impact ballistic theory, therefore, the following remarks have to be made clear, with regard to the combined effects of the annual aberration of moonlight and the travel time of moonlight, from the Moon to Earth:

- The co-moving Moon does not exhibit any sign of planetary aberration, due to Earth's orbital velocity, around the Sun, not because the annual aberration of moonlight is absent, but because the travel time of moonlight and the annual aberration of moonlight balance each other out and make the optical image of the Moon coincide, exactly, with the instantaneous position of the Moon, as observed from the moving Earth.

- The shifting of the optical image of the Moon, to the backward direction, with respect to Earth's orbital velocity vector, $\boldsymbol{v}$, by an angle equal to $\boldsymbol{\Delta\theta}$, due to the travel time of moonlight, renders, at the same time, a small portion of the hidden far side of the Moon equal to $\boldsymbol{\Delta\theta}$ visible from Earth, as well.

- The annual aberration of moonlight takes the backward-shifted optical image of the Moon, caused by the travel time of moonlight, from the Moon to Earth, and rotates it, in the forward direction of Earth's orbital velocity vector, $\boldsymbol{v}$, by an angle equal to $\boldsymbol{\Delta\theta}$; but it does not change, in any way, the direction of moonlight velocity, the direction of the orbital velocity of the Moon, around Earth, or the visible angular size of an angle of $\boldsymbol{\Delta\theta}$ from the far side of the Moon.

- Since the maximum value of $\Delta\theta$ is equal to about 20.5 arc-seconds, over the course of a lunar month, the travel time of moonlight, from the Moon to Earth, contributes an amount of about 41 arc-seconds, to the total libration of the Moon, as observed from Earth.

- As in the case of the co-moving Moon, the travel time of light and light aberration are, always, present and bound to balance each other out, in each and every case, in which the light source and the light receiver are moving with the same speed in the same direction.

- Whenever the velocity of starlight, from the optical images of stars, at their true positions, and the velocity of moonlight, from the backwardly shifted optical image of the Moon, are unequal, the optical images of those stars appear to be projected, on the Moon's disk, during lunar occultations, by a very small angle, as seen from Earth.

3. The Aberration of Moonlight Due to Earth's Velocity Relative to CMBR:

As concluded earlier, neither the travel time of moonlight, from the Moon to earth, nor the aberration of moonlight, due to Earth's orbital velocity, around the Sun, can lead, directly or indirectly, to the apparent projection of occulted stars, on the Moon's disk, as reported by G. Airy and others.

And the same conclusion can be demonstrated to be true, in the case of the travel time of moonlight, from the Moon to Earth, and the secular aberration of moonlight, due to Earth's velocity, relative to the Cosmic Microwave Background Radiation (CMBR).

Nonetheless, the expected contribution, to the libration of the Moon, by the combined effect of the travel time of moonlight, from the Moon to earth, and the secular aberration of moonlight, due to Earth's velocity, relative to the Cosmic Microwave Background Radiation (CMBR), is much larger and more significant than the small amount of about 41 arc-seconds, due to Earth's orbital velocity, around the gravitational center of the solar system.

Earth's velocity, relative to the Cosmic Microwave Background Radiation (CMBR), as measured by the COBE satellite [***Ref. #16***], is within the following range:

$$
\begin{aligned}
&v = 370600 \pm 400 \quad \mathrm{ms}^{-1} \\
&(l,b) = \left(264.31^o \pm 0.17^o, 48.05^o \pm 0.10^o\right) \\
&RA = 11^h\ 12^m, \quad Dec = -7.20^o
\end{aligned}
$$

where ***v*** is the velocity of Earth, relative to the CMBR; ***l*** is the galactic longitude; ***b*** is the galactic latitude; ***RA*** is the right ascension; and ***Dec*** is the equatorial declination.

And, therefore, by inserting the above data, into this trigonometric formula:

$$\sin(\Delta\theta) = \frac{v}{c}\sin(\theta)$$

where ***v*** is Earth's velocity, relative to the CMBR; ***c*** is the velocity of reflected sunlight, from the Moon; and ***θ*** is the position of the Moon, at the time of reflection;

the maximum value of the angle, ***Δθ***, for both the secular aberration of moonlight, and the backward shift of the Moon's optical image, due to the combined effects of Earth's velocity, relative to the CMBR, and the finite velocity of reflected sunlight, from the Moon, can be obtained, in accordance with the assumption of constant speed of light, as defined within the framework of the classical wave theory:

$$v = 371000 \quad \text{ms}^{-1}$$
$$c = 299792458 \quad \text{ms}^{-1}$$
$$\theta = 90^{o}$$
$$\Delta\theta = 255.25746 \;\; \text{arcseconds}$$

where ***Δθ*** denotes the angular amount of the backward shift, due to the time of flight of moonlight, from the Moon to Earth; as well as the angular amount of the forward shift, caused by the secular aberration of moonlight, due to Earth's velocity, relative to the CMBR; respectively.

And consequently, over the course of a lunar month, the combined effect of the travel time of moonlight, from the Moon to Earth, and the secular aberration of moonlight, contributes, to the total libration of the Moon, as observed from Earth, an amount of about:

$$2\Delta\theta = 510.51492 \;\; \text{arcseconds}$$

And likewise, a similar numerical result, can be computed, within the framework of the elastic-impact ballistic theory, by using the following trigonometric relation:

$$\sin(\Delta\theta) = \frac{v}{c'}\sin(\theta)$$

where θ is the position of the Moon, at the time of reflection; v is the velocity of Earth, relative to the CMBR; and c' denotes the velocity resultant of moonlight, c', with respect to observers, on Earth, where its numerical values are, within the velocity range, which starts with the minimum velocity resultant, c'_{min}:

$$c'_{\min} = \sqrt{\left[c - 1997\right]^2 + \left[1022 - (465.278)\cos(\lambda)\right]^2} \ \ \text{ms}^{-1}$$

and ends with the maximum velocity resultant, c'_{max}:

$$c'_{\max} = \sqrt{\left[c + 1997\right]^2 + \left[1022 - (465.278)\cos(\lambda)\right]^2} \ \ \text{ms}^{-1}$$

where c is the muzzle velocity of light.

And accordingly, by inserting the same numerical data, into the above formula, the maximum value of the angle, $\Delta\theta$, for both the secular aberration of moonlight, and the backward shift of the Moon's optical image, due to the combined effects of Earth's velocity, relative to the CMBR, and the finite velocity of reflected sunlight, from the Moon, can be obtained, in accordance with the assumption of ballistic speed of light, as defined within the framework of the elastic-impact ballistic theory:

$$\begin{aligned} &v = 371000 \ \ \text{ms}^{-1} \\ &c' = 299792687.56 \ \ \text{ms}^{-1} \\ &\theta = 90^o \\ &\Delta\theta = 255.25727 \ \ \text{arcseconds} \end{aligned}$$

where $\Delta\theta$ denotes the angular amount of the backward shift, due to the time of flight of moonlight, from the Moon to Earth; and the angular amount of the forward shift, caused by the secular aberration of moonlight, due to Earth's velocity, relative to the CMBR, respectively.

And correspondingly, over the course of a lunar month, the combined effect of the travel time of moonlight, from the Moon to Earth, and the secular aberration of moonlight, contributes, to the total libration of the Moon, as observed from Earth, an amount of about:

$$2\Delta\theta = 510.51454 \ \ \text{arcseconds}$$

From now on, however, since it's quite clear that, on the basis of the ballistic assumption, the maximum value of c', within the above numerical range, can change the value of $\Delta\theta$ by no more than ***0.001"***, both the rotational velocity of the Sun, around its axis, as well as the orbital velocity of the Moon, around the earth, will, no longer, be taken into account, in the rest of this discussion.

4. The Effect of Moonlight Aberration on the Moon's Angular Diameter:

In accordance with the assumption of constant speed of light, as defined within the framework of the classical wave theory, as well as in accordance with the assumption of ballistic speed of light, as defined within the framework of the elastic-impact ballistic theory, upon shifting, in the forward direction, the backwardly shifted image of the Moon, under the effect of the secular aberration of moonlight, the angular diameter of the Moon, as observed from Earth, must change by a certain amount, depending on the position of the Moon, with respect to the velocity vector of the earth.

And therefore, in order to calculate, in a precise manner, the amounts of change, in the length of the angular diameter of the Moon, due to the secular aberration of moonlight, it's necessary to reformulate the general equation, for light aberration, in terms of the actual angle θ', instead of the observed angle θ, through the use of the following trigonometric identity:

$$\sin(\theta' - \theta) = \sin(\theta')\cos(\theta) - \cos(\theta')\sin(\theta)$$

where θ' is the position of the backwardly shifted image of the Moon; and θ is the position of the forwardly shifted image of the Moon, respectively.

In this general formula, for the aberration of moonlight:

$$\sin(\Delta\theta) = \frac{v}{c}\sin(\theta)$$

$\Delta\theta$ stands for $(\theta' - \theta)$.

And subsequently, by applying to the above formula, the aforementioned trigonometric identity, the general equation, for calculating the amounts of change, in the angular diameter of the Moon, due to the secular aberration of moonlight, in the case of Earth's velocity, relative to the CMBR, and the annual aberration of moonlight, in the case of Earth's orbital velocity, relative to the barycenter of the solar system, respectively, can be obtained:

(1.) Variations in the Moon's angular diameter due to the secular aberration of moonlight:

Let D_1 denote the angular diameter of the backwardly shifted image of the Moon, due to the displacement made by the earth, during the travel time of moonlight, from the Moon to Earth.

And let D_2 denote the angular diameter of the forward-shifted image of the Moon, caused by the secular aberration of moonlight, due to Earth's velocity, v, relative to the CMBR.

And so, in order to find out by how much the angular length of $\boldsymbol{D_1}$ changes, upon transforming it into $\boldsymbol{D_2}$, under the process of moonlight secular aberration, it's necessary, first of all, to apply the above trigonometric identity — ***sin(θ′ – θ)*** — to the standard formula, for computing light aberration, and to rewrite it, in the following form:

$$\tan(\theta) = \frac{\sin(\theta')}{\cos(\theta') + \frac{v}{c}}$$

where $\boldsymbol{\theta}$ stands for the observed position of the transformed point, on $\boldsymbol{D_2}$, and whose given position, on $\boldsymbol{D_1}$, is equal to $\boldsymbol{\theta'}$.

Since the Moon, as observed from Earth, is an extended astronomical object, the above formula, for transforming backward-shifted images into forward-shifted images, has to be utilized twice, for each given numerical value of the angle $\boldsymbol{\theta'}$: Firstly, for transforming the upper end of $\boldsymbol{D_1}$, into the upper end of $\boldsymbol{D_2}$; and secondly, for transforming the lower end of $\boldsymbol{D_1}$, into the lower end of $\boldsymbol{D_2}$.

The average numerical value, below, for the angular separation, between the two ends of the angular diameter of the backward-shifted optical image of the Moon, as subtended at the average distance of Earth from the Moon, will be taken for granted, throughout the following calculations:

$$D_1 = 0.5166667^o = 31' = 1860''$$

For instance, if the given position of the upper end of $\boldsymbol{D_1}$, $\boldsymbol{\theta'_1}$, is equal to this angular value:

$$\theta_1' = \theta'$$

then the computed position of the lower end of $\boldsymbol{D_1}$, $\boldsymbol{\theta'_2}$, must be equal to this angular value

$$\theta_2' = \theta' - 0.5166667^o$$

for all given numerical values of $\boldsymbol{\theta'}$, within the forward celestial hemisphere, with respect to Earth's velocity, $\boldsymbol{v}$, relative to the CMBR:

And accordingly, by replacing $\boldsymbol{\theta'}$ with the above values of ($\boldsymbol{\theta'_1}$ & $\boldsymbol{\theta'_2}$), respectively, in the general trigonometric formula, below, for calculating the apparent position of a point light source, from the standpoint of a moving platform:

$$\tan(\theta) = \frac{\sin(\theta')}{\cos(\theta') + \frac{v}{c}}$$

the observed position of the upper end of ***D_2***, ***θ_1*** and the observed position of the lower end of ***D_2***, ***θ_2***, can be, readily, obtained.

And subsequently, by subtracting the computed numerical value of the lower end of ***D_2***, ***θ_2***, from the computed numerical value of the upper end of ***D_2***, ***θ_1***, the numerical value of the forward-shifted angular diameter of the Moon, ***D_2***, can be, readily, obtained, as well:

$$D_2 = \theta_1 - \theta_2$$

And, finally, by subtracting the computed angular diameter of the forward-shifted image of the Moon's disk, ***D_2***, from the given angular diameter of the backward-shifted image of the Moon's disk, ***D_1***, the angular amount, ***ΔD***, by which the angular diameter of the Moon is shortened, under the process of the secular aberration of moonlight, due to Earth's velocity, ***v***, relative to the CMBR, can be calculated, in each and every case, throughout the forward hemisphere of the celestial sphere; i.e.,

$$\Delta D = D_1 - D_2$$

For example, upon the insertion, into the aforementioned general trigonometric formula, for computing the apparent positions of point light sources, under the process of light aberration, the following set of data, regarding the backward-shifted optical image of the Moon's disk:

$$c = 299792458 \ \ \text{ms}^{-1}$$
$$v = 371000 \ \ \text{ms}^{-1}$$
$$D_1 = 31' = 1860''$$
$$\theta_1' = 30^o$$
$$\theta_2' = 29.4833333^\circ$$

the computed set of numerical results, regarding the forward-shifted optical image of the Moon's disk, under the process of moonlight aberration, is the following:

$$\theta_1 = 29.9645855^\circ$$
$$\theta_2 = 29.4484736^\circ$$
$$D_2 = \theta_1 - \theta_2 = 1858.0028736''$$
$$\Delta D = D_1 - D_2 = 1.997''$$

in accordance with which the calculated angular diameter of the forward-shifted optical image of the Moon's disk is less than the given angular diameter of the backward-shifted optical image of the Moon's disk, by an amount of about ***2.0*** seconds of arc.

The computed numerical values, for the difference, ΔD, between the given angular diameter of the backward-shifted image of the Moon's disk, D_1, and the calculated angular diameter of the forward-shifted image of the Moon's disk, D_2, for a number of angles, around the forward direction of Earth's velocity vector, v, relative to the CMBR, are summarized in the last column, in **Table #1**, below:

θ'_1	θ'_2	θ_1	θ_2	D_2	ΔD
0.0000000º	*0.5166667º*	*0.0000000º*	*0.5160281º*	*1857.7011600"*	*2.2989164"*
10.0000000º	*9.4833333º*	*9.9877025º*	*9.4716652º*	*1857.7340879"*	*2.2659121"*
20.0000000º	*19.4833333º*	*19.9757773º*	*19.4597118º*	*1857.8356991"*	*2.1643009"*
30.0000000º	*29.4833333º*	*29.9645855º*	*29.4484736º*	*1858.0028736"*	*1.9971263"*
45.0000000º	*44.4833333º*	*44.9499066º*	*44.4336940º*	*1858.3650987"*	*1.6349013"*
60.0000000º	*59.4833333º*	*59.9386326º*	*59.4222885º*	*1858.8387332"*	*1.1612668"*
90.0000000º	*89.4833333º*	*89.9290952º*	*89.4124322º*	*1859.9867735"*	*0.0132265"*

Table #1: ***The calculated results of ΔD, for the given angles, within the range ($0^o \leq \theta'_1 \leq 90^o$).***

It should be pointed out, within the current context, that, in the backward hemisphere of the celestial sphere, with respect to Earth's velocity vector, v, relative to the CMBR, the position angle of the upper end of the angular diameter of the backward-shifted optical image of the Moon's disk, θ'_1, is, by definition, always, less than the position angle of the lower end of the angular diameter of the backward-shifted optical image of the Moon's disk, θ'_2.

And correspondingly, by subtracting the computed numerical value of the upper end of D_2, θ_1, from the computed numerical value of the lower end of D_2, θ_2, the numerical of the forward-shifted angular diameter of the Moon, D_2, can be, readily, obtained, in accordance with the following equation:

$$D_2 = \theta_2 - \theta_1$$

And, subsequently, by subtracting the angular diameter of the backward-shifted image of the Moon's disk, D_1, from the angular diameter of the forward-shifted image of the Moon's disk, D_2, the angular amount, ΔD, by which the angular diameter of the Moon is increased, under the process of the secular aberration of moonlight, due to Earth's velocity, v, relative to the CMBR, can be computed, in each and every case, throughout the backward hemisphere of the celestial sphere; i.e.,

$$\Delta D = D_2 - D_1$$

For example, upon inserting, into the aforementioned general trigonometric formula, for computing the apparent positions of point light sources, under the process of light aberration, the following set of data, regarding the backward-shifted optical image of the Moon's disk :

$$c = 299792458 \ \ \text{ms}^{-1}$$
$$v = 371000 \ \ \ \text{ms}^{-1}$$
$$D_1 = 31' = 1860''$$
$$\theta_1' = 119.4833333^o$$
$$\theta_2' = 120.0000000^o$$

the computed set of numerical results, regarding the forward-shifted image of the Moon's disk, under the process of moonlight aberration, is the following:

$$\theta_1 = 119.4215732^o$$
$$\theta_2 = 119.9385566^o$$
$$D_2 = \theta_2 - \theta_1 = 1861.1404429''$$
$$\Delta D = D_2 - D_1 = 1.1404429''$$

in accordance with which the calculated angular diameter of the forward-shifted optical image of the Moon's disk, in this case, is larger than the given angular diameter of the backward-shifted optical image of the Moon's disk, by an amount of about ***1.1*** seconds of arc.

The computed numerical values, for the difference, $\boldsymbol{\Delta D}$, between the given angular diameter of the backward-shifted image of the Moon's disk, $\boldsymbol{D_1}$, and the calculated angular diameter of the forward-shifted image of the Moon's disk, $\boldsymbol{D_2}$, for a number of angles, around the backward direction of Earth's velocity vector, $\boldsymbol{v}$, relative to the CMBR, are summarized in the last column, in **Table #2**, below:

θ'_1	θ'_2	θ_1	θ_2	D_2	ΔD
109.4833333º	*110.0000000º*	*109.4164610º*	*109.9333431º*	*1861.7752949''*	*0.7752949''*
119.4833333º	*120.0000000º*	*119.4215732º*	*119.9385566º*	*1861.1404429''*	*1.1404429''*
134.4833333º	*135.0000000º*	*134.4327021º*	*134.9498188º*	*1861.6202243''*	*1.6202243''*
149.4833333º	*150.0000000º*	*149.4472902º*	*149.9645096º*	*1861.9895965''*	*1.9895965''*
159.4833333º	*160.0000000º*	*159.4584538º*	*159.9757209º*	*1862.1615657''*	*2.1615657''*
169.4833333º	*170.0000000º*	*169.4703760º*	*169.9876725º*	*1862.2676610''*	*2.2676610''*
179.4833333º	*180.0000000º*	*179.4826932º*	*180.0000000º*	*1862.3046131''*	*2.3046131''*

Table #2: ***The calculated results of ΔD, for the given angles, within the range ($90^o < \theta'_2 \le 180^o$).***

(2.) Variations in the Moon's angular diameter due to the annual aberration of moonlight:

The same procedures, above, for calculating $\boldsymbol{\theta'_1}$, $\boldsymbol{\theta'_2}$, $\boldsymbol{D_2}$, and $\boldsymbol{\Delta D}$, under the process of secular aberration of moonlight, due to Earth's velocity, $\boldsymbol{v}$, relative to the CMBR, are carried out, in the exact same way, for calculating $\boldsymbol{\theta'_1}$, $\boldsymbol{\theta'_2}$, $\boldsymbol{D_2}$, and $\boldsymbol{\Delta D}$, under the process of annual aberration of moonlight, due to Earth's orbital velocity, $\boldsymbol{v''}$, relative to the barycenter of the solar system.

But, as expected, however, the numerical values of the computed results, for $\boldsymbol{\theta'_1}$, $\boldsymbol{\theta'_2}$, $\boldsymbol{D_2}$, and $\boldsymbol{\Delta D}$, are, relatively, less significant, in the case of Earth's motion, relative to barycenter of the solar system, than the numerical values of the computed results, for the same parameters, in the case of Earth's motion, relative to the CMBR.

For example, upon the insertion, into the following general trigonometric formula, for computing the apparent positions of point light sources, under the process of annual light aberration:

$$\tan(\theta) = \frac{\sin(\theta')}{\cos(\theta') + \frac{v''}{c}}$$

the following set of data, related to the backward-shifted optical image of the Moon's disk:

$$\begin{aligned}
c &= 299792458 \quad \text{ms}^{-1} \\
v'' &= 29780 \qquad \text{ms}^{-1} \\
D_1 &= 31' = 1860'' \\
\theta'_1 &= 30^o \\
\theta'_2 &= 29.4833333^o
\end{aligned}$$

the calculated set of numerical results, regarding the forward-shifted optical image of the Moon's disk, under the process of moonlight aberration, is the following:

$$\begin{aligned}
\theta_1 &= 29.9971545^o \\
\theta_2 &= 29.4805324^o \\
D_2 &= \theta_1 - \theta_2 = 1859.8395848'' \\
\Delta D &= D_1 - D_2 = 0.1604''
\end{aligned}$$

in accordance with which the calculated angular diameter of the forward-shifted optical image of the Moon's disk, in this case, is less than the given angular diameter of the backward-shifted optical image of the Moon's disk, by an amount of about ***0.16*** seconds of arc.

The computed numerical values, for the difference, ΔD, between the given angular diameter of the backward-shifted optical image of the Moon's disk, D_1, and the calculated angular diameter of the forward-shifted optical image of the Moon's disk, D_2, for a number of angles, around the forward direction of Earth's orbital velocity vector, v'', relative to the gravitational center of the solar system, are summarized in the last column, in **Table #3**, below:

θ'_1	θ'_2	θ_1	θ_2	D_2	ΔD
0.0000000^o	0.5166667^o	0.0000000^o	0.5166153^o	$1859.8152570''$	$0.1847430''$
10.0000000^o	9.4833333^o	9.9990118^o	9.4823957^o	$1859.8179183''$	$0.1820817''$
20.0000000^o	19.4833333^o	19.9980536^o	19.4814352^o	$1859.8261110''$	$0.1738896''$
30.0000000^o	29.4833333^o	29.9971545^o	29.4805324^o	$1859.8395848''$	$0.1604152''$
45.0000000^o	44.4833333^o	44.9959758^o	44.4793456^o	$1859.8687651''$	$0.1312349''$
60.0000000^o	59.4833333^o	59.9950713^o	59.4784305^o	$1859.9068887''$	$0.0931111''$
90.0000000^o	89.4833333^o	89.9943085^o	89.4776421^o	$1859.9991486''$	$0.0008514''$

Table #3: ***The calculated results of ΔD, for given angles, within the range ($0^o \leq \theta'_1 \leq 90^o$).***

And similarly, for position angles, in the backward direction, with respect to Earth's orbital velocity vector, v, around the barycenter of the solar system, the numerical value of the forward-shifted angular diameter of the Moon, D_2, is obtained, in accordance with the following equation:

$$D_2 = \theta_2 - \theta_1$$

In addition, the angular amount, ΔD, by which the angular diameter of the Moon is increased, under the process of the annual aberration of moonlight, due to Earth's orbital velocity, v'', relative to gravitational center of the solar system, can be computed, through the use of this equation:

$$\Delta D = D_2 - D_1$$

As an example, upon the insertion, into the aforementioned general trigonometric formula, for computing the apparent positions of point light sources, under the process of annual light aberration, the following set of data, concerning the backward-shifted optical image of the Moon's disk:

$$c = 299792458 \ \ \text{ms}^{-1}$$
$$v'' = 29780 \qquad \text{ms}^{-1}$$
$$D_1 = 31' = 1860''$$
$$\theta_1' = 179.4833333^o$$
$$\theta_2' = 180.0000000^o$$

the computed set of numerical results, regarding the forward-shifted optical image of the Moon's disk, under the process of moonlight aberration, is the following:

$$\theta_1 = 179.4832820^o$$
$$\theta_2 = 180.0000000^o$$
$$D_2 = \theta_2 - \theta_1 = 1860.1818369''$$
$$\Delta D = D_2 - D_1 = 0.1847797''$$

in accordance with which the calculated angular diameter of the forward-shifted optical image of the Moon's disk, in this case, is larger than the given angular diameter of the backward-shifted optical image of the Moon's disk, by an amount of about ***0.18*** seconds of arc.

The computed numerical values, for the difference, $\boldsymbol{\Delta D}$, between the given angular diameter of the backward-shifted optical image of the Moon's disk, $\boldsymbol{D_1}$, and the calculated angular diameter of the forward-shifted optical image of the Moon's disk, $\boldsymbol{D_2}$, for a number of given angles, around the backward direction of Earth's orbital velocity vector, $\boldsymbol{v''}$, relative to the gravitational center of the solar system, are summarized in the last column, in **Table** ***#4***, below:

θ'_1	θ'_2	θ_1	θ_2	D_2	ΔD
109.4833333º	*110.0000000º*	*109.4779676º*	*109.9946516º*	*1860.0623951″*	*0.0623951″*
119.4833333º	*120.0000000º*	*119.4783787º*	*119.9950708º*	*1860.0916499″*	*0.0916499″*
134.4833333º	*135.0000000º*	*134.4792724º*	*134.9959752º*	*1860.1300568″*	*0.1300568″*
149.4833333º	*150.0000000º*	*149.4804430º*	*149.9971540º*	*1860.1596005″*	*0.1596005″*
159.4833333º	*160.0000000º*	*159.8413384º*	*159.9980532º*	*1860.1733479″*	*0.1733479″*
169.4833333º	*170.0000000º*	*169.4822944º*	*169.9990116º*	*1860.1818269″*	*0.1818269″*
179.4833333º	*180.0000000º*	*179.4832820º*	*180.0000000º*	*1860.1847797″*	*0.1847797″*

Table ***#4*****:** ***The calculated results of*** ΔD***, for the given angles, within the range*** $(90^o < \theta'_2 \leq 180^o)$***.***

The following points should be made explicit, with regard to the variations of the angular diameter of the Moon, under the process of moonlight aberration:

A. For position angles, within the range of $(0^o \leq \theta'_1 \leq 90^o)$, with respect to Earth's velocity vector, $\boldsymbol{v}$, relative to the CMBR, as well as Earth's orbital velocity vector, $\boldsymbol{v''}$, relative to the barycenter of the solar system, the angular diameter of the backward-shifted optical image of the Moon's disk, under the process of the travel time of moonlight, is, always, greater than the angular diameter of the forward-shifted optical image of the Moon's disk, under the process of both secular aberration and annual aberration of moonlight. And that is because, within this specific range, the position angles, with the larger forward shifts, are, always, shifted towards the position angles, with the smaller forward shifts.

B. For position angles, within the range of $(90^o < \theta'_2 \leq 180^o)$, with respect to Earth's velocity vector, $\boldsymbol{v}$, relative to the CMBR, as well as Earth's orbital velocity vector, $\boldsymbol{v''}$, relative to the barycenter of the solar system, the angular diameter of the backward-shifted optical image of the Moon's disk, under the process of the travel time of moonlight, is, always, less than the angular diameter of the forward-shifted optical image of the Moon's disk, under the process of both secular aberration and annual aberration of moonlight. And that is because, within this specific range, the position angles, with the larger forward shifts, are, always, shifted away from the position angles, with the smaller forward shifts.

C. Every position angle within the range of $(0^o \leq \theta'_1 \leq 90^o)$, except the position angle of (0^o), can be rotated (360^o), around Earth's velocity vector, $\boldsymbol{v}$, relative to the CMBR, as well as around Earth's orbital velocity vector, $\boldsymbol{v''}$, relative to the barycenter of the solar system, to form a circle, along the periphery of which, the forward shifts, under the process of the secular aberration of moonlight, and under the process of annual aberration of moonlight, remain constant and, exactly, the same.

D. Every position angle within the range of $(90^o \leq \theta'_2 \leq 180^o)$, except the position angle of (180^o), can be rotated (360^o), around Earth's velocity vector, $\boldsymbol{v}$, relative to the CMBR, as well as around Earth's orbital velocity vector, $\boldsymbol{v''}$, relative to the barycenter of the solar system, to form a circle, along the periphery of which, the forward shifts, under the process of the secular aberration of moonlight, and under the process of annual aberration of moonlight, remain constant and, precisely, the same.

E. Whenever the position of the center of the backward-shifted optical image of the Moon's disk, and the position of the center of the forward-shifted optical image of the Moon's disk, coincide with each other, at the angle of (0^o), the angular radius of the backward-shifted image increases, by an amount of about ***1.5*** seconds of arc, under the process of the travel time of moonlight; while, at the same time, the angular radius of the forward-shifted image decreases, by an equal amount, under the process of the secular aberration of moonlight, due to the velocity of the earth, $\boldsymbol{v}$, relative to the CMBR.

F. Whenever the position of the center of the backward-shifted optical image of the Moon's disk, and the position of the center of the forward-shifted optical image of the Moon's disk, coincide with each other, at the angle of (180^o), the angular radius of the backward-shifted image decreases, by an amount of about ***1.5*** seconds of arc, under the process of the travel time of moonlight; while, at the same time, the angular radius of the forward-shifted image increases, by the same amount, under the process of the secular aberration of moonlight, due to the velocity of the earth, ***v***, relative to the CMBR.

G. Each time, the position of the center of the backward-shifted optical image of the Moon's disk, and the position of the center of the forward-shifted optical image of the Moon's disk, coincide with each other, at the angle of ***(0º)***, the angular radius of the backward-shifted optical image increases, by an amount of about ***0.1*** seconds of arc, under the process of the travel time of moonlight; while, at the same time, the angular radius of the forward-shifted optical image decreases, by an equal amount, under the process of the annual aberration of moonlight, due to the velocity of the earth, ***v″***, relative to the barycenter of the solar system.

H. Each time, the position of the center of the backward-shifted optical image of the Moon's disk, and the position of the center of the forward-shifted optical image of the Moon's disk, coincide with each other, at the angle of ***(180º)***, the angular radius of the backward-shifted optical image decreases, by an amount of about ***0.1*** seconds of arc, under the process of the travel time of moonlight; while, at the same time, the angular radius of the forward-shifted optical image increases, by the same amount, under the process of the secular aberration of moonlight, due to the velocity of the earth, ***v″***, relative to the barycenter of the solar system.

5. The Apparent of Projection of Stars on the Moon's Disk:

As indicated earlier, no calculation of the apparent projection of stars, on the Moon's disk, is, theoretically, possible, in all of the cases, in which the velocity of moonlight and the velocity of starlight are either equal, or assumed to be equal.

On the basis of the ballistic assumption, however, it's possible, in principle, for the computed predictions of the apparent projection of stars, on the Moon's disk, to be carried out, in all cases, in which the velocity of moonlight and the velocity of starlight have either different measured numerical values, or different given numerical values, from each other.

There are two major cases, which have to be taken into consideration, in the current discussion:

A.) The case, in which the velocity of moonlight is greater than the velocity of starlight:

Let moonlight travel, from the Moon to Earth, at the muzzle velocity of light, ***c***.

And let the star, to be occulted by the Moon, recede, directly, from the solar system, at the velocity, ***v′***; and hence, the velocity of its starlight is equal to (***c - v′***).

And, let Earth travel with the velocity, ***v***, relative to the CMBR.

And it follows, therefore, that the secular aberration of moonlight, due to Earth's motion, relative to the CMBR, can be computed through the use of the following equation:

$$\sin\left(\Delta\theta_M\right)=\frac{v}{c}\sin\left(\theta_M\right)$$

where $\boldsymbol{\theta_M}$ is the apparent position of the Moon, as observed from Earth, at the time of reception.

While, the secular aberration of starlight, due to Earth's motion, $\boldsymbol{v}$, relative to the CMBR, can be calculated through the use of the following equation:

$$\sin\left(\Delta\theta_S\right)=\frac{v}{c-v'}\sin\left(\theta_S\right)$$

where $\boldsymbol{\theta_S}$ is the apparent position of the star, as observed from Earth, at the time of reception.

And accordingly, the angle of the apparent projection of the receding star, on the Moon's disk, $\boldsymbol{\Delta\theta'}$, can be obtained, on the basis of the ballistic assumption, by using this equation:

$$\Delta\theta'=\Delta\theta_S-\Delta\theta_M$$

where $\boldsymbol{\Delta\theta_S}$ is the secular aberration of starlight; and $\boldsymbol{\Delta\theta_M}$ is the secular aberration of moonlight, due to Earth's velocity, $\boldsymbol{v}$, relative to the CMBR, respectively.

The above computed angle, $\boldsymbol{\Delta\theta'}$, varies, periodically, by a small amount, $\boldsymbol{\Delta\theta''}$, due to the annual aberration of starlight and the annual aberration of moonlight, caused by Earth's orbital velocity, $\boldsymbol{v''}$, relative to the barycenter of the solar system, both of which can be calculated by using the following trigonometric formulas:

$$\sin\left(\Delta\theta''_M\right)=\frac{v''}{c}\sin\left(\theta''_M\right)$$

where $\boldsymbol{\theta''_M}$ is the apparent position of the Moon, with respect to Earth's orbital velocity vector, $\boldsymbol{v''}$, at the time of reception.

$$\sin\left(\Delta\theta''_S\right)=\frac{v''}{c-v'}\sin\left(\theta''_S\right)$$

where $\boldsymbol{\theta''_S}$ is the apparent position of the receding star, with respect to Earth's orbital velocity vector, $\boldsymbol{v''}$, at the time of reception.

And, therefore, the computed apparent projection of the receding star, on the Moon's disk, is $\Delta\theta'$:

$$\Delta\theta'' = \Delta\theta''_S - \Delta\theta''_M$$

where $\Delta\theta''_S$ is the annual aberration of starlight; and $\Delta\theta''_M$ is the annual aberration of moonlight, due to Earth's orbital velocity, v'', relative to the barycenter of the solar system, respectively.

In the case of the frequently occulted bright star, Aldebaran (Alpha Tauri), for example, by inserting into the above equations, the following velocity values:

$$v' = 54260 \text{ ms}^{-1}$$
$$c = 299792458 \text{ ms}^{-1}$$
$$v = 371000 \text{ ms}^{-1}$$
$$v'' = 29780 \text{ ms}^{-1}$$

these two maximum numerical values, for $\Delta\theta'$ and $\Delta\theta''$, respectively, can be obtained:

$$\Delta\theta' = 46.21 \text{ milliarcseconds}$$
$$\Delta\theta'' = 3.71 \text{ milliarcseconds}$$

where $\Delta\theta'$ denotes the angular amount of the apparent projection of the receding star, on the Moon's disk, due to the secular aberration of moonlight and the secular aberration of starlight, caused by Earth's velocity, v, relative to the CMBR; and $\Delta\theta''$ denotes the angular amount of the apparent projection of the same receding star, on the Moon's disk, due to the annual aberration of moonlight and the annual aberration of starlight, caused by Earth's velocity, v'', relative to the gravitational center of the solar system.

The computed numerical values of the apparent projection of a number of receding stars, occulted, frequently, by the Moon, as calculated in accordance with the ballistic assumption, in the case of Earth's velocity, v, relative to the CMBR, and, in the case of Earth's orbital velocity, v'', relative to the barycenter of solar system, respectively, are listed in the last column, in **Table #5**, below.

Stars (*Bayer designation*)	*+v* (*m/s*)	*Δθ′ ±Δθ″* (*milliarcseconds*)
Aldebaran (α Tau)	*54260*	*46.21 ± 3.71*
Rho Leonis (ρ Leo)	*42000*	*35.77 ± 2.87*
Gamma Tauri (γ Tau)	*38700*	*32.96 ± 2.65*
71 Tauri (71 Tau)	*38300*	*32.61 ± 2.62*
Mu Piscium (μ Psc)	*34190*	*29.11 ± 2.34*
60 Cancri	*25380*	*12.61 ± 1.73*
119 Tauri (CE Tau)	*23750*	*20.22 ± 1.62*
Electra (17 Tauri)	*10900*	*9.28 ± 0.74*
Lambda Ceti (λ Cet)	*10200*	*8.69 ± 0.70*
Regulus (α Leo)	*6300*	*5.36 ± 0.43*
Beta Virginis (β Vir)	*4100*	*3.49 ± 0.28*
16 Tauri	*2900*	*2.47 ± 0.20*
Eta Virginis (η Vir)	*2300*	*1.96 ± 0.18*
Spica (α Vir)	*1000*	*0.85 ± 0.07*
Nu Piscium (ν Psc)	*760*	*0.65 ± 0.05*

Table #5: *The computed apparent projection of several receding stars, on the Moon's disk.*

***B.*) The case, in which the velocity of moonlight is less than the velocity of starlight:**

Let moonlight travel, from the Moon to Earth, with the muzzle velocity of light, c.

And let the star, to be occulted by the Moon, approach, directly, the solar system, at the velocity, v'; and hence the velocity of its starlight is equal to $(c + v')$.

And, let Earth travel with the velocity, v, relative to the CMBR.

And it follows, therefore, that the secular aberration of moonlight, due to Earth's motion, relative to the CMBR, can be computed through the use of the following equation:

$$\sin(\Delta\theta_M) = \frac{v}{c}\sin(\theta_M)$$

where θ_M is the apparent position of the Moon, as observed from Earth, at the time of reception.

While, at the same time, the secular aberration of starlight, due to Earth's motion, relative to the CMBR, can be calculated through the use of the following equation:

$$\sin(\Delta\theta_S) = \frac{v}{c+v'}\sin(\theta_S)$$

where θ_S is the apparent position of the star, as observed from Earth, at the time of reception.

And accordingly, the apparent projection of the approaching star, on the Moon's disk, $\Delta\theta'$, can be obtained, on the basis of the ballistic assumption, by using this equation:

$$\Delta\theta' = \Delta\theta_M - \Delta\theta_S$$

where $\Delta\theta_S$ is the secular aberration of starlight; and $\Delta\theta_M$ is the secular aberration of moonlight, due to Earth's velocity, v, relative to the CMBR, respectively.

The above computed angle, $\Delta\theta'$, varies, periodically, by a small amount, $\Delta\theta''$, due to the annual aberration of starlight and the annual aberration of moonlight, caused by Earth's velocity, v'', relative to the barycenter of the solar system, which can be calculated by using the following formulas:

$$\sin(\Delta\theta''_M) = \frac{v''}{c}\sin(\theta''_M)$$

where θ''_M is the apparent position of the Moon, with respect to Earth's orbital velocity vector, v'', at the time of reception; and:

$$\sin\left(\Delta\theta''_S\right) = \frac{v''}{c+v'}\sin\left(\theta''_S\right)$$

where $\boldsymbol{\theta''_S}$ is the apparent position of the approaching star, with respect to Earth's orbital velocity vector, $\boldsymbol{v''}$, at the time of reception.

And, consequently, the angular difference, $\boldsymbol{\Delta\theta''}$, between the annual aberration of starlight and the annual aberration of moonlight, due to Earth's orbital velocity, $\boldsymbol{v''}$, relative to the barycenter of the solar system, can be calculated by using the following formula:

$$\Delta\theta'' = \Delta\theta''_M - \Delta\theta''_S$$

where $\boldsymbol{\Delta\theta''_S}$ is the annual aberration of starlight; and $\boldsymbol{\Delta\theta''_M}$ is the annual aberration of moonlight, due to Earth's orbital velocity, $\boldsymbol{v''}$, relative to the barycenter of the solar system, respectively.

In the case of the frequently occulted star, Beta Capricorni (β Cap), for instance, by inserting into the above equations, the following velocity values:

$$v' = -19000 \quad \text{ms}^{-1}$$
$$c = 299792458 \quad \text{ms}^{-1}$$
$$v = 371000 \quad \text{ms}^{-1}$$
$$v'' = 29780 \quad \text{ms}^{-1}$$

these two maximum numerical values, for $\boldsymbol{\Delta\theta'}$ and $\boldsymbol{\Delta\theta''}$, respectively, can be obtained:

$$\Delta\theta' = 16.18 \quad \text{milliarcseconds}$$
$$\Delta\theta'' = 01.30 \quad \text{milliarcseconds}$$

where $\boldsymbol{\Delta\theta'}$ denotes the angular amount of apparent projection of the approaching star, on the Moon's disk, caused by the secular aberration of moonlight and the secular aberration of starlight, due to Earth's velocity, $\boldsymbol{v}$, relative to the CMBR; and $\boldsymbol{\Delta\theta''}$ denotes the angular amount of apparent projection of the same approaching star, on the Moon's disk, caused by the annual aberration of moonlight and the annual aberration of starlight, due to Earth's velocity, $\boldsymbol{v''}$, relative to the barycenter of the solar system.

The apparent projection of a number of approaching stars, occulted, frequently, by the Moon, as calculated on the basis of the ballistic assumption, is listed in **Table *#6***, below.

Stars ***(Bayer designation)***	***-v (m/s)***	***Δθ′ ±Δθ″ (milliarcseconds)***
Beta Capricorni (β Cap)	***-19000***	***16.18 ± 1.30***
Theta Aquarii (θ Aqr)	***-13770***	***11.72 ± 0.94***
Lambda Aquarii (λ Aqr)	***-10460***	***8.91 ± 0.71***
Lambda Geminorum (λ Gem)	***-7400***	***6.3 ± 0.51***
Sigma Librae (σ Lib)	***-4200***	***3.58 ± 0.29***
Antares (α Scorpii)	***-3400***	***2.89 ± 0.23***
Theta Ophiuchi (θ Oph)	***-2000***	***1.7 ± 0.14***

Table #6: ***The computed apparent projection of several approaching stars, on the Moon's disk.***

With regard to the computed angles, $\Delta\theta'$ and $\Delta\theta''$, on the basis of the ballistic assumption, as defined within the framework of the elastic-impact ballistic theory, for the apparent projection of stars, on the Moon's disk, in **Table #5** and **Table #6**, above, the following aspects should be pointed out:

I. The maximum numerical values of $\Delta\theta'$ have been calculated, for the position of 90^o, with respect to Earth's velocity, v, relative to the CMBR, for each star.

II. The maximum numerical values of $\Delta\theta''$ have been calculated, for the position of 90^o, with respect to Earth's orbital velocity, v'', relative to the barycenter of the solar system, for each star.

III. For each star, the numerical value of $\Delta\theta''$ is added to the numerical value of $\Delta\theta'$, when Earth's orbital velocity vector, v'', relative to the barycenter of the solar system, and Earth's velocity vector, v, relative to the CMBR, are pointing in the same direction.

IV. For each star, the numerical value of $\boldsymbol{\Delta\theta''}$ is subtracted from the numerical value of $\boldsymbol{\Delta\theta'}$, when Earth's orbital velocity vector, $\boldsymbol{v''}$, relative to the barycenter of the solar system, and Earth's velocity vector, $\boldsymbol{v}$, relative to the CMBR, are pointing in the opposite direction of each other.

V. The apparent position, $\boldsymbol{\theta_S}$, for each star, with respect to Earth's velocity vector, $\boldsymbol{v}$, relative to the CMBR, is constant and unchanging with time; while, by contrast, the apparent position, $\boldsymbol{\theta''_S}$, with respect to Earth's orbital velocity vector, $\boldsymbol{v''}$, relative to the barycenter of the solar system, is changing, continuously, in a periodic manner, with time.

VI. The great circle, around Earth's velocity vector, $\boldsymbol{v}$, relative to the CMBR, divides the celestial sphere into two equal halves: A forward hemisphere and backward hemisphere.

VII. In the forward hemisphere, receding stars are projected on the upper limb of the Moon; while approaching stars are projected on the lower limb of the Moon.

VIII. In the backward hemisphere, approaching stars are projected on the upper limb of the Moon; while receding stars are projected on the lower limb of the Moon.

6. Concluding Remarks:

As established, in the present discussion, within the framework of the classical wave theory, neither the aberration of moonlight, nor the aberration of starlight can be utilized, in any manner, to explain away the apparent projection of occulted stars, on the Moon's disk, since its basic theoretical assumption, about the propagation of light, implies, necessarily, that the velocity of moonlight and the velocity of starlight, in vacuum, are the same and equal to each other, under all conceivable circumstances.

And consequently, the discarded Airy's hypothesis of refrangibility, according to which whenever starlight is less refrangible than moonlight, the apparent projection of the star takes place, at the upper limb of the Moon; and, conversely, whenever moonlight is less refrangible than starlight, the apparent projection of the star takes place, at the lower limb of the Moon [***Ref. #1.b***], cannot be made viable, once again, as suggested at the start of the current investigation, by merely combining its computed predictions with similar predictions, calculated, on the basis of the aberration of starlight and the aberration of moonlight, within the framework of the classical wave theory.

Nonetheless, it seems, in principle, it's still possible, for the highly intuitive hypothesis of refrangibility, to be made workable, within the framework of the classical wave theory, by combining the effect of Airy's refrangibility with the effect of Fresnel's Drag Coefficient [***Ref. #20***], inside moving air currents in Earth's atmosphere, during lunar occultations; although the complicated nature of such moving air currents makes any attempt at investigating the combined effect, in this case, extremely, difficult.

In regard to the second type of observations, labeled as *'Class B'*, in Airy's report; i.e., the occasional hanging of stars, to the Moon's limb, at the start of various lunar occultations **[*Ref. #1.a*]**, it's quite possible to be accounted for, entirely, on the basis of the assumption of constant speed of light, as well as on the basis of the assumption of ballistic speed of light, by, simply, taking into account, the rotational velocity of Earth, around its axis, at the location of the observatory, in question, along with the orbital velocity of the Moon, around the Earth, at the instant of occultation.

And that is, evidently, because the rotational velocity of Earth varies with the geographical latitude, in accordance with this equation:

$$v_L = 465.278 \times \cos(\lambda) \ \text{ms}^{-1}$$

where v_L is Earth's rotational velocity, at the geographical latitude, in question; ***(465.278 ms⁻¹)*** is the rotational velocity of Earth, at the equator; and λ is the angle of the geographical latitude.

And since the average orbital velocity of the Moon, around Earth, v_M, is equal to ***(1022 ms^{-1})***, it can be shown that it's possible, for the vector sum of ***[v_L & v_M]***, at certain locations, at certain times, to keep stars hanging to the Moon's limb, for a short period of time, at the start of various lunar occultations.

Moreover, it's, also, possible, for the rapidly varying vector sum of ***[v_L & v_M]***, on certain occasions, to make an occulted star reappear, for a short interval of time, after being disappeared behind the Moon's disk.

And, once again, is it, somehow, conceivable, in principle, at least, for the orbital velocity of the Earth-Moon system, ***v''***, around the barycenter of the solar system, to be assigned, somehow, to moonlight, and for the velocity, ***v***, relative to the CMBR, to be assigned to moonlight as well as to starlight, with respect to the reference frame of the moving of Earth, on the assumption of constant speed of light, as defined within the framework of the classical wave theory, as well as on the assumption of ballistic speed of light, as defined within the framework of the elastic-impact ballistic thoery, respectively?

Clearly, neither the orbital velocity of the Earth-Moon system, ***v''***, around the barycenter of the solar system, nor the velocity of Earth, ***v***, relative to the CMBR, can be assigned, even in principle, to moonlight, starlight, or any other type of light, on the basis of the assumption of constant of light, as defined within the framework of the classical wave theory.

And as a result, there is a very minute mismatch, between the backward shift of the optical image of the Moon, due to the travel time of moonlight, from the Moon to Earth, and the forward shift of the optical image of the Moon, due to moonlight aberration, as computed in accordance with the assumption of constant speed of light, as defined within the framework of the classical wave theory, in the case of ***v***, and in the case of ***v''*** as well. And that is because Earth travels a bit farther, during the slightly longer travel time of moonlight, from the Moon to Earth, than the standard ***[d/c]***, in the stationary case.

By comparison, however, the travel time of moonlight, from the Moon to Earth, as calculated in accordance with the assumption of ballistic speed of light, as defined within the framework of the elastic-impact ballistic theory, in the case of ***v***, and in the case of ***v''***, is, always, equal to the travel time of moonlight, ***[d/c]***, as, if the Moon and Earth remain, all the time, at rest, relative to each other. And consequently, there can be no mismatch, between the backward shift of the optical image of the Moon, due to the travel time of moonlight, from the Moon to Earth, and the forward shift of the optical image of the Moon, due to moonlight aberration, on the basis of the ballistic assumption of the speed of moonlight and the speed of starlight.

But, more precisely, why does the velocity resultant of light, ***[c']***, have to be replaced with the muzzle velocity of light, ***[c]***, in the general equation, for moonlight aberration, in the case of the velocity, ***v***, relative to CMBR, and in the case of the orbital velocity, ***v''***, relative to the barycenter of the solar system, for all calculations, on the basis of the the assumption of ballistic speed of light?

It's, definitely, because, in all cases, in which the light source and the light receiver are moving with the same speed in the same direction, the muzzle velocity of light, ***[c]***, must replace the velocity result of light, ***[c']***, in the general ballistic formula, for computing light aberration; since the velocity component, ***v***, in the velocity resultant ***[c']***, for incident light, and the velocity of the light receiver, ***v***, cancel each other out, during the time of reception; leaving, only, the measurable velocity component ***[c]*** of the velocity resultant, in question, within the aforementioned formula, for light aberration.

In conclusion, therefore, on the basis of the ballistic assumption, it's possible, for occulted stars, as shown earlier, to be projected, on the Moon's disk, if, and only, if the peculiar velocity of each occulted star, with respect to Earth, is unequal to zero. And since the peculiar velocities of stars, regularly occulted by the Moon, do not exceed ***100*** km/s, the computed differences, between moonlight aberration and starlight aberration, in this regard, are, relatively, small; and hence, although the path of the apparent projection of a star, on the Moon's disk, can be greater than a few seconds of arc, its maximum angular value, in the transverse direction, has to be, in the cases under discussion, less than ***50*** milliseconds of arc. And that is, probably, one of the main reasons why almost all of the reported observations of apparent projection of stars, on the Moon's disk, were being carried out, exclusively, through the employment of very large telescopes, such as those telescopes being, routinely, used by G. B. Airy, C. Messier, and other astronomers, throughout the 18th century and 19th century.

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