The Relationship of Planetary Orbits in the Solar System

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1. Introduction

The Titius-Bode law [1] was proposed in the 18th century as a formula for describing planetary distances in the Solar System. Although this law today holds only historical significance due to a lack of physical basis and relatively large prediction errors, particularly for outer planets, its mention serves as a useful introduction to the topic of orbital element relationships.

The aim of this paper is to present a simple and accurate representation of the semi-major axis ratios of planets in the Solar System, using Mars as a reference point and mathematical constants associated with π\pi. The methodology employed is inspired by the principles in *The Theory of Natural Philosophy* by Ruđer Bošković [2] and the *Theory of Unity of the Whole and Its Parts* [3], further emphasizing the importance of these theoretical approaches in modern understanding of nature.

2. Relationships

The Titius-Bode law [1] is an empirical formula describing planetary semi-major axes in the Solar System:

$$a_n = 0.4 + 0.3 * 2^n \qquad (1)$$

Where:

- a_n : semi-major axis of a planet from the Sun (in astronomical units, AU),
- n: ordinal number of the planet (Mercury corresponds to $n=-\infty$, Venus to $n=0$, Earth to $n=1$, Mars to $n=2$, etc.).

This law provides approximate values, often with errors exceeding several percent [1]. Therefore, this paper proposes an alternative approach with coefficients ***k***, which represent ratios of the planets' semi-major axes relative to reference orbits of Earth and Mars:

Table: Orbits of Solar System Planets

Planet	*Semi-major axis* *a [m]*	*Coefficient k* *k*	k= k=	*az=k*a(Earth)* *am=k*a(Mars)*	*Re%* *Re%*
Mercury	**5,79092270E+10**	**(1+(2π)^1/4)^-1**	**0,38711**	**5,791115E+10**	***0,003***
Venus	1,08209475E+11				
Earth	**1,49598023E+11**	***1***	1,0000	**1,495980E+11**	***0,000***
Mars	**2,27943824E+11**	***1***	**1,0000**	**2,279438E+11**	***0,000***
Jupiter	**7,78340821E+11**	***2+2^0.5***	**3,4142**	**7,782489E+11**	***-0,012***
Saturn	**1,43352698E+12**	***2π=***	**6,2832**	**1,432213E+12**	***-0,092***
Uranus	**2,87524663E+12**	***4π=***	**12,5664**	**2,864427E+12**	***-0,376***
Neptune	**4,49506083E+12**	***2π^2=***	**19,7392**	**4,499431E+12**	***0,097***
Pluton	***5,90637627E+12***	***4π^2=***	**39,4784**	**5,905893E+12**	***-0,008***

Analysis and Implications

1. **Accuracy for Outer Planets:** For outer planets (from Jupiter onward), coefficients ***k*** provide values with relative errors (***Re***) below 0.1%. This demonstrates the consistency of the relationships based on coefficients associated with ***π\pi***.
2. **Formula Differences:** A clear distinction is observed between inner and outer planets.
3. **Venus**: A reference semi-major axis tied to π\piπ that would achieve the same level of accuracy could not be found, though hints of a connection with $\boldsymbol{lg_2(3)}$ exist.
4. **Jupiter**: The relationship is linked to the constant 2.
5. **Saturn, Uranus and Neptune:** form an orderly sequence of relationships related to ***π\pi***.
6. **Pluto:** is also included in the Table because it continues the sequence of the previous ones but not towards Mars but towards the Earth.
7. **Solar System Dichotomy:** Inner planetary orbits are relatively close, while outer planetary orbits progressively widen with distance from the Sun. Coefficients ***k*** explain this dichotomy, while orbital resonances are a consequence [4].
8. **Physical Interpretation:** Coefficients ***k*** point to orbital resonances, which contribute to the stability of the Solar System and shape the specific characteristics of planetary orbits [2].
9. **Broader Application:** The defined coefficients can be applied to analyze and compare orbital structures in other stellar systems, providing new insights into orbital mechanisms.

3. Conclusion

The proposed empirical relationships for the semi-major axes of planets in the Solar System provide a more precise alternative to the classical Titius-Bode law. The results of this study highlight the importance of utilizing mathematical constants, such as ***π***\pi, and ratios based on reference orbit units to better understand the dynamics of the Solar System.

"Semi-major axes, as virtual indicators, exhibit clear mathematical relationships that real aphelia and perihelia do not possess. Notably, this seems to be a general rule that applies across various structures, including quantum physics, where virtual points are often utilized."

A particular advantage of using Mars' semi-major axis as a reference value lies in the accuracy of the results obtained, especially for the outer planets. However, for the inner planets (Mercury and Venus), no suitable reference semi-major axis was found that would yield the same level of precision. For the outermost object, Pluto, Earth proved to be a more suitable reference value.

Future research could focus on:

- Applying these methods to exoplanetary systems,
- Studying the impact of gravitational resonances on planetary orbit stability,
- Conducting a more detailed analysis of the effects that smaller bodies, such as satellites and asteroids, have on planetary orbit dynamics.

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References

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[2] Boscovich, R.: 1758, *Philosophiae naturalis theoria redacta ad unicam legem virium in natura existentium*, Vienna (first edition); 1763, Venice (second edition); 1922 and 1966, *A Theory of Natural Philosophy*, Open Court, London, and The Massachusetts Institute of Technology, M.I.T. Press, Cambridge (respectively); 1974, *The Theory of Natural Philosophy Reduced to One Unique Law of the Forces Existing in Nature* (bilingual: Latin and Croatian), Liber, Zagreb.

[3] *The Theory of Unity of the Whole and its Parts*, viXra.org e-Print archive, viXra: 1703.0152.

[4] https://en.wikipedia.org/wiki/Orbital_resonance