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A little forgotten story: Einstein, Burali Forti and Boggio.

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1. - Many years ago, when I was a student, I devoted myself for some time at the collection of used scientific books: I had discovered that Naples was a open-air mine, with its booksellers in the basements of the streets close to the University. My father was generous; and he helped me a long way relative, Mario Pascal, the rational mechanic son of Ernesto, famous analyst, who gave me bibliographic news (often for knowledge of the authors of those "sacred" texts. I was ignorant, but tireless reader or bibliomaniac. Among the many, I found some books that intrigued me a lot (and that today are found only in libraries of old institutes): a *Space and Time (according to the views of A. Einstein)* by Guido Castelnuovo, published by Zanichelli in 1923 – by which many years later I had the honor of writing a short preface, on request of the Enriques, for the anastatic reprint (1981)-; essay, *Espaces courbes, critique of relativity*, by Cesare Burali Forti and Tommaso Boggio, published by Andrea Viglongo of Turin in 1924 (perhaps better known as "STEN" - Printing *Company Editrice Nazionale)*; in addition, by Tullio Levi Civita, the Issues of classical and relativistic mechanics (Zanichelli, on conference texts held in Spain in January-February 1921), as well as his lectures Fundamentals of relativistic mechanics, written by Enrico Persico, the my teacher (Zanichelli, 1928); by Roberto Marcolongo, *Relativity*, published with Principato (Messina) in 1921; finally, by Augusto Occhialini (father of the unforgettable Giuseppe-Beppo) *reason and spirit of relativity*, published with Le Monnier (Florence) in 1922 and based on lectures promoted by none other than the Legal Institute (!) of the regia Unversity of Sassari. I want to talk especially about the first two of these books the

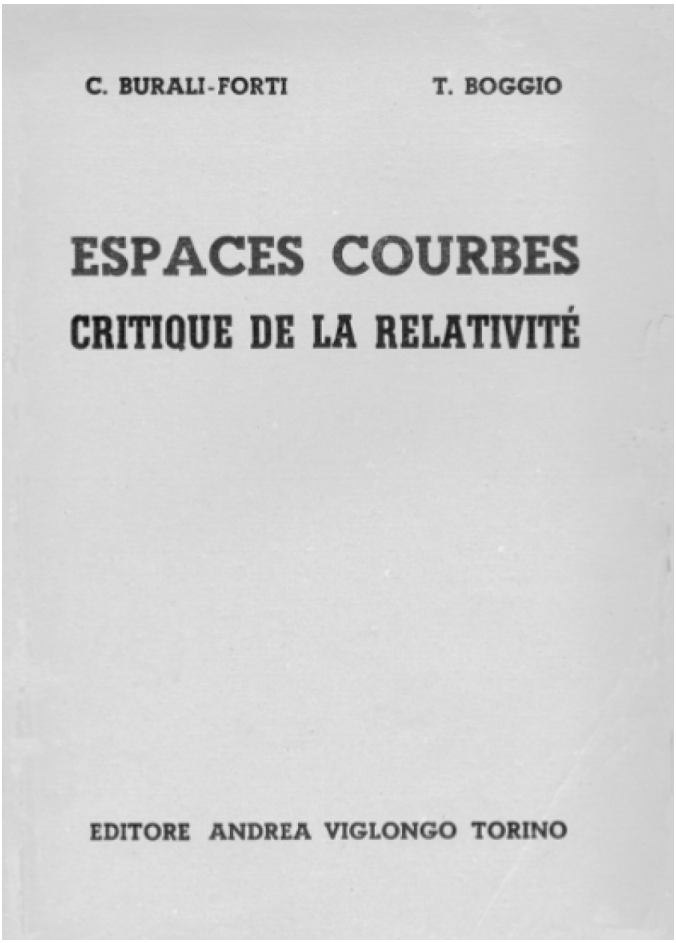

Fig. 1. - Cover of the book by C. BURALI FORTI and T. BOGGIO published by Viglongo (National Publishing Printing Company) in 1924.

curiosity came to me from two different ideas: the first (the relative one in Castelnuovo), is that his text appeared as an essay of "high disclosure", of those that publishers have long rejected because the author gives himself some formula. The second, since the "preface general", appeared as a poisonous libel against Albert's ideas Einstein. Now, thanks to Mario Pascal's patient explanations, I learned immediately that Guido Castelnuovo was a great of mathematics of the '900 and that his dissemination effort was the best that could be produced at the time (good even much later, when the anastatic came out); but even of Burali Forti and Boggio you could not speak badly: the first remains headed, in the history of logic, an important antinomy ([1]), a second it can be recognized to have been a

([1]) The antinomy enunciated in 1897 by Burali Forti, appreciated by Bertrand Russell, it is about the nature of the "ordinal maximum"; who was interested in this problem of logic could consult the *History of Logic* by Corrado Mangione and Silvio Bozzi, Garzanti 1993

rational mechanic of value (when the holders of chairs of rational mechanics had leading roles in the Italian Academy). Therefore, the contents of two books were also supposed to be symptoms of a contrast in the community scientific of the time. I became curious and read the invective as well by Boggio and Burali, filing the thing in my head for more than 50 years. And here is that the anniversary of 2005, 100 years of relativity, is a good excuse to exhume those opinions, trying to understand why the initial path of relativity was so tiring even beyond the simple violations of common sense, that is, of an identified realism with the ideas of Newton work).

Among mathematicians, geometry seems to be the most natural contact and historically more remote with the surrounding world. Space is intangibly around us, so that one can have a representation of it abstract mental, as they like; time is already approaching to the cases of life, because it marks evolution and phenomena: mechanics rationals use complex kinematic notions to "geometrize it" as far as they can ("geometry of motion" will be called, more nobly, the kinematics of Gian Antonio Maggi ([2])). The grande Feli Klein Klein will take geometry to extraordinary levels ([3]), also of abstractionism, overcoming intuitive conceptions; and the Italians Gregorio Ricci Curbastro and Tullio Levi Civita will build a differential geometry, at first met with skepticism, which will allow results unprecedented in any space and will be indispensable for the Einstein's formulation of general relativity.

The boundary line on which mathematicians and physicists separate is "matter". Matter is somewhat far from the rigor of structures symbolic, it is "dirty": space contains it, time accompanies it, but they are containers and markers that are clean, intangible, unrelated to their contents. Perhaps, any scientist of the 800 would have thought that if even matter had reached its poorest and most degraded forms, the space and time that have always contained and accompanied it the space

([2]) G. A. Maggi, *Movement Geometry*, Zanichelli, 1913 (three editions, until 1927)

([3]) I. M. Yaglom, F. Klein and S. Lie, Birkhauser, 1988

and time that have always contained and accompanied it they would have kept their properties unchanged. Absolute space and time. Euclidean, as their simplest and most intuitive model, good for every point where there is an observer. And here in the bastion of certainties comforted by common sense, comes this German Jew, Albert Einstein, who claims that matter is "a measure of all things". An extraordinary novelty for Guido Castelnuovo; a horror for Cesare Burali Forti and Tommaso Boggio. Such a thing they do not even name it, pouring their wits on a defense of the splendor of geometry.

Castelnuovo has no repugnance for material bodies. His book takes the moves from Galilei, also to claim to this exceptional character, rightly, the dual role is the "father" of relativity this special statement, already clearly foreshadowed in the famous speech of the ship in the Dialogue over the two greatest systems of the World ([4]), which that of the father of relativity called general with the utterance of the "Equivalence principle" at least for the motion of gravity in proximity of the earth's surface (independence from the mass of acceleration of fall: legendary experiment of the Tower of Pisa). Castelnuovo he appears enthusiastic about the new ideas with which Einstein embraces the whole physics then known including extreme conditions, up to the borders marked by the speed of light and the immensity of the cosmos. It's not certain the use of non-Euclidean geometries to embarrass him, indeed: that the physics uses notions that mathematicians have already thought of just because thinkable, reassures him of the importance of mathematics itself.

The triggering cause of the work of Burali Forti and Boggio seems to be an article by Carlo Somigliana, another illustrious name of physics-mathematics Italian, pure "understood in a classic way, [...] indifferent to the rising and to develop [...] of new relativistic and quantum theories" ([5]). The article, *On the Fundamentals of Relativity*, appeared in "Scientia" (the magazine of Federigo Enriques, Eugenio Rignano and Paolo Bonetti he had provoked the ire of Benedetto Croce and Giovanni Gentile for "theft of philosophy"), vol. XXXIV,

([4]) Scrpra Dialogues the two Greatest Systems of the World, Day Two

([5]) Cf. the biography written by Bruno Barberis in *La Facolta di Scienze MFN of Turin, 1848-1998*, Tomo II ("The Teachers"), edited by Silvia Roero, Subalpine Deputation of History Patria, Turin 1999.

1923. It seems that Somigliana was convinced (and he also wrote it in a note of the Reports Lincei, series V, vol. XXXI, 1922) that "to the relativist explanation of a phenomenon, we can associate another, net the basics of theory of relativity are very unsafe". With these premises, Burali and Boggio rely on the illustrious Augusto Murri who, in his Lectures of the Medical Clinic of 1906 he wrote: "There are a number of people who he enjoys believing the incredible, indeed he feels happy only when he fails to understand nothing of what he believes". Their wise man, who from here takes the however, it is written with notations that today appear very "heavy" and, at least for physicists, obsolete and obscure (from the point of view of identification the underlying symmetries).

Affiliation with the great Giuseppe Peano seems to play a role decisive: Burali and Boggio, as if their goal were definitions of a formal language, from the first pages (and there are it will be 250!) they point out and eviscerate notions of which it is difficult to he senses relevance to the proposed goal: unmasking absurdity logic of relativity. It will have to be studied intensively for 212 pages the "absolute computation of general homographs in n-dimensional spaces", which serves them to "eliminate" the elements of the ordinary ([6]) computation absolute differential (coordinates, covariance and contravariance - of which "prove complete uselessness" -, symbols of Riemann and of Christoffel, etc.) before arriving at a chapter V, *Critique de la relativité*. Criticism is deliberately (and correctly) called "mathematical", having already been given a "physical" criticism in the preface (according to the ideas of Somigliana) and not intending (God scampi and liberi) give it a "philosophical" critique. Only notes lighten sometimes the text bringing back verses by Renato Fucini indeed often tasty ([7]).

([6]) Of the unnamed (with all due respect to bibliographic rigour!) Gregorio Ricci Curbastro and Tullio Levi-Civita, so dear to Einstein.

([7]) For example, from *The Creation of Man:*

"I know from a good source that the Creator /after making the worms and the firmament /decided to to be a man in a moment / of a bad mood / but when he had done it / and, beautiful alive, to remember it he saw, / he said to himself dancing like crazy / - Rascal world, at least now we laugh!"

Burali and Boggio did not doubt for a moment that those verses in the near future, they may turn against the it would soon appear, pure pedantry: theirs. Of course, you can ask if, dropped the oblivion on these criticisms and acquired by now the content of special and general relativity, it is worth taking still with Boggio and Burali for their slip. I think so, what worth it, for a reason that perhaps does not yet collect its due attention in the history of mathematics. Everyone knows the story of the Hamilton quaternions: it is a story that can be summarized as a "clash of notations" between Hamilton's (succumbing) quaternions and of his few followers and vector calculus, now widely spread and winning, with use of matrices representing comfortably non-objects commutative. Something similar had happened in Italy: Tullio Levi Civita and Gregorio Ricci Curbastro were obviously not very loved by some of their colleagues, such as Boggio and Burali, who had very focused on a vector calculus (already called " vector calculus italian" or "all'italiana") which used hybrid notations, much less effective than those of tensor calculus. What Boggio and Burali stated in addition, overdoing and pulling Einstein into the ball, was that their calculation had a more solid character of truth than that of opponents, eliminating conventional elements of geometry and using exclusively absolute concepts, independent of explicit coordinates. Here, it seems to me of extreme interest I dealt with the history of the development of notations: the discovery of notations of high "intelligibility a vista" must be counted among the great achievements of mathematics. The example of the equations of Maell like those used by Minkoski symmetries of their deep structure, is, in this regard, striking.

2. - Of course, Boggio and Burali were not alone: among the physicists, here were opponents of rank, such as Michele La Rosa and Quirino Majorana (the uncle of the more famous Ettore) and others less prominent. As stated with coupons reasons, some of these physicists produced, with their experiments intended to disprove relativity, among the most accurate experimental proofs of it. But here I

would rather speak again of mathematicians than gods Italian physicists, especially remembering Roberto Marcolongo, Attilio Palatini and Carlo Cattaneo. A case of exceptional intellectual honesty is undoubtedly that of Roberto Marcolongo: despite being one of the authors, with Burali Forti, of the "Italian vector calculus", in his treatise *Relativities* of 1921 already cited gives full credit to the calculation absolute differential even if it tries to represent some results through the favorite homographies. So too, call the celebrities "Lorentz" and not "Voigt-Lorentz" coordinate transformations as Somigliana had suggested for the pure purpose of insinuating that, without use innovative relativistic concepts, analogous transformations were been introduced in a strictly classical context by the German Woldemar Voigt based on certain elastic properties of a "luminiferous aether". In short, Marcolongo is an excellent example, perhaps not enough recognized, of high-quality physicist-mathematician ([8,9]).

Remaining to the strict interests for relativity, especially the one called General, Attilio Palatini, professor at Pavia, is one of the few physicists' Italian mathematicians who still figure, as the author of results strongly innovative, in international treaties ([10]): was in fact the author (in 1919) of the variational Hamiltonian formulation of the equations of Einstein. Carlo Cattaneo, a professor in Rome, also contributed in more recent times (1960-1970) with important works on hydrodynamics in general relativity. Finally, Tullio cannot be ignored Regge (theoretical physicist, in Turin) who came out with one remaining work in 1961 then among the classics of general relativity ([11]).

3. - Obviously, the two central figures in the relationship between athematics Italian and Einstein's general relativity are Gregorio Ricci Curbastro and Tullio Levi-Civita: as we have seen, they are also the main initial target of colleagues . I leave them aside

([8]) Among his pupils, Mario Pascal and Antonio Carrelli, in Naples.

([9]) The figure of Marcolongo is well contextualized in *Italian mathematics after the Unification (The years between the two world wars)* edited by S. DiSieno, A. Guerraggio, P. Nastasi; Marcos y Marcos, 1998

([10]) C.W. Misner, K.S. Thorne, J.A. Wheeler, *Gravitation*, Freeman, 1973

([11]) *General Relativity without coordinates*, Nuovo Cimento, **19,** 558, (1961)

incompetent objections, which also abounded, in Italy, on relativity. If anything, I am referring to Federigo's reticent distrust Enriques or the hesitant acceptance of Guido Fubini. The same Levi-Civita should not consider himself a defender of Einstein's ideas if not after a long intellectual labor: of course, he himself had the advantage of the transparency of that absolute differential calculus which with Ricci they had rigorously built. Ricci passed away at 72 years as early as 1925; Levi-Civita was able to build relationships a lot close to the community of physicists including (particularly Persico) found convinced supporters of the new ideas that bestowed on him the due acknowledgment for providing Einstein with the instrument necessary to formalize the general conception. Unfortunately in the 1938, Levi-Civita is expelled from the University due to racial laws and he will not survive this abuse for long: he will die in 1941, at 68 years old. But on the story of the creators of the absolute differential calculus and of their relationship with Einstein much has already been written and I do not it seems appropriate to go further; I therefore refer you to the volumes cited to notes (9, 10) for further historical information. But it is also the case with point out the thick essay by Abraham Pais, *Subtle is the Lord*, published by Boringhieri in 1986.

APPENDIX

Vector homography

Many, perhaps, are no longer familiar with the notions and notations corresponding to the homographies and hyperhomographies dear to Burali Forti, Boggio and, to some extent, Marcolongo. I summarize here briefly some useful information for those wishing to tackle the reading of "vintage books". The most exemplarily clear text among those available is undoubtedly that of Bruno Finzi and Maria Pastori, *Tensor calculus and applications*, published by Zanichelli in 1949: the chap. III, "Vectoral Homographs", is a comprehensive summary of the basic concepts.

In any linear vector space, a homography is an operator τ which transforms one vector into another. It is therefore referable to a matrix or a tensor, of rank 2, always trivially decomposable into a symmetric and an antisymmetric part:

$$A_{ik} = (A_{ik} + A_{ki})/2 + (A_{ik} - A_{ki})/2 \equiv S_{ik} + E_{ik}$$

In the case of three-dimensional vector spaces, the transcription of is convenient this operator in the following form:

$$\tau = \sigma + \varphi \wedge$$

where a coincides with the symmetrical part and is called the *dilatation* of τ and φ or is called the *vector* of the axial part of the homography, being $\varphi^1 = E_{32} = - E_{23}$, etc.; in general, the Italian convention of the is followed vector product corresponding to the Ricci tensor: $E_{ik} = \varepsilon_{kij}\varphi^j$. Important notions are those of the invariants: linear, quadratic and cubic. These are the current properties of invariance under transformations units corresponding to the trace of A, of A^2 and of A^3 as well as the determinant of A. The hyperhomographies, then, correspond then simply to tensors of rank 3 (but also, generalizing, from 3 upward). Thus, a rank 3 hyperomography will transform a vector in a homography, etc. Then there is the whole apparatus relating to diagonalization of the matrices that automatically corresponds to the construction of canonical quadratic forms, with identification of "aces". I insist on the fact that, today, notations have acquired a better and transparency, as is easily found especially in the texts of *Mathematical Methods for Physicists* ([12]).

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Italian to English translation using Google Translate and Yandex Translate by Thomas E. Miles. Other Ritz & La Rosa related files located by Robert Fritzius at web site: http://shadetreephysics.com/ with other relating at Gen. Sci. Journal: https://gsjournal.net/

([12]) C. BERNARDINI, O. RAGNISCO, P. M. SANTINI, *Mathematical Methods of Physics*, Carocci 1993.