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## New contribution to the Ballistic Theory of "variable stars".

### Explanation of the phenomenon for U Geminorum type and Cluster type stars.

By ***M. La Rosa***.

**1** . The objections of Mr. *Bernheimer*[1]) and Mr. *Salet*[2]) against mine from the application of the Ballistic principle theoretical knowledge gained on the propagation of light sketch of the phenomenon of "variability" have me for an in-depth study of the peculiarities causes, which this sketch lets us foresee for the case that the constant *Kb* of the fundamental equation becomes much larger than *r*.

The results of such an investigation are very been satisfying because they take me to a simple and immediate explanation of the behavior of the two groups "variable stars" led to this day in a dense remained shrouded in mystery.

The importance of such agreement emphasize is superfluous. It's a new set of facts the direct and informal in favor of the application of the Ballistic principle on the light speaks, one application, based on the floor of the quantistic presents theories as something easy and natural.

By me discussing the divergence between the variability type, that of the Ballistic Theory is foreseen and he who observes (according to *Bernheimer*). should have been reserved for a later communication, I just want to present the analysis that led me to the announced has led to results.

I will start from the investigation on the conditions of disappearance of the equation:

$$\chi + a\cos\chi = K$$

whereupon the investigation into the positions of one in his orbit orbiting star from which the emitted light arrives at the observer at the same time, runs out; one Investigation that because of the applications they find can, has an interest in and for itself. Then I will explain the method to be followed when calculating the light curves in the most general possible case (any *a*, that is, the superposition of the from any number of positions of coming light) can follow, and its application to a concrete case ($a$ = 10π) perform what will give me opportunity, the perfect analogy in behavior between those to be foreseen for capital a to prove light curves and the light curves,

[1]) Z. f. Phys. **36**.302, 1926. [2]) C. R. **183**.1263, 1926.

which the observation for the variable dated for a long time Type U Geminorum and those found in star cluster's variables of a special character (cluster type) has revealed.

**2**. It is worth remembering that the basic equation of my sketch of the Ballistic Theory of variable stars [3])

$$T = K\tau_0 + t + Kb\tau_0 \cos \omega t$$

is, which with the introduction of corresponding variables shape

$$y = x + a \cos x \tag{1}$$

or

$$\theta = u - e \sin u \tag{1'}$$

assumes.

In this second form it is good to astronomers known but has only been studied in the case in which $e<1$, because then they use the extremely simple connection represents, in the movement of the planets between the time $\theta$, the eccentric anomaly $u$ and the eccentricity $e$ the train exists.

It is further recalled that equation (1) graphically - in orthogonal Cartesian coordinates - becomes an "oblique sinusoid" (see Fig. 1) is drawn within of stripes,

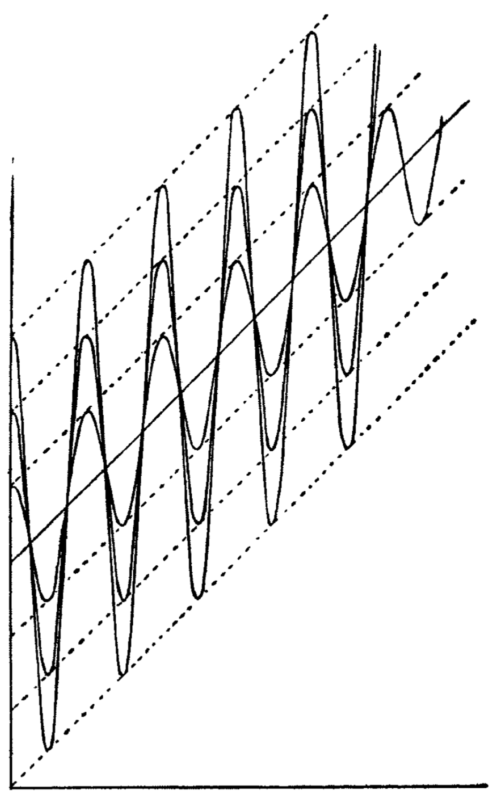

**Fig. 1**

[3]) The symbols have the following meaning: $t$=time of departure of light from the star in its orbit, $T$=time of arrival at Observer, $\tau_0$=orbital period of the star (in a circular orbit), $\omega=2\pi/\tau_0$ =angular velocity of the star, $b$=ratio of tangential speed $v$ of the star to the normal speed of light $c$, $K=\Delta/c\tau_0$, where $\Delta$ is the distance star·to the observer. The case of the elliptical orbit is described by *C. Cannata* in R. Acc. Lincei has been treated.

the straights

$$y = x + a \qquad y = x - a \tag{2}$$

has edges. These are in infinitely many points tangents to the curve, namely the upper one in the abscissa points:

$$x = 2n\,\pi$$

the lower in the points:

$$x = (2n + 1)\,\pi$$

with $n$ as any integer.

If one then puts in (1) $x \pm 2n\,\pi$ with whole $n$ at the place of $x$, you get:

$$y(x + 2n\,\pi) = \pm 2n\,\pi + y(x)\,. \tag{3}$$

This equation tells us that at constant amplitude intervals $2\pi$ the curve is reproduced identically, where only the following change takes place: the new arc turns out to be a relative to the previous one $y$-axis parallel translation by the constant amount shifted from $2\pi$.

This circumstance allows us to look at the periodic features commonly used terminology to operate by we as period within an amplitude interval 2π contained curve section, as "amplitude" the factor $a$ of the periodic element, with the straight line as the axis of the curve $y=x$, which is the axis of the strip, as the phase at point $P$ the difference[1])

$$x - 2n\,\pi$$

denote where $x$ is the abscissa of $P$ and n is the ordinal number is the period to which the point $P$ belongs, assuming that that one of that period, which begins at $x = 0$, the index zero returns. In a word, we can in many ways treat equation (1) as if it were a periodic [2]) were. As long as one assumes $a<1$, equation (1) is always growing as it is not possible of condition

$$y^{\cdot} = 1 - a\sin x = 0$$

to suffice, while the second differential $y^{\cdot\cdot}$ always >0 remains; as soon as $a>1$, on the other hand, it shows an infinite number of maxima and infinitely many minima. Of these, the first have the coordinates:

$$x_{M,n} = \alpha + 2n\,\pi \qquad y_{M,n} = 2n\,\pi + (\alpha + a\cos\alpha)$$

the others:

$$x_{m,n} = (2n+1)\pi - \alpha \qquad y_{m,n} = (2n+1)\,\pi - (\alpha + a\cos\alpha)$$

where $n$ denotes any integer (including zero), and α is the smallest value of $x>0$ that satisfies equation (3) enough; namely

$$\alpha = \operatorname{arc}\sin(1/a) \tag{4'}$$

there $0 < \alpha < \frac{\pi}{2}$ .

Having said that, let us proceed to examine which is the number of points in which a straight line the equation $y = K$ intersects the curve, or more precisely, for a given value of $a$ to examine how this number at varying $K$ changes.

[1]) So that the terminology introduced is analogous to that used for the sinusoidal functions, one should designate the ratio $(x\text{-}2n\pi)/2\pi$ as the phase and our difference as the angular value of the phase.

[2]) For this reason, we will say that equation (1) is an "improper periodic" function.

Thanks to the "improper periodicity" from $y$ it is clear that it suffices the investigation to be limited to a single period, that is $K$ as variable in an amplitude interval ($2\pi$) since the results found for one period on the others periods can be extended. And we want $K$ as variable between $a$ and $a + 2n$. We first emphasize that the edges of the strip on the lines $y = K$ separate a segment of constant length whose ends have the abscissas $K - a$, $K + a$, and that the points you are looking for - these are the roots of the equation $x + a\cos x = K$ are-all must be on this segment.

Let us begin by assuming that $K = a$, i.e., that is that the straight line meets the curve at first touch - index 0 - cut with the top edge. If $n$ is the number of minima lying to the left of the end $2a$ of our segment, so we can claim that the number of intersections we are looking for $2n + 1$ is since every minimum to two such points leads without the first touch mentioned above to count that lies to the left of the first maximum and those therefore to the right of the immediate axis $y$ previous minimums.

The assumed condition that the index minimum ($n$ - 1) be the last one falling to the left of $2a$ leads to the others that the ordinate of this minimum is the last the $\leq a$ is. This becomes inequality

$$y_{m,n-1} = (2n-1)\pi - (\alpha + a\cos\alpha) \leq a$$

or

$$(2n-1)\pi \leq a + y_{M,1}.$$

To give us an accurate idea of what what happens to the number of roots when we use $K$, starting from $a$, let it grow constantly, it is indicated to keep in mind the distances that the first maximum and the index minimum $n$ of the line $y{=}a$ have.

These distances are

$$\delta = y_{M,1} - a = \alpha + a(\cos\alpha - 1)$$

and

$$\eta = y_{m,n} - a = (2n+1)\pi - (y_{M.1} + a)\,.$$

It is immediately obvious that in the case where $\delta<\eta$ would be no change in the number of roots $2n$+1 as $K$ grows from $a$ to $a+\delta=y_{M,1}$ (including the extremes) can expect, because during the straight line in its gradual rise always the crest of the first maximum intersects, it still succeeds not to touch the index minimum $n$. Against must we expect a reduction of this number by 2 units, if $K$, $y_{M,1}$ exceeding that between this value and the value $y_{m,n}$. included interval runs through.

Yes, there is then an amplitude interval

$$\eta - \delta = y_{m,n} - y_{M,1} = (2n+1)\pi - 2y_{M,1} > 0$$

within which the number of roots decreases to $2n$ - 1, to return to $2n$ + 1 when $K$ equals $y_{m,n}$. This value is maintained throughout the remainder of the period, i.e. in the interval ($y_{m,n}$ $a$ + $2\pi$), unchanged since inside of the segment $K - a$, $K + a$ no loss of maxima yet a gain in minimums more can be obtained.

In short, if (2$n$-1) is the largest odd integer, which of the condition

$$(2n-1)\pi \leq a + y_{M,1}$$

enough, and if we moreover

$$(2n+1)\pi > 2y_{M,1}$$

have or at all

$$(2n-1)\pi \leq a + y_{M,1} < 2y_{M,1} < (2n+1)\pi$$

the searched number of roots varies between (2$n$ + 1) and (2$n$ - 1). It is (2$n$ + 1) in the interval ($a$, $a$ + δ), (2$n$ - 1) in the interval ($a$ + δ, $a$ + η) and again becomes (2$n$ + 1) in ($a$ + η, $a$ + ω). Basically, in each period we have two intervals, in one of which is the number of roots (2$n$ + 1), and that itself by 2$x$ - η to the left of touching the top edge and by δ to the right of it.

In particular, in the case where

$$(2n-1)\pi = a + y_{M,1}$$

i.e., $y_{\mathrm{m,n}} = a$, the number of roots 2$n$ + 1 in the interval δ and becomes 2$n$ - 1 in all remaining parts because the new minimum will only be touched at the end of the period.

It remains to examine the cases where $\delta \geq \eta$.

In the first of these it happens that the straight line $y$=$K$ at the same time tangent to the curve in the 1st maximum ($K = y_{M,1} = a + \delta$) and in the index minimum $n$ becomes; the number of points of intersection then increases from (2$n$ + 1) to (2$n$ + 3) - by counting the touches as colons - and immediately returns to (2$n$ + 1) as soon as $K$ surpasses $y_{m,n}$. In general, one has 2$n$ + 1 root, which only at $K = y_{M,1}$ becomes (2$n$+3).

In the other case, δ>η, there is a whole amplitude interval (δ - η) = ($y_{M,}$ -$y_{m,n}$), in which the number of roots increasing from (2$n$ + 1) to (2$n$ + 3) to return to (2$n$ + 1), as soon as $K$ $y_{M,1}$ exceeds. This value is evident until the end of the interval. In short, in this case things take the following course: while $K$ from $a$ to $y_{m,n}$ varies - barring this extreme - is the number of roots 2$n$ + 1, it becomes 2$n$ + 3, while $K$ from $y_{m,n}$ to $y_{M,1}$ varies - extremes included - and returns for the throughout the remainder of the period to 2$n$ +1.

In summary: always assumed that (2$n$-1) be the largest, odd integer for which one

$$(2n-1)\pi \leq y_{M,1} + a$$

has, so we'll get when we

$$\delta - \eta = y_{M,1} - y_{m,n} = 2y_{M,1} - (2n+1)\pi \geq 0$$

have that the number of roots is between (2$n$ + 1) and (2$n$ + 3) sways. The two conditions together result sure that this is the case when you have:

$$(2n-1)\pi < y_{M,1} + a < (2n+1)\pi \leq 2y_{M,1}.$$

Finally, the analysis carried out shows that that in order to find the desired number of roots, if a is given, the ordinate of the 1st maximum suffices using the formula

$$y_{M,1} = \alpha + a\cos\alpha = \arcsin\left[1/a \cdot \sqrt{(a^2-1)}\right]$$

to calculate and get the numbers ($a$+$y_{M,1}$)/π and 2$y_{M,1}$/π to build; then wavers when among them a certain odd integer ($2n + 1$) is included, the number of roots between this and the following odd one number in the manner described; in the opposite if the number you are looking for fluctuates between the following two the calculated numbers comprise integers.

In particular, in the case where $(a+y_{M,1})/\pi$ itself an odd integer would be, ($2n$ - 1), the number of roots $2n + 1$ in the interval ($a$, $y_{M,1}$) and becomes ($2n$ - 1) in the remaining part of period; in the case where $2y_{M,1}/\pi$ is the odd integer ($2n$ - 1), the number sought is constant ($2n$ - 1) except at one point (at $K=y_{M,1}$), where they $2n+1$ becomes.

**3**.· Let's now get to the actual calculation of the roots of the equation

$$y = x + a \cos x = K \tag{6}$$

step where $K$ is a number between $a$ and $a + 2\pi$, so we can very advantageously of through the relationship (**3**) expressed property that allows us to use the investigation on the one after the intersections of certain straight lines parallel to the $x$ axis only with the first one period[1]) of the curve $y(x)$.

For the sake of greater clarity, let's take these roots as known and arranged in ascending order. They were:

$$x_1, x_2, x_3, \cdots x_{2n+1}.$$

We notice that the first three or the 1. belong solely to the 1st period of the curve, namely each since $K \leq y_{M,1}$ or $K > y_{M,1}$, and that the following belonging to two and two to the succeeding periods.

So suppose that the two roots $x_4$ and $x_5$, belonging to the 2nd period, so we will discuss

$$x_4 = 2\pi + x_{4\cdot 0} \qquad x_5 = 2\pi + x_{5\cdot 0}\ .$$

Equation (3) tells us that

$$y(x_4) = 2\pi + y(x_{4\cdot 0})$$

is, from which, since $y(x_4) = K$, one gets:

$$y(x_{4\cdot 0}) = K - 2\pi$$

and as well:

$$y(x_{5\cdot 0}) = K - 2\pi\ .$$

The latter tells us that in order to find the two roots $x_4$ and $x_5$, it suffices to use the equation line $y=K-2\pi$ to draw and their intersections with the first period to look for the curve. So, basically, we get the angular values of the phases $x_{4.0}$, $x_{5.0}$, with the help of which one immediately gets the two values we are looking for by simply add the number $2\pi$.

In a perfectly analogous way we find that, to get the roots $x_6$, $x_7$, the straight-line $y=K$ is sufficient $-2\cdot 2\pi$ to draw the abscissas of the intersection points $x_{6.0}$, $x_{7.0}$ just look with the first period of the curve and to add the number $2\cdot 2\pi$ to them, etc.

In short, we will get all the required roots, by taking the intersection points of the first period of the curve with the equation degrees:

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[1]) It is necessary to clearly distinguish this "first period of the curve" we are talking about and the first period of function $y(x)$. In fact, the former includes all points whose $x$ falls between 0 and $2x$, while the latter includes all points whose $y$ falls between $a$ and $a + 2\pi$, and may have points whose $x$ belong to the interval $0 < (2n + 1)\pi$.

$$y = K$$
$$y = K - 2\pi$$
$$y = K - 2 \cdot 2\pi$$
$$\cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot \qquad (7)$$
$$\cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot$$
$$y = K - n \cdot 2\pi$$

search and for the phases found in this way the corresponding ones add multiples of $2\pi$ that are in the second term of (7) happen.

After the investigation is so formulated, it is to find the phases you want, it is best to construct tables in which the values of $y$ are assumed for values of $x$ entered for the interval $(0,2\pi)$ are.

The construction of such panels is not practically difficult because it suffices an ordinary trigonometric table, the values of the cosine – from 30' to 30' – with to multiply the constant $a$ and that of the arc, expressed in parts of the radius. Yes, in regarding the use of our function for astronomical purposes can produce a certain number be useful of auxiliary tables showing the products of the values of the function cos $x$ with the successive natural ones numbers from 1-9 included, so that when you use them easily the product $a \cos x$ for any given one calculate the value of $a$ and thus the table of

$$y = x + a \cos x$$

for the desired value of $a$ can form [1]).

**4**· Moving on to the calculation of the Ballistic Theory of "Variable" anticipated light curves we prefer to choose a concrete example that not only gives an opportunity to show how specifically the considerations set out above are applied, but also to eliminate a misunderstanding in which I myself regarding the choice of the upper limit to which the product $Kb$ is subjected would have to be, so that the phenomenon of variability is still perceptible, am addicted. So, we want on purpose choose the case where $a$ = 10 and therefore $Kb$ those value has been intuitive in my previous work due to contemplations as the limit had been contemplated, where the variability would vanish.

The relationship between observation time and departure time the rays of the star:

$$T = K\tau_0 + t + Kb\tau_0 \cos \omega t$$

becomes in the assumed case:

$$y = x + 31.41592 \cos x$$

where $$y = \omega(T - k\tau_0) \qquad x = \omega t\ .$$

So, the first maximum of $y$ has instead of the through

$$\alpha = \arcsin(1/10\pi) = 1°49'26.7"$$

given value of $x$, and the value of the ordinate is $y_{M,1}$ if 31.43205.

The numbers $(a+y_{M,1})/\pi$ and $2y_{M,1}/\pi$ consequently have the values 20.0054 and 20.0108, which us as they are both between 19 and 21 indicate that the number of

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[1]) It is obvious that if the first leads to three points of intersection, the last will not be able to give any if the number of roots 2n + 1.

distinct positions of the star from which simultaneously rays reach the observer themselves between 19 and 21 emotional.

The overall picture seen by the observer is thus formed by the superposition of these 19 or 21 elementary images (since the distance between star and observer and the radius of the orbit the separation of these images do not allow), and there will be a brightness have equal to the sum of the brightnesses of the elementary images.

If one wants to construct the light curve by points, so you will have to proceed in the following way:

It is appropriately a certain number of values of $T$ (observation time), that is determined by $y$; let these be $y_0, y_1, y_2 ... y_i$.

Corresponding to each of the same for example, for $y_1$ etc., the values $x_1$ etc. are searched for that satisfy the equation

$$x + a \cos x = y_i$$

fulfill, or rather the corresponding phases with help of the equations $x + a\cos x = y_i - m \cdot 2\pi$, which with the help of the prepared panel and in the manner already indicated happens. Let $x_{i0}, x_{i1}, x_{i2} ... x_{i, 2n+1}$ the 21 (or 19) be like this found numbers.

One then calculates according to each of these phases the absolute value of the derivatives d$y$/d$x$, since these, by coinciding with d$T$/d$t$, one at each point takes on a value that is inversely proportional to the brightness of the sub-image and therefore us for the measurement of the same can serve.

So, if you take the inverse values of the 21 numbers $y'(x_{ij})$ and adding them together gives a number that is proportional to the apparent magnitude of that observed at moment $T_j$ image is and is given by

$$T(T_i - K\tau_0) = y_i .$$

The somewhat laborious calculations are mine assistant Dr. *G. Petrucci* has been executed to whom I am responsible express my thanks and praise.

I give some examples below:

If one takes the amplitude a as the first value $y_0$ of $y$ self = 31.41592 ..., and one therefore thinks of that first point of contact of the curve with the upper edge of the if you draw a line parallel to the $x$-axis, you get as phases of their intersections with the curve the following numbers:

| | $x$ | $J$ | $x$ | $J$ | $x$ | $J$ |
|---|---|---|---|---|---|---|
| 1. Periode | 0°0′ 0″ | 1 | 3°38′ 0″ | 1.0526 | 325°24′ 0″ | 0.0529 |
| 2. » | — | — | 38 51 | 0.0540 | 310 59 | 0.0400 |
| 3. » | — | — | 55 19 50 | 0.0402 | 295 45 | 0.0342 |
| 4. » | — | — | 68 48 | 0.0353 | 284 1 | 0.0318 |
| 5. » | — | — | 81 3 30 | 0.0333 | 272 46 | 0.0209 |
| 6. » | — | — | 92 57 30 | 0.0331 | 261 38 45 | 0.0311 |
| 7. » | — | — | 104 58 45 | 0.0339 | 250 10 | 0.0328 |
| 8. » | — | — | 117 43 | 0.0372 | 357 51 15 | 0.0134 |
| 9. » | — | — | 132 20 | 0.0450 | 223 37 | 0.0442 |
| 10. » | — | — | 152 12 30 | 0.0733 | 204 0 | 0.0762 |

After these were found, the absolute ones arose values of $1/y'$ given in the attached table in the columns labeled $J$ (partial light intensities). are; after all, that would be right now $T_0 = K\tau_0 + Kb\tau_0$ (which we take as the starting point of the times $T$ will be taken) observed total brightness is calculated. In the case cited, it can be seen that these sum is 2.84. In other words, in that moment $T=T_0$ becomes the star, as a result of assumed Ballistic propagation of light, the observers with a magnitude of 2.84 appear when he should have shown if himself that propagated light at a constant speed, or if the star does not have an orbit described.

It is still very important to draw attention to that to form this total brightness the 21 partial images contribute in very different degrees.

To a large extent, the two first of the three pictures belonging to the 1st period; and the first with the value 1, the other with the value 1.0526, while the remaining 19 images do this one below the other little different posts of the size of some hundredths deliver, so that while the two first for themselves a contribution of 2.05 to the brightness of the overall picture alone deliver, the other 19 together do not achieve a contribution of 0.8!!

The brightnesses are calculated in the same way been assigned to the observer in the first column times indicated in the table below. These times are based on the moment indicated above $T_0$, determined and in fractions of the period (i.e., at assuming $\tau_0 = 1$).

| | J | | J |
|---|---|---|---|
| 0.000000 | 2.8411 | 0.487500 | 2.1113 |
| 0.004838 | Max. | 0.49490 | Max. |
| 0.013888 | 1.0035 | 0.51388 | 1.48744 |
| 0.027776 | 0.7845 | 0.52776 | 1.28407 |
| 0.035552 | 0.7842 | 0.666717 | 0.97692 |
| 0.083333 | 0.7802 | 0.83333 | 0.98390 |
| 0.250000 | 0.7656 | 0.97222 | 1.28424 |
| 0.472222 | 0.7794 | 0.98610 | 1.48110 |
| 0.486060 | 0.7818 | 1.00000 | 2.84110 |

The method described encounters a serious difficulty when it comes to evaluating the maximums brightness corresponding to the light curve is applied (i.e., if the values of $K$ are the ordinates of the 1st maximum and the last minimum to the left of $2a$ become).

In fact, since then among the $x$; the abscissa of the maximum (or the minimum) figures, one of the $y'$ null, which entails that one corresponding elementary image would have to ascribe an infinite brightness. It would in this. case seem much more appropriate that brightness of this "frame" from the one outside the derive the value of the ratio $\Delta y/\Delta x$ taken from the limiting value. But also, the numbers obtained in this way are strongly dependent on the special value that one attaches to the (arbitrary) $\Delta x$, so that in the absence of of a criterion for the choice of *Δx* is not possible to rely one of to rely on one of those that can be calculated.

So, in order to derive the values of the maxima, one must proceed differently. It is readily evident that the best criterion is the following: Graphically the curve due to the already found points (i.e., draw them without the maxima drag) and fix the latter on it, considering that the total area between the curve, the axis of the *x* and the extreme ordinates must be equal to the rectangle, that the period and the effective magnitude 1 of the star has sides. This is because the two areas or the total amounts of light represent that in a period to the observer in the Ballistic and in the ordinary hypothesis must arrive and the evident energetic requirements (1st principle) must be the same.

Applying this criterion to the one in question concrete case we have as values for the two sought maximum received the numbers 4.06 and 4.92.

In short, proportionate in spite of what is given to the constant a large value (10) leads us to the Ballistic hypothesis to foresee a light curve that not only still clearly exhibits the phenomenon of variability, but possesses the qualities that said phenomenon observed in many cases.

In fact, the projected light curve shows two very pronounced, extremely brusque and very short-lived maxima in the phase of ascent, which is divided by two very flat minima are separated, where the brightness is long remains almost constant over time. The amplitude of the brightness change in the calculated example is from the magnitude of two degrees in the scale of stellar magnitudes, and their phases are practically distributed as follows:

Two long intervals in which the brightness is noticeable is diminished with the values it has in the two minima (0.78 and 0.92 respectively), totaling 60/100 of the period; the two phases of decrease that include 20/100 that the slow increase to 15/100 and the two maxima, which develop almost entirely in the remaining 5/100 of the period, although most of the climb to the maximum and descent in an extremely short time.

The same results are found if much larger values are assigned to the constant *a*, like me by an indicative calculation for the case $a = 200\pi$ have been able to determine. Everything recorded in the light curve is, is *a* lower duration of the maxima and a corresponding increase in their brightness. We can so conclude that no upper bound for the constant *a* (and thus for *Kb*) exists beyond which the phenomenon of variability is no longer evident. The dichotomy between the implications of Ballistic Theory and the observations cited by *Bernheimer* and *Salet* is therefore not essential and is limited to a divergence in the form of the light curve that is easily turned off can be.

The properties of the Ballistic Theory for these large values of *a* anticipated light changes occurs, in fact do not coincide with those the variables of the Algol type, rather follow very closely in the case of the phenomenon of variability some stars in near those who are among the 3rd group variables, Class IV of the *Pickering* classification, i.e., the as "cluster type" designated group (which is the most numerous of all is) and the long-period variables of Class II B of the same classification have been observed[1]).

From a treatise by *S. I. Bailey*[2]) we bring in figures 2, 3, 4 some light curves from to that type belonging stars and also the following table, in the case of the phenomenon of variability in some of the most prominent stars observed in the star clusters M5 the phases are shown schematically.

| | | | |
|---|---|---|---|
| Duration | Of the | Maximum | 0/100 |
| " | " | Minimum | 40/100 |
| " | " | Descent | 50//100 |
| " | " | Rise | 10/100 |
| | | | 100/100 |

This table and those light curves clearly show the parallelism of behavior between these cases of observation and the theoretically derived curve.

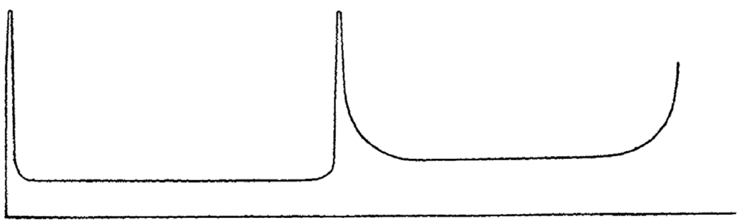
Fig. 2


Fig. 3

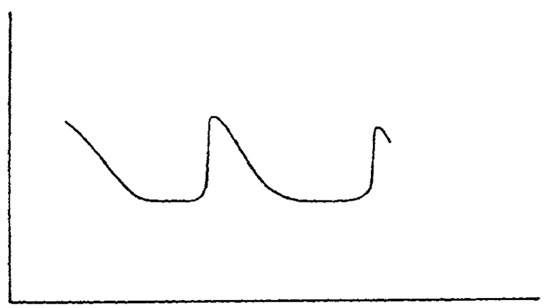
Fig. 4

Analogous properties are possessed by that in SS Cygni (Fig. 3) (the most typical and strangest of the variables of the the light curve observed in SS Cygni (Fig. 3) (the most typical and strangest of *Pickering*'s Class IIB variables) has analogous properties, which also two long intervals of light constancy (minimum), two almost instantaneously flaring maxima, etc. It must also be further borne in mind that the Ballistic Theory has good reserves to those of each the mutable of these important and mysterious group that has escaped any attempt to explain it to this day is to be able to explain presented phenomena in more detail, namely by using: 1. the most appropriate wise choice of the constant *a*; 2. the ellipticity the orbit (which we have here assumed to be circular for simplicity); 3. the orientation of the same in relation to the line of sight, etc. It seems possible to explain the different durations of the two maxima, the different speeds of the two descents, etc., in this way.

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[1]) Compare *K.Schiller*, Introduction to the Study of Changed. Stars, A. Barth, Leipzig, 1923.

[2]) ApJ 10.260 (1899).