IS PROF. HOWARD EVES WORNG IN EQUATING THE AREA OF CIRCLE TO THE AREA OF RECTANGLE?

(546th Proof on Rho)

Hippocretes of Chios (450 B.C) has squared a semicircle and a full circle along with a lune. He has equated the sum of the areas of three lunes and a semicircle with the area of a trapezium. Similarly, he has equated the sum of the areas of a lune and a full circle with the sum of the areas of a triangle and a hexagon.

We have one more evidence for the squaring of a circle. Here, a circle in area is equated to the area of a rectangle. This evidence is obtained from the classic on π.

π : A Biography of the World's
Most Mysterious Number
By
Alfred S. Posamentier & Ingmar Lehman
Page 293, 2004
Prometheus Books
59, John Glen Drive
Amherst, New York, 14228-2197

This author, humbly expresses his grateful thanks to the authors, **Prof. Alfred S. Posamentier** and **Prof. Ingmar Lehman** and the Publisher M/s. Pormetheus Books, New York for letting the world of mathematics to have a great opportunity to see that squaring of circle is not impossible, though the view about it is otherwise in the mathematical establishment. I thank them again and again. This author believes people will feel very happy that there is a clear evidence

of squaring of circle, questioning the **false idea** of calling π as a transcendental number. A latest evidence, indeed. Two great mathematicians have been seen by the world of mathematics again, when some thing different is going on in the Pi world. We are very fortunate to see **Francesco Bonaventura Cavalieri** (1598-1647) and **Howard Eves**, our legendary. **F.B. Cavalieri** is an Italian mathematician and is famous for his **Cavalieri's principle.** It states that

"Two solid figures are equal in volume if a randomly selected plane cuts both figures in equal areas".

Our mathematics historian professor **Howard Eves** developed highly ingenious and very simple proof for the Cavalieri's principle. For this proof he was awarded, 1992 **"George Polya Award"**. His proof says that *"there exists a tetrahedron which has the same volume as a given sphere"*, or, as he says, where the two solids are "Cavalieri Congruent".

Thus, we have two **very recent** evidences questioning 1. The validity of upper limit of π, i.e. less than 1/7 of **Archimedes** and 2. The impossibility of squaring a circle of **James Gregory** (1660) of Scotland and **C.L.F. Lindemann** (1882) of Germany.

Professor **C.H. Edwards Jr** and Professor **David E. Penny** of the University of Georgia, Athens, have given in Page No. 295, of their very voluminous classic.

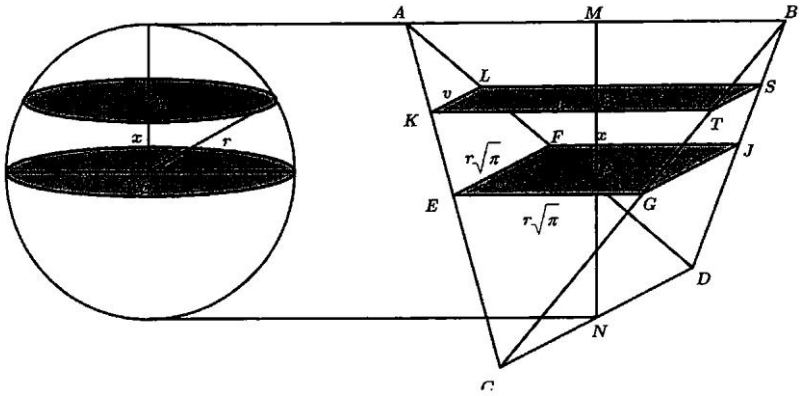
Calculus and Analytic Geometry

2nd Edition, 1986

and published by Prentice-Hall International that **"π lies between 3.133259323 ≈ 3.133 and 3.14659265 ≈ 3.147".**

Archimedean upper value is 3 1/7 = 22/7 = 3.142. The official value 3.141592653 is in total agreement with the upper limit 3.142 of **Archimedes.**

But, the latest finding for upper limit of π is 3.1465… There is a clear cut opinion on the value of π and the concept of squaring a circle. Let us see the proof of our Professor **Howard Eves.**



(By courtesy of authors and publisher)

$$UV = \pi (r + x) (r - x) = \pi (r^2 - x^2)$$

"Thus the area of circle H and the area of rectangle LSTK are equal. So by Cavalieris' theorem, the two volumes must be the same".

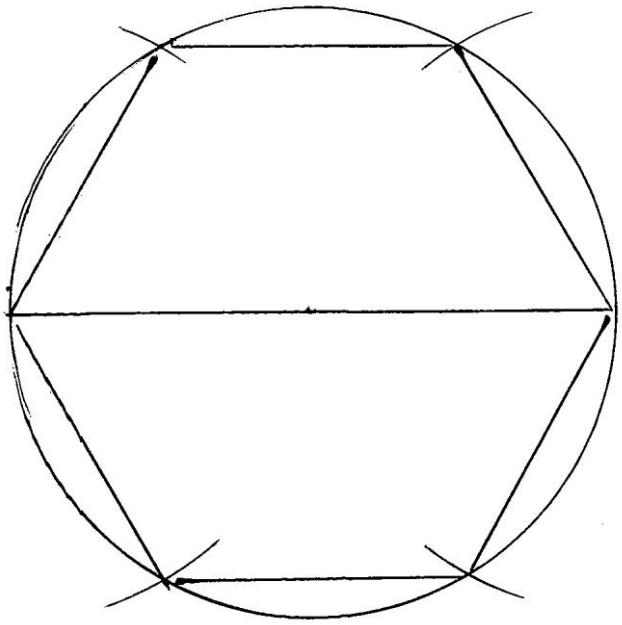
For details, the above book on π and / or

2. **Howard Eves** "Two supporting Theorems on Cavalieri Congruence's". College Mathematics Journal 22, No. 2 (March 1991): 123-124.

I beg of you Sirs, the above two are crucial evidences for the probable **accurate** value of π and a clear evidence for the squaring of circle. They imply further, that 3.14159265358… is a **far lower value** and π is **not** a transcendental number.

–Author

We call 3.14159265358…. and π, a transcendental number. Is it apt to call this number a transcendental ?



We inscribe a regular polygon in a circle say with 6 sides. We double the number of sides and continue it, till the gap between the inscribed polygon and the circumference disappears, leaving no gap what so ever. We imagine the inscribed polygon ultimately becomes a circle. **Ludolph Van Ceulen** (1610) of Germany has obtained 35 decimals of 3.14159265358… from the inscribed polygon having 2^{62} sides. It means 4, 611, 686, 018, 427, 387, 904 sides.

However, according to **J. Houston Banks** et al of the book **Geometry** (1972), in Page 409,

"…. Yet no matter how many times we repeat this process the perimeter of the polygon will never actually reach the circumference."

Oxford dictionary says

Transcendental = going beyond human knowledge

In the process we know the perimeter of the inscribed polygon grows and grows, but it reaches an **ending** because the circle obstructs the growth of the polygon. In otherwords, it is limited and not limitless. Further, the surd $\sqrt{3}$ is used to calculate the perimeter of the polygon. Number of decimals do **not** change from the beginning of 6 sides to 10^{62} sides. Only **value** changes. The number of decimals remain unchanged, because $\sqrt{3}$ stands till the end. The Japan mathematical society in its **Encyclopaedic Dictionary of Mathematics,** in Page 1310, says "*The theory of transcendental members is, however, far from complete. There is no general criterion that can be utilized to characterize transcendental numbers*".

Secondly, which one increases its extent in the above diagram ? Is it inscribed polygon or circle ? Which one really a transcendental entity ? Is it polygon or circle ? The circle remains static. It observes silently the coming of smaller polygon slowly towards it i.e. circle. The one (circle) that is obstructing the continuous growth of the polygon is being called a transcendental !

By the by, is there any entity that can be called, **transcendental ?** Yes, there is, i.e. **Space** and nothing else. How ? Let us imagine. The **Cosmos** consists of two entities. They are, physical universe comprising of planets, stars, solar families, galaxies, clusters of galaxies; and two: radiation. The radiation is often called electromagnetic radiation. Let us do this in imagination. Let us go and close the eyes Sir, to our imagination, very slowly. Let us think our Sun, other planets have disappeared leaving Earth untouched. We stand in our imagination. Next, other solar families, galaxies, clusters of galaxies have disappeared one by one. And finally, our Earth has also disappeared leaving us intact. What exists ? It is **Space**. Space has no physical

quality. So, no concept of "distance". The space between two bodies is distance. No two bodies and no distance. Here, now, we are in space. Yes, in space, if we say so, we are **wrong**. How ? We are **not** in space. If we say, we are "in space", it implies that universe is a large container or bowl of astronomical magnitude. Then, a question comes. What is **beyond** the container ? No answer Sir. **Albert Einstein** has rightly said that

"Physical objects are not in space, but these objects are spatially extended. In this way the concept, 'empty space' loses its meaning."

There is thus a clarity on "space" in saying not "**in space**" but **"spatially extended"** of planets, stars etc. Let us come back Sir, from our imagination.

"Science" is called a **self correcting** subject. It appears that it is not the case with mathematics. Tomorrow science may correct the following hypothesis. Mathematics, ultimately certifies, if correct. What is that ? Here is a brief **"thought experiment"**. Either "imagination" or "intuition", both are necessary in mathematics just like, they in science. Did anybody think of, how God controls the World ? The universe is very big. Its distance is measured in **"light years".**

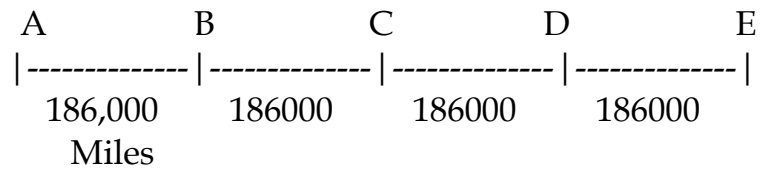
If God, say is with us. He gets a message that some accident has happened on the other side/ boundary of the physical world, say distance is trillions of kilometers or one thousand light years away. If He starts instantly to go to the accidental spot with the speed of an electromagnetic ray, He takes more time. By the time He goes, nothing He can do there. In this imagination, our boy was standing in an open ground, thinking about His administrative capability looking high above into the sky.

A **divine form** suddenly appeared before this boy and enquired about his problem. The boy narrated what was in his mind. Immediately the divine form carried the boy on His shoulders, went with such a high speed, to a far of place, say to the distance of one thousand light years, and returned, leaving the boy and disappeared. The boy looked at his watch and found, he took just less than a minute for his up and down journey with the divine form. The boy thought and thought for many years and came to the following conclusion. What is the conclusion ?

1. The velocity of the light may not be fixed / constant.
2. Or, there must be a **non**-electromagnetic radiation whose velocity of travel **doubles successively** in its onward passage along the distance, for every unit of distance of velocity.

$$\text{i.e. } 1 \rightarrow 2 \rightarrow 4 \rightarrow 8 \rightarrow 16 \rightarrow 32 \ldots\ldots \rightarrow$$

In other words, the speed of travel by the God, doubles successively, for every **unit** of distance travelled in the first second of the first unit. For example



For light ray to travel from A to E, it takes four seconds. But God travels this distance between A and E in **less than two seconds.**

How ? First unit of distance (186000 miles) takes one second, second unit half a second, third unit quarter second and fourth unit one 8^{th} second.s

To end the discussion, the surd $\sqrt{3}$ which is used in arriving at 3.14159265358… is itself **never ending in its decimal form**. Inscribed polygon is a finite entity drawn in another finite entity : circle. So, naturally, it has to be represented by a finite number. As $\sqrt{3}$ is an **exact number** in its surd form, $\sqrt{3}$ is suffice. Calling either 3.14159265358 of polygon or circumference or area of circle or circle constant π, the term "transcendental" becomes **redundant**. And, to be accurate nothing in the world is transcendental except **Space**.

Finally, as the mathematical giant **Prof. Howard Eves** has equated the area of a circle to the area of a rectangle or in otherwords, called, squaring a circle, the π number can **no more be called a transcendental number**. Further, the concept of "Squaring of Circle" tells us that π is a finite number, representing either area or circumference of a circle, and can be **equated** with that of either square or rectangle or triangle or trapezium. The new π value $\frac{14-\sqrt{2}}{4}$ has squared exactly a circle, it has circled a square exactly, it has constructed a triangle whose area is equal to that of a given circle, it has squared arbelos of **Archimedes**. $\frac{14-\sqrt{2}}{4}$ stands at every step for **exactness** with all the straight-lined geometrical constructions. Though all the constructions are **very elementary** in nature, that does not mean the quality of this work is substandard.

"medeis ageometrtos eisito"

- Plato

("Let no one without geometry enter here" is at the portals of the academy of **Plato**)

Thank GOD

Sarva Jagannadha Reddy
13,February 2020