the general science journal

Intriguing Aspects of Ionization of Hydrogen Atom with Photon and Electron

Pavle I. Premović
Laboratory for Geochemistry, Cosmochemistry and Astrochemistry,
University of Niš, pavleipremovic@yahoo.com, Niš, Serbia

The simplest of all atoms is the hydrogen atom (H-atom), consisting of a single proton (H^+) in the nucleus and a single electron (e^-). It is a fundamental system in quantum physics.

In elementary physics, an H-atom in the ground state can be ionized by monochromatic radiation of photon energy 13.6 eV ($\approx 2.2 \times 10^{-18}$ J). This energy has the ultraviolet photon with a wavelength of approximately 91 nm. If the total photon energy, $E_p = h\nu$, is greater than that of ionization, 13.6 eV, the excess energy is converted into the kinetic energy of the ejected electron, KE_e, which can be expressed mathematically as:

$$E_p = 13.6 \text{ eV} + KE_e = h\nu$$

where h (= 6.63×10^{-34} J sec) is Planck's constant and ν is the photon frequency.

Let us assume that an incident electron strikes an H- atom, transferring a part of its kinetic energy to the kinetic energy of the electron of this atom, KE_H. So, we have,

$$KE_H = 1/2(m_e\upsilon_e^2) \quad \ldots (1)$$

where m_e (9.1×10^{-31} kg) is the electron mass and υ_e is the speed of this electron. Let us assume that this energy is high enough to ionize the H- atom, exceeding 13.6 eV and the electron of the H-atom will be ejected with a kinetic energy KE_e, as in the photon case above. Or

$$h\nu = 1/2(m_e\upsilon_e^2) \ (= 13.6 \text{ eV} + KE_e)$$

Dividing this equation by the speed of light c ($\approx 3 \times 10^8$ m sec^{-1}), we have

$$h\nu/c = m_e\upsilon_e^2/2c.$$

We know that $c/\nu = \lambda$, where λ is the photon wavelength. So, $h/\lambda = m_e\upsilon_e^2/2c$ or

$$\lambda = 2hc/m_e\upsilon_e^2.$$

Elementary quantum physics says that the wavelength $h/m_e\upsilon_e$ is the wavelength of the electron of the H-atom before ionization (or ejection), λ_i, then after a bit of algebra, we get

$$\lambda/\lambda_i = 2c/\upsilon_e \quad \ldots (2).$$

In the common non-relativistic case, when $c >> \upsilon_e$,

$$\lambda >> \lambda_i.$$

Physicists usually assume that the massive particles with $\upsilon/c \leq 0.1$ (or $c \geq 10\upsilon_e$) are "non-relativistic". Thus,

$$\lambda_i \leq 0.05\,\lambda \quad \ldots (3).$$

Suppose now that we are ionizing the H-atom with an ultraviolet light having a photon of the energy 25 eV ($\approx 4 \times 10^{-18}$ J), having a wavelength of about 50 nm. Employing eqn. (2), we find that for this wavelength and $\upsilon_e \approx 0.1c$

$$\lambda_i \approx 2.5 \text{ nm}.$$

Moreover, introducing into eqn. (2) the values for λ_i (≈ 2.5 nm) and for c ($\approx 3 \times 10^8$ m sec^{-1}), we estimate that

$$\upsilon_e \approx 3 \times 10^7 \text{ m sec}^{-1}.$$[1]

This speed is non-relativistic. Obviously, the serious problem is that the electron has a wavelength of about 2.5 nm, which is in the wavelength range of the extreme ultraviolet (EUV) radiation, bordering on soft X-rays, and it is moving with a non-relativistic speed of about 3×10^7 m sec^{-1}.

[1] In fact, $\lambda_i \leq 2.5$ nm and $\upsilon_e \leq 3 \times 10^7$ m sec^{-1}.