the general science journal

The First Order Differential Equation in Mathematics and Physics

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The most general first-order differential equation can be written as dy/dx = f(x,y) (or dy/dx = y'), where dx and dy represent infinitesimals of the variables x and y. This equation is widely applied across various science disciplines, ranging from physics to chemistry, engineering and biology. In this communication, we will consider its application in the simplest examples: the instantaneous speed (kinematics) and the first-order rate of radioactive decay (nuclear chemistry).

Elementary kinematics defines the instantaneous speed of a massive particle, υ, at a particular instant of time. Mathematically, it is defined as the first derivative of position with respect to time:

$$\upsilon = dx/dt \quad \ldots (1)$$

where dx mathematically represents an infinitesimally small change in position of this particle during an infinitesimally small change of time, dt. Since υ has an exactly defined value, dx and dt must also have exactly defined values. In other words, dx and dt can have extremely small values but not be infinitely small and undefined. Therefore, it is much more "consistent" to express the instantaneous speed with the equation scientifically:

$$\upsilon = \Delta x/\Delta t$$

where now Δx and Δt denote the corresponding extremely small displacements in position of the massive particle during the extremely small interval of time Δt, but keeping in mind that these are both of finite value.

Similar examples to the above can be found in physics, chemistry, engineering, biology, and other scientific fields, but these are beyond the scope of this communication.

Radioactive decay is a first-order reaction and it is expressed by the following:

$$d\mathcal{N}/dt = -\lambda\mathcal{N} \quad \ldots (1)$$

where $\mathcal{N}$ is the initial number of radioactive nuclei, λ is the decay constant and $d\mathcal{N}/dt$ is the decay rate. Regardless of the negative sign of its right-hand side, this equation can be written in the following form:

$$d\mathcal{N}/dt = K\lambda$$

where $K = -\lambda\mathcal{N}$ is constant since $\mathcal{N}$ can also be considered a constant. Considering the previous case, it is clear that $d\mathcal{N}$ and dt cannot be infinitely small but extremely small and finite.

Exponential (integrated) form of eqn. (1) is

$$\mathcal{N}_t = \mathcal{N}e^{-\lambda t}$$

where $\mathcal{N}_t$ is the number of radioactive nuclei at time t. The validity of this equation has been confirmed in all cases of the decays of radioactive substances, so it can be considered as a reliable approximation even though d$\mathcal{N}$ and dt are extremely small but not infinitely small and it can be treated as infinitely small for mathematical purposes.

The Heisenberg uncertainty principle applied to position and momentum is often expressed by the following relation:

$$dpdx \geq \hbar/2\pi$$

where $\hbar$ is reduced Planck's constant ($\approx 1.05 \times 10^{-34}$ J sec), and dp and dx represent the uncertainty in momentum and position. These uncertainties in a particular case have exact values, so the Heisenberg uncertainty principle cannot be expressed as a product of dp and dx since these are mathematically indefinite infinitesimals. Therefore, dp and dx could be extremely small or large but finite. So, it is scientifically correct to define the Heisenberg uncertainty principle by the relation:

$$\Delta p\Delta x \geq \hbar/2\pi$$

bearing in mind Δp and Δx are both of finite value.