

Squaring of circle and arbelos and the judgment of arbelos in choosing the real Pi value (Bhagavan

XY length can be obtained by applying **Altitude theorem**

$$XY = \sqrt{BX \times XT} = \sqrt{\left(\frac{14 - \sqrt{2}}{40}\right) \times \left(\frac{2 + \sqrt{2}}{40}\right)}$$

$$\frac{\sqrt{26 + 12\sqrt{2}}}{40} = XY$$

27. **Triangle BXY**

$$BX = \frac{14 - \sqrt{2}}{40}, \quad XY = \frac{\sqrt{26 + 12\sqrt{2}}}{40}$$

BY can be obtained by applying **Pythagorean Theorem**

$$BY = \sqrt{BX^2 + XY^2} = \sqrt{\left(\frac{14 - \sqrt{2}}{40}\right)^2 + \left(\frac{\sqrt{26 + 12\sqrt{2}}}{40}\right)^2} = \frac{\sqrt{14 - \sqrt{2}}}{10}$$

BY is the required side of the square whose area is equal to the area of the arbelos of **Archimedes**.

$$\text{Side} = \frac{\sqrt{14 - \sqrt{2}}}{10} = a$$

$$\text{Area of the square on BY} = a^2 = \left(\frac{\sqrt{14 - \sqrt{2}}}{10}\right)^2 = \frac{14 - \sqrt{2}}{100} \text{ of S.No. 16}$$

= Area of arbelos

Part-IV (The Judgment on the Real Pi value)

In this paper, the **correctness** of the area of the arbelos of **Archimedes** can be **confirmed**. How? Here are the following steps.

28. **New π value** $\frac{14 - \sqrt{2}}{4}$ gives area of the arbelos as $\frac{14 - \sqrt{2}}{100} = 0.12585786437$. Whereas the **official**

π value 3.14159265358... gives the area of the arbelos as $\frac{\pi d^2}{4} = 3.14159265358 \times d \times d \times \frac{1}{4}$

$$d = GH = \frac{2}{5} \text{ of S.No. 3}$$

$$3.14159265358 \times \frac{2}{5} \times \frac{2}{5} \times \frac{1}{4} = 0.12566370614$$

Thus, the following are the two different values for the **same area** of the arbelos.

Official π value gives = 0.12566370614

New π value gives = 0.12585786437

29. Diameter of the arbelos circle GH = d = $\frac{2}{5}$

$$\text{Square of the diameter} = d^2 = \frac{2}{5} \times \frac{2}{5} = \frac{4}{25}$$

$$\text{Reciprocal of the square of the diameter} = \frac{1}{d^2} = \frac{1}{\frac{4}{25}} = \frac{25}{4}$$

30. Area of arbelos, if **multiplied** with $\frac{25}{4}$ we get the area of the inscribed circle in the ABCD square

$$\text{Area of the circle} = \frac{\pi d^2}{4}$$

$$d = a = 1, \quad \pi = \frac{14 - \sqrt{2}}{4}$$

$$= \frac{14 - \sqrt{2}}{4} \times 1 \times 1 \times \frac{1}{4} = \frac{14 - \sqrt{2}}{16}$$

31. **Area of the arbelos \times reciprocal of the square of the arbelos circle's diameter = Area of the inscribed circle in ABCD square**

$$\frac{14 - \sqrt{2}}{100} \times \frac{25}{4} = \frac{14 - \sqrt{2}}{16}$$

S.No. 16 S.No.29 S.No. 30

32. Let us derive the following formula from the dimensions of square ABCD
ABCD square, AB = side = a = 1

AC = BD = diagonal = $\sqrt{2}a = \sqrt{2}$, Perimeter of of ABCD square = 4a

Perimeter of ABCD square

$$\text{Half of 7 times of AB side of square} - \frac{1}{4} \text{ th of diagonal}$$

$$= \frac{4a}{7a} = \frac{4}{7} = \frac{4}{7} = \frac{4}{7} = \frac{16}{14 - \sqrt{2}}$$

33. In this step, above 2 steps (S.No. 29 and 32) are brought in.

$$\text{Arbelos area} \times \frac{25}{4} \times \frac{16}{14 - \sqrt{2}} = \text{Area of the ABCD square, equal to 1.}$$

As there are **two** values representing for the **same area** of the arbelos, let us verify, with the both the values, which is ultimately the correct one.

Arbelos area of official π value 3.14159265358

$$0.12566370614 \times \frac{25}{4} \times \frac{16}{14 - \sqrt{2}} = 0.99845732137 \text{ and}$$

$$\text{Arbelos area of new } \pi \text{ value } \frac{14 - \sqrt{2}}{4}$$

$$\frac{14 - \sqrt{2}}{100} \times \frac{25}{4} \times \frac{16}{14 - \sqrt{2}} = 1$$

This process is done by understanding the actual and exact **interrelationship** among, 1. area of the ABCD square, 2. area of the inscribed circle in ABCD square and, 3. area of the arbelos of **Archimedes**.

34. For questions "**why**", "**what**" and "**how**" of each step, the known mathematical principles are **insufficient**, unfortunately.

So, as the exact area of ABCD square equal to 1 is obtained finally with new π value. The new π value equal to

$$\frac{14 - \sqrt{2}}{4} \text{ is confirmed as the real } \pi \text{ value. This is the Final Judgment of arbelos of } \mathbf{Archimedes}.$$

III. CONCLUSION

This study, proves, that squaring of a circle is **not** impossible, and no more an **unsolved geometrical problem**. The belief in its (squaring of circle) impossibility is due to choosing the wrong number 3.14159265358... as π

value. The new π value $\frac{14 - \sqrt{2}}{4}$ has done it. The arbelos of **Archimedes has also chosen the real π value** in association with the inscribed circle and the ABCD superscribed square.

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Durga Method of Squaring A Circle

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Abstract: Squaring of circle is an unsolved problem with the official π value 3.1415926... with the new π value $1/4 (14 - \sqrt{2})$ it is done in this paper.

Keywords: Exact Pi value = $1/4 (14 - \sqrt{2})$, Squaring of circle, Hippocrates squaring of lunes.

I. Introduction

Squaring a circle is defined as constructing a square having an area equal to that of a given circle. It is also called as quadrature of the circle.

This concept has been there from the days of **Rhind Papyrus** (1800 B.C) written by a scribe named **Ahmes**. **Hippocrates of Chios** (450 B.C) has squared lunes, full circle and semicircle along with lunes. He fore saw the algebraic nature of the π value. π value 3.1415926... has failed to find a place for it in the squaring of lunes. Though the World of Mathematics has accepted his squaring of lunes, they became silent for why 3.1415926... is a misfit in his constructions. Further, there is a false opinion that **Hippocrates** could not square a circle. However, **Hippocrates did** square a full circle and a semicircle **along** with a lune. In both the cases – squaring a lune, squaring a circle along with a lune – the new π value, $\frac{14 - \sqrt{2}}{4}$ has explained perfectly well the constructions of **Hippocrates**. Thus the propositions of **Hippocrates** which remained **theoretical** all these 2400 years, have become **practical constructions** with the discovery of $\frac{14 - \sqrt{2}}{4}$. It is clear therefore, we have

misunderstood Hippocrates because, we believed 3.1415926... as the value of π . I therefore **apologize to Hippocrates on behalf of mathematics community** for the past mistake done by us. And **to atone the academic sin committed by us, I bow my head and dedicate the explained parts** (for details: **Pi of the Circle**, last chapter: Latest work, Pages from 273 to 281) **to Hippocrates**, in www.rsreddy.webnode.com

James Gregory (1660) has said squaring of circle is impossible. His view has been confirmed by **C.L.F. Lindemann** (1882) based on **Euler's** formula $e^{i\pi} + 1 = 0$. **Von K. Weierstrass** (1815-1897) and **David Hilbert** (1893) have supported the proof of Lindemann by their proofs.

S. Ramanujan (1913) has squared a circle upto some decimals of 3.1415926... **Prof. Underwood Dudley** doesn't accept **Lindemann's** proof because this is based on numbers which are approximate in themselves.

Now, the exact π value is discovered. It is $\frac{1}{4}(14 - \sqrt{2})$. It is an algebraic number. The following is the procedure how to square a circle.

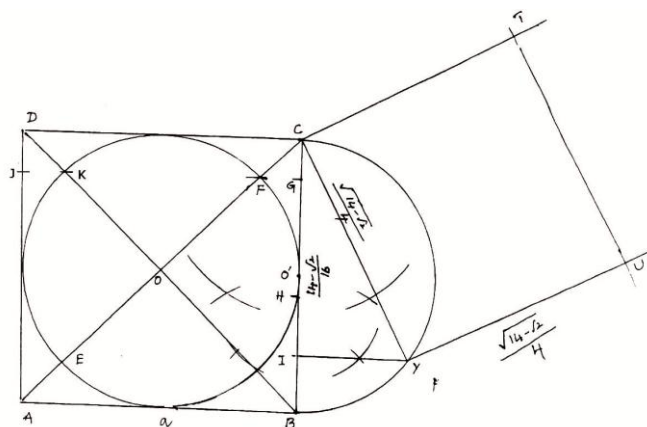
II. Procedure

We have to obtain a side of the square whose value is $\frac{1}{2}\sqrt{\pi}$; when $\pi = \frac{14 - \sqrt{2}}{4}$, then

$$\frac{1}{2}\sqrt{\pi} = \frac{1}{2}\sqrt{\frac{14 - \sqrt{2}}{4}} = \frac{\sqrt{14 - \sqrt{2}}}{4}$$

CD = a, OK = OF = radius = $\frac{a}{2}$, FK = $\frac{\sqrt{2}a}{2}$, JK = FG = GC,

$$GC = (JG - KF) \frac{1}{2} = \left(a - \frac{\sqrt{2}a}{2} \right) \frac{1}{2} =$$



$$\left(\frac{2-\sqrt{2}}{4}\right)a; GB = BC - GC = a - \left(\frac{2-\sqrt{2}}{4}\right)a = \left(\frac{2+\sqrt{2}}{4}\right)a;$$

$$\text{Bisect GB. } GH = HB = \left(\frac{2+\sqrt{2}}{8}\right)a. \text{ Bisect HB. } \left(\frac{2+\sqrt{2}}{16}\right)a = HI = BI$$

$$CI = BC - BI = a - \left(\frac{2+\sqrt{2}}{16}\right)a = \left(\frac{14-\sqrt{2}}{16}\right)a = \frac{\pi}{4}; \text{ Area of the circle} = \left(\frac{14-\sqrt{2}}{16}\right)a^2$$

CB = diameter = a;

Draw a semicircle on CB, with radius $\frac{a}{2}$ and center O'; $CO' = O'B = \frac{1}{2}$ where a = 1

Draw a perpendicular line on CB at I, which meets semicircle at Y. Apply altitude theorem to obtain IY length.

$$IY = \sqrt{CI \times IB} = \sqrt{\left(\frac{14-\sqrt{2}}{16}\right)\left(\frac{2+\sqrt{2}}{16}\right)} = \frac{\sqrt{26+12\sqrt{2}}}{16}$$

Connect YC which is the side of the square CYUT whose area is equal to that of the inscribed circle in the square ABCD.

Apply Pythagorean theorem to get CY from the triangle CIY.

$$\text{Side of the square CY} = \sqrt{CI^2 + IY^2} = \sqrt{\left(\frac{14-\sqrt{2}}{16}\right)^2 + \left(\frac{\sqrt{26+12\sqrt{2}}}{16}\right)^2} = \frac{\sqrt{14-\sqrt{2}}}{4}$$

$$\text{Area of the square CYUT} = \left(\frac{\sqrt{14-\sqrt{2}}}{4}\right)^2 = \frac{14-\sqrt{2}}{16} = \text{area of the inscribed circle in the square ABCD.}$$

III. Conclusion

$\frac{14-\sqrt{2}}{4}$ is the exact π value of circle. Hence, squaring of circle is done now. The misnomer "Circle squarer" will sink into oblivion. **Hippocrates** will now gets his deserving **throne of greatness** though delayed unfortunately for 2400 years.

Pythagorean way of Proof for the segmental areas of one square with that of rectangles of adjoining square

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Abstract: It is universally accepted that 3.14159265358... as the value of π . It is thought an approximation, at its last decimal place. It is a transcendental number and squaring a circle is an unsolved problem with this number. A new, exact, algebraic number $\frac{14-\sqrt{2}}{4} = 3.14644660942...$ is derived and verified with a proof that is followed for Pythagorean theorem. It is proved here squaring of circle and rectification of circumference of a circle are possible too.

Keywords: Pythagorean thorem, circle, square, π

I. Introduction

Official π value is 3.14159265358... It is obtained from the Exhaustion method, which is a geometrical method. This method involves the inscription of a polygon in a circle and increased the sides of the polygon, until the inscribed polygon touches the circle, leaving no gap between them. The value 3.14159265358... is actually the length of the perimeter of the inscribed polygon. And it is **not** the value of circle. There was no method till yesterday to measure the circumference of a circle, directly or indirectly.

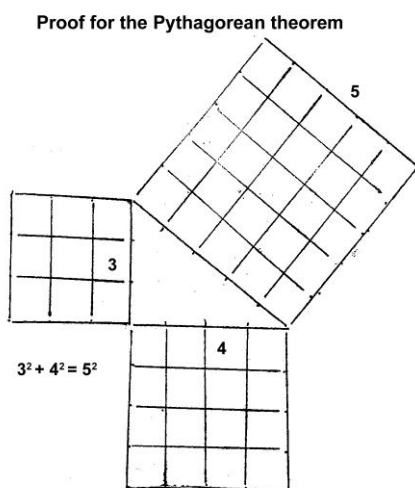
3.14159265358... has four characteristics: 1) It represents polygon, 2) It is an approximation, 3) It is a transcendental number and 4) It says squaring a circle is impossible. And such a number is attributed and followed as π of the circle based on limitation principle, because of the impossibility of calculating, the length of the circumference of circle and in such a situation this field of mathematics has been thriving for the last 2000 years.

From 1450 **Madhavan** of South India and a galaxy of later generations of mathematicians have discarded geometrical construction and have introduced newly, the concept of **infinite series**.

In this paper geometrical constructions are approached again, for the derivation of π value. New π value has been derived. It is $\frac{14-\sqrt{2}}{4} = 3.14644660942...$ It is an exact value, an algebraic number and makes squaring of circle possible and done too.

II. Procedure

Siva method for the area of the circle of 1st square ABCD



Thus constituent entities of two sides = constituent of entities of hypotenuse

Construction procedure

Draw a square ABCD. Draw two diagonals. 'O' is the centre. Inscribe a circle with centre 'O' and radius $\frac{1}{2}$. E, F, H and J are the mid points of four sides. Join EH, FJ, FH, HJ, JE and EF. Draw four arcs taking A, B, C and D as centres and radius $\frac{1}{2}$. Now the circle square nexus is divided into 32 segments. Number them 1 to 32. 1 to 16 segments are called S_1 segments. 17 to 32 segments are called S_2 segments. 17 to 24, S_2 segments are outside the circle. 25 to 32, S_2 segments are inside the circle.

Draw KP, a parallel line to the side DC which intersects diagonals at M and N.

Square = ABCD

Side = AB = 1 = EH = diameter

Areas of S_1 and S_2 segments

$$S_1 = \frac{6-\sqrt{2}}{128}; S_2 = \frac{2+\sqrt{2}}{128}$$

$16S_1 + 16S_2 =$ Area of square

$$16\left(\frac{6-\sqrt{2}}{128}\right) + 16\left(\frac{2+\sqrt{2}}{128}\right) = 1$$

$$16S_1 + 8S_2 = \text{Area of circle} \quad 16\left(\frac{6-\sqrt{2}}{128}\right) + 8\left(\frac{2+\sqrt{2}}{128}\right) = \frac{14-\sqrt{2}}{16}$$

Square has 32 constituent parts

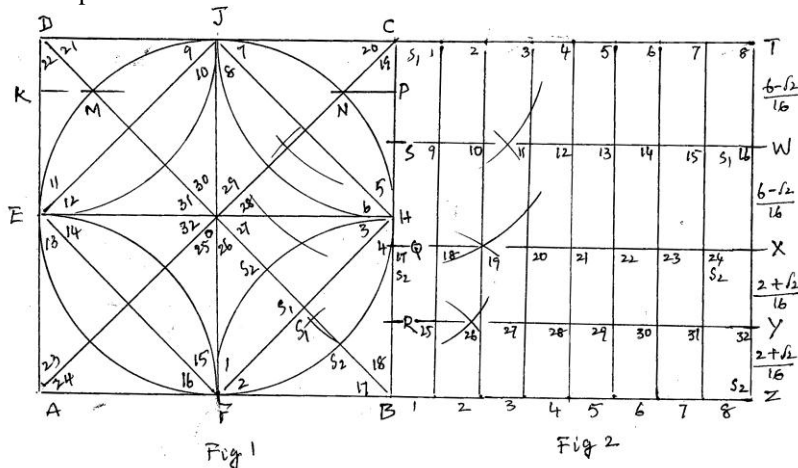


Fig-1: Segmental areas calculated; Fig-2: Areas of Rectangles are calculated Both values are same

This method is taken from the book **Pi of the Circle** of this author (available at www.rsreddy.webnode.com).

In this method there are two squares of same sides. First square has an inscribed circle divided into 32 segments of two dimensions called S_1 and S_2 segments, each category of 16 in number. And areas of these segments are calculated using the following two formulas

$$S_1 = \frac{a^2}{32}(\pi - 2) \quad \text{and} \quad S_2 = \frac{a^2}{32}(4 - \pi)$$

which are obtained by solving two equations (in Square 1)

$$16 S_1 + 16 S_2 = a^2 = \text{Area of the square (Eq.1)}$$

$$16 S_1 + 8 S_2 = \pi a^2/4 = \text{Area of the inscribed circle (Eq.2)}$$

This method is called as Siva method. In the present method: Siva Kesava method, second square is joined to the 1st square. One side CB is common to both the squares.

The second square is similarly divided, as in the case of 1st square, into 32 rectangles. Rectangles are also of two dimensions each category of 16 numbers. The areas of each type of rectangle is equal to S_1 and S_2 segments of the 1st square. **These rectangles are formed, based on the division of common side of the both the squares.** The areas of rectangles agree **cent percent** with the above two formulas of Siva method, where π

value is $\frac{14-\sqrt{2}}{4}$. Thus, the division of 1st square is **exactly duplicated** in the second square, except for the difference, in the 1st square, 32 segments are curvy linear, and in the 2nd square, 32 segments are rectangles, naturally, of straight lines.

Now let us see how the common side CB is divided.

1. Squares 1 = ABCD, 2 = BZTC
2. Side = diameter of the inscribed circle = 1
3. KP = Parallel side to the side DC
4. OM = ON = radius $\frac{1}{2}$
5. MON = triangle; MN = hypotenuse = $\frac{\sqrt{2}}{2}$
6. $DK = KM = NP = PC = \frac{KP - MN}{2} = \left(1 - \frac{\sqrt{2}}{2}\right)\frac{1}{2} = \frac{2-\sqrt{2}}{4}$
7. So, $CP = \frac{2-\sqrt{2}}{4}$, $PB = CB - CP = 1 - \left(\frac{2-\sqrt{2}}{4}\right) = \frac{2+\sqrt{2}}{4}$

Pythagorean way of Proof for the segmental areas of one square with that of rectangles of

8. Bisect PB. $PB \rightarrow PQ + QB \rightarrow QR + RB = \frac{2+\sqrt{2}}{4} \rightarrow \frac{2+\sqrt{2}}{8} \rightarrow \frac{2+\sqrt{2}}{16}$
9. $QB = \frac{2+\sqrt{2}}{8}$, $CB = 1$, $CQ = CB - QB = CQ = \left(1 - \frac{2+\sqrt{2}}{8}\right) = \frac{6-\sqrt{2}}{8}$
10. Bisect CQ $\rightarrow CS + SQ = \frac{6-\sqrt{2}}{8} \rightarrow \frac{6-\sqrt{2}}{16}$
11. We have started with side = 1, divided next, into $CP = \frac{2-\sqrt{2}}{4}$, and $PB = \frac{2+\sqrt{2}}{4}$. In the second step PB is bisected into $PQ = \frac{2+\sqrt{2}}{8}$ and $QB = \frac{2+\sqrt{2}}{8}$. In the third step $CQ = \frac{6-\sqrt{2}}{8}$ is bisected into $CS = \frac{6-\sqrt{2}}{16}$ and $SQ = \frac{6-\sqrt{2}}{16}$.
12. After divisions, finally we have CB Side divided into 4 parts
 $CS = \frac{6-\sqrt{2}}{16}$, $SQ = \frac{6-\sqrt{2}}{16}$, $QR = \frac{2+\sqrt{2}}{16}$ and $RB = \frac{2+\sqrt{2}}{16}$
13. 2nd Square BZTC is divided horizontally into four parts: CS, SQ, QR and RB.
14. Now BZ side of 2nd square is divided into 8 parts. So, each length is 1/8.
15. Finally, the 2nd square is divided into 16 rectangles of one dimension equal in area to S_1 segments of 1st square and 16 rectangles of 2nd dimension, equal in area to S_2 segments of 1st square.
16. Square BZTC consists of first two rows are of S_1 and 3rd & 4th rows are of S_2 segments.
17. Area of each rectangle = S_1 segments of 1st square = $\frac{6-\sqrt{2}}{16} \times \frac{1}{8} = \frac{6-\sqrt{2}}{128}$
- Sides of rectangles of 1st & 2nd rows = $TW = WX = \frac{6-\sqrt{2}}{16}$ and other side = $\frac{1}{8}$
18. Area of each rectangle = S_2 segment of 1st square = $\frac{2+\sqrt{2}}{16} \times \frac{1}{8} = \frac{2+\sqrt{2}}{128}$
- Sides of rectangles of 3rd & 4th rows = $XY = YZ = \frac{2+\sqrt{2}}{16}$ and other side = $\frac{1}{8}$
19. The areas of 16 S_1 and 16 S_2 segments of 1st square
 $16 \left\{ \frac{a^2}{32} \times (\pi - 2) \right\} + 16 \left\{ \frac{a^2}{32} \times (4 - \pi) \right\} = 1 = \text{Area of the 1st square; where } a = 1$
20. The area of all the 32 rectangles.
 $16 \left(\frac{6-\sqrt{2}}{128} \right) + 16 \left(\frac{2+\sqrt{2}}{128} \right) = 1 = \text{Area of the 2nd square}$
21. The area of the inscribed circle in the 1st square = 16 S_1 segments + 8 S_2 segments
 $= 16 \left(\frac{6-\sqrt{2}}{128} \right) + 8 \left(\frac{2+\sqrt{2}}{128} \right) = \frac{14-\sqrt{2}}{16} = \text{Area of the circle in the 1st square}$
22. The areas of the 1st, 2nd and 3rd rows of rectangles of 2nd square.
 $8 \left(\frac{6-\sqrt{2}}{128} \right) + 8 \left(\frac{6-\sqrt{2}}{128} \right) + 8 \left(\frac{2+\sqrt{2}}{128} \right) = \frac{14-\sqrt{2}}{16}$
23. Thus area of the circle from 1st and 2nd squares is = $\frac{14-\sqrt{2}}{16} = \frac{\pi d^2}{4}$
- $\therefore \pi = \frac{14-\sqrt{2}}{4}$ where side = diameter = $d = 1$

24. When π is equal to $\frac{14-\sqrt{2}}{4}$, the length of the inscribed circle in the 1st square is $= \pi d = \pi a = \frac{14-\sqrt{2}}{4} \times 1 = \frac{14-\sqrt{2}}{4}$ where side = diameter = 1

25. Perimeter of the rectangle QXWTCS is equal to the circumference of the inscribed circle in the 1st square.

$$QX = 1 = TC; XW = WT = CS = SQ = \frac{6-\sqrt{2}}{16}$$

$$QX + XW + WT + TC + CS + SQ = 1 + \frac{6-\sqrt{2}}{16} + \frac{6-\sqrt{2}}{16} + 1 + \frac{6-\sqrt{2}}{16} + \frac{6-\sqrt{2}}{16} = \frac{14-\sqrt{2}}{4}$$

In the first square we have seen that the length of the circumference of the inscribed circle is the **outer edge** of the 16 S_1 segments. In the 2nd square also the outer edges of the 1st and 2nd rows of 16 rectangles are equal to $\frac{14-\sqrt{2}}{4}$.

26. Thus, Siva Kesava Method supports the π value $\frac{14-\sqrt{2}}{4}$ obtained by earlier Gayatri, Siva, Jesus methods.

27. And also, the curvy linear 16 S_1 and 16 S_2 segments of 1st square are all squared in the 2nd square.

III. Conclusion

Two squares of same sides are drawn with one common side. Circle is inscribed in one square. Areas of square and its inscribed circle are calculated from their constituent curvy linear segments. The correctness of areas of constituent segments are verified with that of the areas of rectangles of the adjoining square. All the values thus are proved correct.

**SQUARING OF CIRCLE, SQUARING OF SEMI CIRCLE AND SEMI
CIRCLE EQUATED TO RECTANGLE AND TRIANGLE
(Euclid's Knowledge of Algebraic Nature of Circle and its Pi)**

Introduction

Till 1450 of Madhavan of Kerala, India, π number had its base in the **Exhaustion method of Archimedes**. With Madhavan and **independently**, John Wallis (1660) of England and James Gregory (1660) of Scotland, π number has become a **special number**, dissociated from geometrical constructions almost. Radius or diameter or circumference has become nothing in its relation with π constant then onwards. From here, π constant has travelled choosing a new path of its further journey.

Even in the Exhaustion method, the derivation of π value 3.1415926358... is only **partial**. Here, the perimeter of the polygon is divided by the diameter of the circle. In other words, 3.1415926358 is not a pure one. It is a **hybrid value**. Its outcome is based on the perimeter of the polygon and the diameter of the circle.

$$\frac{\text{Perimeter of the polygon}}{\text{Diameter of the circle}}$$

From 1450 onwards, even this partial association with polygon has been slowly cut off permanently. Ultimately, 3.1415926358 has gained **wings of infinite series** and has grown to the magnitude of **trillions of its decimals**, sitting on the shoulders of **Super Computers**.

Till 1882 of C.L.F. Lindemann, the nature of 3.1415926358 or π constant was not definite i.e., is π an algebraic number or a transcendental number ?

When the situation was like this

“In 1873, Charles Hermite (1822-1901) proved that the number e is transcendental; from this it follows that the finite equation

$$a e^r + b e^s + c e^t + \dots = 0 \quad (6)$$

cannot be satisfied if r, s, t, \dots are natural numbers and a, b, c, \dots are rational numbers not all equal to zero.

In 1882, F. Lindemann finally succeeded in extending Hermite’s theorem to the case when r, s, t, \dots and a, b, c, \dots are algebraic numbers, not necessarily real. Lindemann’s theorem can therefore be stated as follows:

If r, s, t, \dots, z , are distinct real or complex algebraic numbers, and a, b, c, \dots, n are real or complex algebraic numbers, at least one of which differs from zero, then the finite sum

$$a e^r + b e^s + c e^t + \dots + n e^z \quad (7)$$

cannot equal zero.

From this the transcendence of π follows quickly. Using Euler’s Theorem in the form

$$e^{i\pi} + 1 = 0, \quad (8)$$

*we have an expression of the form (7) with $a = b = 1$ algebraic, and c and all further coefficients equal to zero; $s = 0$ is algebraic, leaving $r = i\pi$ as the only cause why (8) should vanish. Thus, $i\pi$ must be transcendental, and since i is algebraic, π must be transcendental.” (Page No.172) Petr Beckmann
“A History of π ”*

What is the base for Euler’s formula $e^{i\pi} + 1 = 0$. Let us see the following Sir,

“But if Euler finished off one chapter in the history of π , he also started another. What kind of number was π ? Rational or irrational? With each new decimal digit the hope that it might be rational faded, for no period could be found in the digits. There was no proof as yet, but most investigators sensed that it was irrational. However, Euler asked a new question: Could π be the root of an algebraic equation of finite degree with rational coefficients? By merely asking the question, Euler opened a new chapter in the history of π , and a very important one, as we shall see. He was also the one who started writing it, for later investigations were based on one of Euler’s greatest discoveries, the connection between exponential and trigonometric functions,

$$e^{ix} = \cos x + i \sin x \quad (16)$$

*Euler discovered a long, long list of theorems. They are known as “Euler’s theorem on...” and “Euler’s theorem of ...” But this one is simply known as Euler’s Theorem.” (Page No. 156 of **A History of π**)*

So, it was settled in 1882 that π **constant** was a transcendental number. Let us observe here Sir, in Euler’s Theorem,

$$e^{i\pi} + 1 = 0$$

π refers to π radians 180° . π constant 3.14... has no right of its participation in theorem. When it participates the theorem itself becomes wrong.

We know Archimedes, Isaac Newton are the two greatest mathematicians. To the place of third position, there is a doubt. Who? Is Carl Friedrich Gauss (1777-1855) or Leonhard Euler (1707-1783)? So, the theorem is from such a greatest mathematician. He can’t be wrong.

Here, now comes a question, how did C.L.F. Lindemann chose Euler’s theorem to prove that π **constant** was a transcendental number?

Because, π constant has no place in the Theorem. But it was called transcendental, showing Euler's theorem.

Are we to accept that

$$\text{Pi radians } 180^\circ = \text{Pi constant } 3.14 ?$$

Does Mathematics accept this ?

There is no iota of doubt if Lindemann calls that polygon number 3.14159265358... is transcendental. He is cent percent right. But he becomes wrong if he calls π constant as transcendental based on Euler's Theorem.

To sum up thus, π constant was attached to polygon based on Exhaustion principle of Eudoxus is the beginning. Then it left the polygon in 1450 with Madavan of India. C.L.F. Lindemann wrongly interpreted Euler's Theorem and called π constant as a transcendental number.

This paper goes back turning the **wheel of Time** and even before Egyptian mathematics, and starts returning in a forward journey and stops at Hippocrates of Chios (450 B.C.), gets the support of squaring of lunes, squaring of semicircle and squaring of full circle and submits itself humbly before the **World of Mathematics** now, that the real π value is $\frac{14-\sqrt{2}}{4} = 3.14644660941\dots$ and is an algebraic number, and it merges with triangle, rectangle, square etc. **revealing**, that circle - square - triangle - rectangle stands as **one whole geometrical entity**.

Sir, **In Figure 5** we see a semi circle on PT converting a rectangle CJQP into a square of its side as QU. How ? Is it possible, if circle were

to be a transcendental entity, to convert rational entities into one form to another form ? **Think over Sir, please !**

Procedure:

Draw a square ABCD. Side = AB = 1 Inscribe a circle with centre 'O' and radius $1/2$. So, Diameter = side = 1

This combined geometrical construction consists of

1. Square
2. Inscribed circle
3. Left side semicircle
4. Triangle
5. Rectangle
6. Right side semicircle (shaded area)
7. Larger semicircle
8. DL Line-segment as the side of the square whose area is equal to the area of the inscribed circle in the ABCD square and
9. QU line-segment as the side of the square whose area is equal to the area of the shaded semi circle.

Let us study one by one

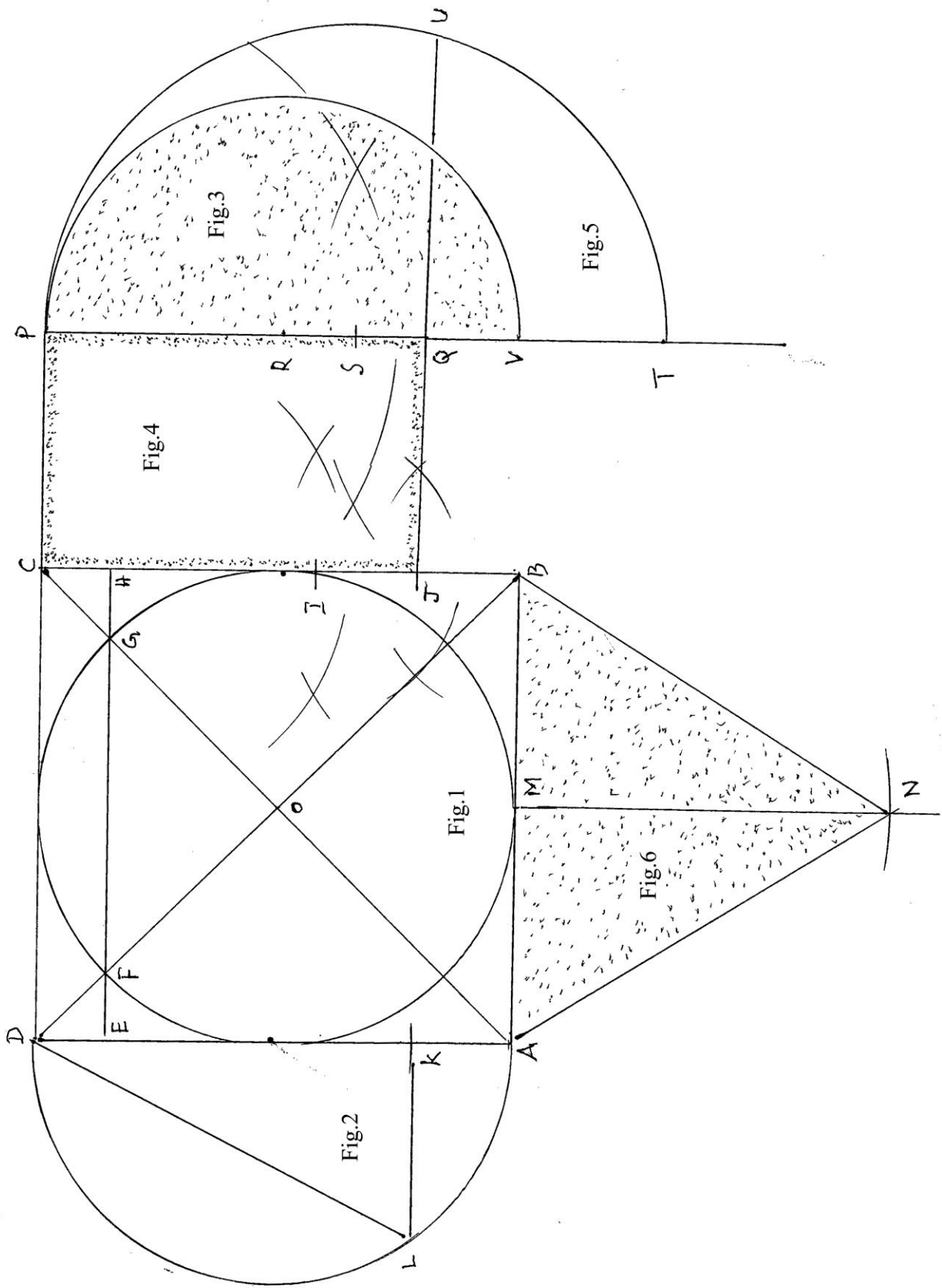
I. Figure 1

10. To obtain a length equal to $\frac{\pi}{4}$, where π is equal to $\frac{14-\sqrt{2}}{4} =$

3.14644660941... the following steps are followed:

$$\frac{\pi}{4} = \text{Quarter circumference of the inscribed circle} = \frac{14-\sqrt{2}}{16}$$

11. EH = side = 1
12. OF = OG = radius = $\frac{1}{2}$



13. FOG = triangle

$$FG = \text{Hypotenuse} = OF \times \sqrt{2} = \frac{1}{2} \times \sqrt{2} = \frac{\sqrt{2}}{2}$$

$$14. \quad EF = GH = HC = \frac{\text{Side} - \text{hypotenuse}}{2} = \left(1 - \frac{\sqrt{2}}{2}\right) \frac{1}{2} = \frac{2 - \sqrt{2}}{4}$$

$$15. \quad HB = CB - CH = \text{Side} - CH = \left(1 - \frac{2 - \sqrt{2}}{4}\right) = \frac{2 + \sqrt{2}}{4}$$

16. Bisect HB **twice**

$$\begin{aligned} HB &\rightarrow HI + IB \rightarrow IB \rightarrow IJ + JB \\ &= \frac{2 + \sqrt{2}}{4} \rightarrow \frac{2 + \sqrt{2}}{8} \rightarrow \frac{2 + \sqrt{2}}{16} \end{aligned}$$

$$17. \quad CB - JB = 1 - \frac{2 + \sqrt{2}}{16} = \frac{14 - \sqrt{2}}{16} = \frac{\pi}{4}$$

$$18. \quad \frac{\pi}{4} = \frac{14 - \sqrt{2}}{16} = CJ$$

II Figure 2

19. AB = Diameter. Draw a semi circle with radius $\frac{1}{2}$.

$$\frac{\pi}{4} = CJ \text{ of Fig.1} = DK \text{ of Fig.2}$$

$$20. \quad AK = \frac{2 + \sqrt{2}}{16}, \quad DK = \frac{14 - \sqrt{2}}{16}$$

Apply altitude theorem to obtain KL

$$KL = \sqrt{AK \times DK} = \sqrt{\frac{2 + \sqrt{2}}{16} \times \frac{14 - \sqrt{2}}{16}}$$

$$KL = \frac{\sqrt{26 + 12\sqrt{2}}}{16}$$

21. DL is the side of the square whose area is equal to the area of the inscribed circle in the ABCD square.

$$22. \quad DK = \frac{14 - \sqrt{2}}{16}, \quad KL = \frac{\sqrt{26 + 12\sqrt{2}}}{16}$$

23. Apply Pythagorean theorem to obtain the length of DL

$$\begin{aligned}\sqrt{DK^2 + KL^2} &= \sqrt{\left(\frac{14 - \sqrt{2}}{16}\right)^2 + \left(\frac{\sqrt{26 + 12\sqrt{2}}}{16}\right)^2} \\ &= \frac{\sqrt{14 - \sqrt{2}}}{4}\end{aligned}$$

Which is the side of the square whose area is equal to the area of the inscribed circle, which is equal to $\frac{1}{2}\sqrt{\pi}$. Where $\pi = \frac{14 - \sqrt{2}}{4}$

III Figure 3 : Semicircle (Shaded area)

24. PV = Diameter = 1

R = centre. Draw a semi circle

With centre R and radius $\frac{1}{2}$.

$$\text{Circle Area} = \frac{\pi d^2}{4}. \quad \text{Semi circle} = \frac{\pi d^2}{8}$$

$$\text{Where } d = 1, \text{ then } \frac{\pi}{8} = \left(\frac{14 - \sqrt{2}}{4}\right) \frac{1}{8} = \frac{14 - \sqrt{2}}{32}$$

IV Figure 4 : Rectangle CJQP

25. $CJ = \frac{\pi}{4} = \frac{14 - \sqrt{2}}{16}$

$$CP = JQ = \frac{1}{2} = \text{radius}$$

Area of the rectangle = CJ × CP

$$= \frac{14 - \sqrt{2}}{16} \times \frac{1}{2} = \frac{14 - \sqrt{2}}{32}$$

26. So, Semicircle on PV = Rectangle CJQP

(S. No. 24)

$$\frac{14 - \sqrt{2}}{32} = \frac{14 - \sqrt{2}}{32}$$

V Figure 5: Squaring a rectangle

27. Rectangle CJQP = $\frac{14-\sqrt{2}}{32}$
28. Diameter PT = $\frac{\pi}{4} + \text{radius} = \frac{14-\sqrt{2}}{16} + \frac{1}{2} = \frac{22-\sqrt{2}}{16}$
29. S is the centre of diameter PT.
Draw a semi circle on PT.
30. Draw a perpendicular line at Q on PT which meets circumference at U. To obtain QU apply altitude theorem.

$$QJ = \frac{1}{2} = QT = \frac{1}{2}, \quad PQ = \frac{14-\sqrt{2}}{16}$$

$$\sqrt{PQ \times QT} = \sqrt{\frac{14-\sqrt{2}}{16} \times \frac{1}{2}}$$

It is a well established fact that a rectangle can be squared when $QT = QJ$

Euclid III. 35 (Book III, Theorem 35)

31. So, it implies, that Euclid must have been aware of the algebraic nature of circle and its π , necessarily.

VI Figure 6 : Semicircle = Triangle

32. AB = Base of the triangle ABN = 1

M = Mid point of AB

$$MN = \frac{\pi}{4} = CJ = \frac{14-\sqrt{2}}{16} = \text{altitude}$$

Area of the triangle ABN

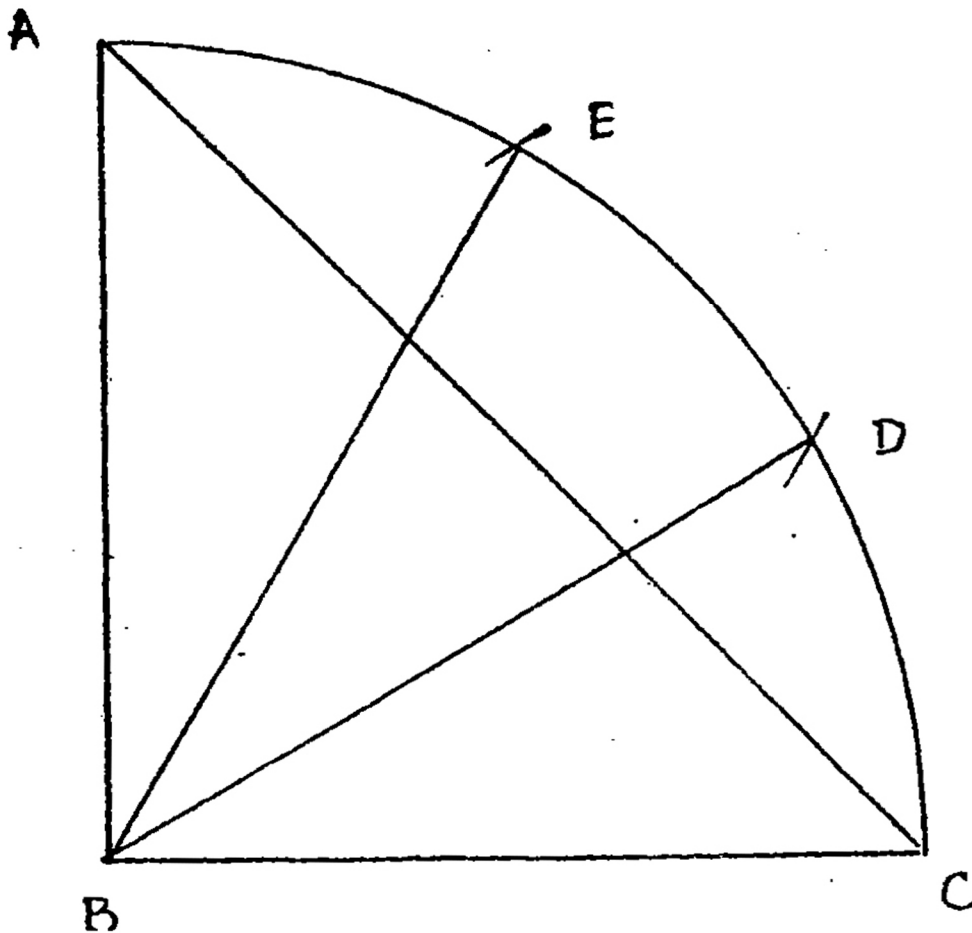
$$\begin{aligned} \frac{1}{2} ab &= \frac{1}{2} \times MN \times AB = \frac{1}{2} \times \frac{14-\sqrt{2}}{16} \times 1 \\ &= \frac{14-\sqrt{2}}{32} \end{aligned}$$

So, area of the semi circle (shaded area) on PV is equal to that of ABN triangle.

APPENDIX

Trisection of an Angle of 90°

A right angle ABC is drawn in which AB and BC are the adjacent sides and AC is the hypotenuse. With the help of a compass, taking B as the centre and arc connecting points A and C is drawn. With the same radius the above arc is intersected by two smaller arcual lengths one from A and the second from C. These two intersecting points on the arc are D and E. when D and E are joined to B we get the angle ABC trisected into $\angle ABE = \angle EBD = \angle DBC = 30^\circ$ each.



24. DUPLICATION OF THE CUBE

A Geometric Construction

A new formula for the arrival of the required side of the duplicated cube is proposed and is $\sqrt{\left(\frac{31+14\sqrt{2}}{32}\right)a^2}$ where a is the given side of a cube.

METHOD:

A square ABCD of a given side is drawn. A circle is inscribed with it. Two diagonals AC and BD are drawn. The diagonals intersect the circle at E, F, G, and H. When four parallel lines through the points E, F, G & H to the four sides are drawn, we get four smaller squares for example, one square is KHJD. One parallel line through EH is IJ which is equal to AD. The sum of the lengths of IE, EH and HJ is equal to the side AD. HM is half the length of HJ.

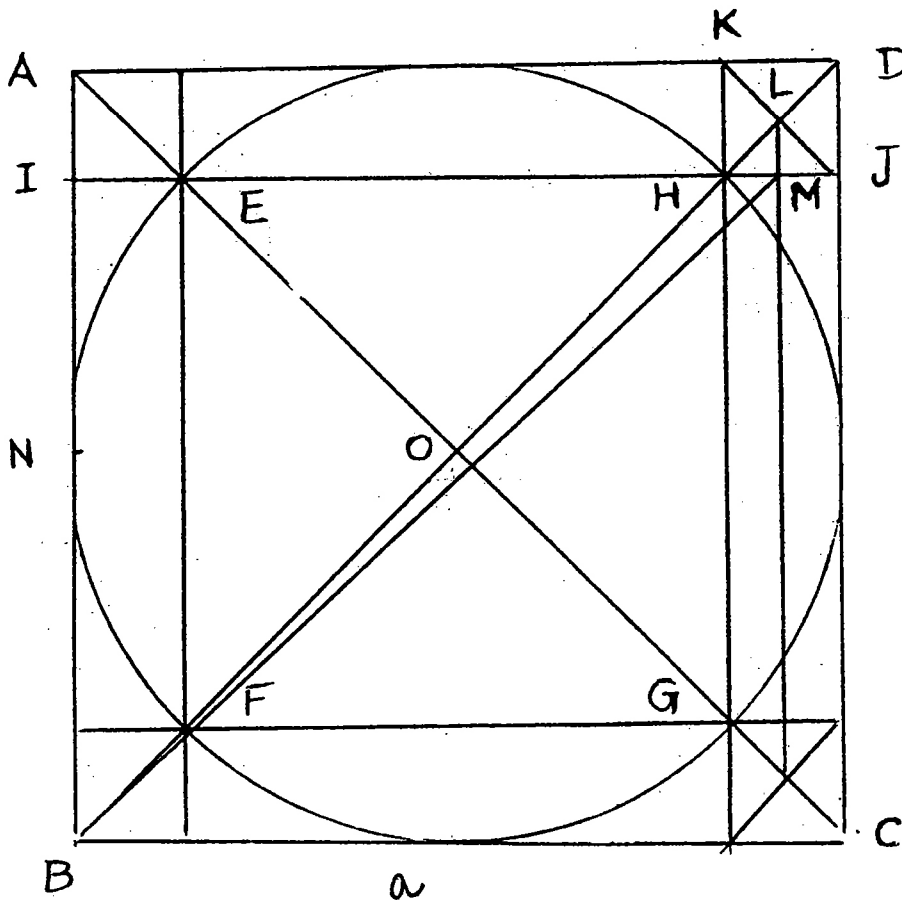


Fig - 43